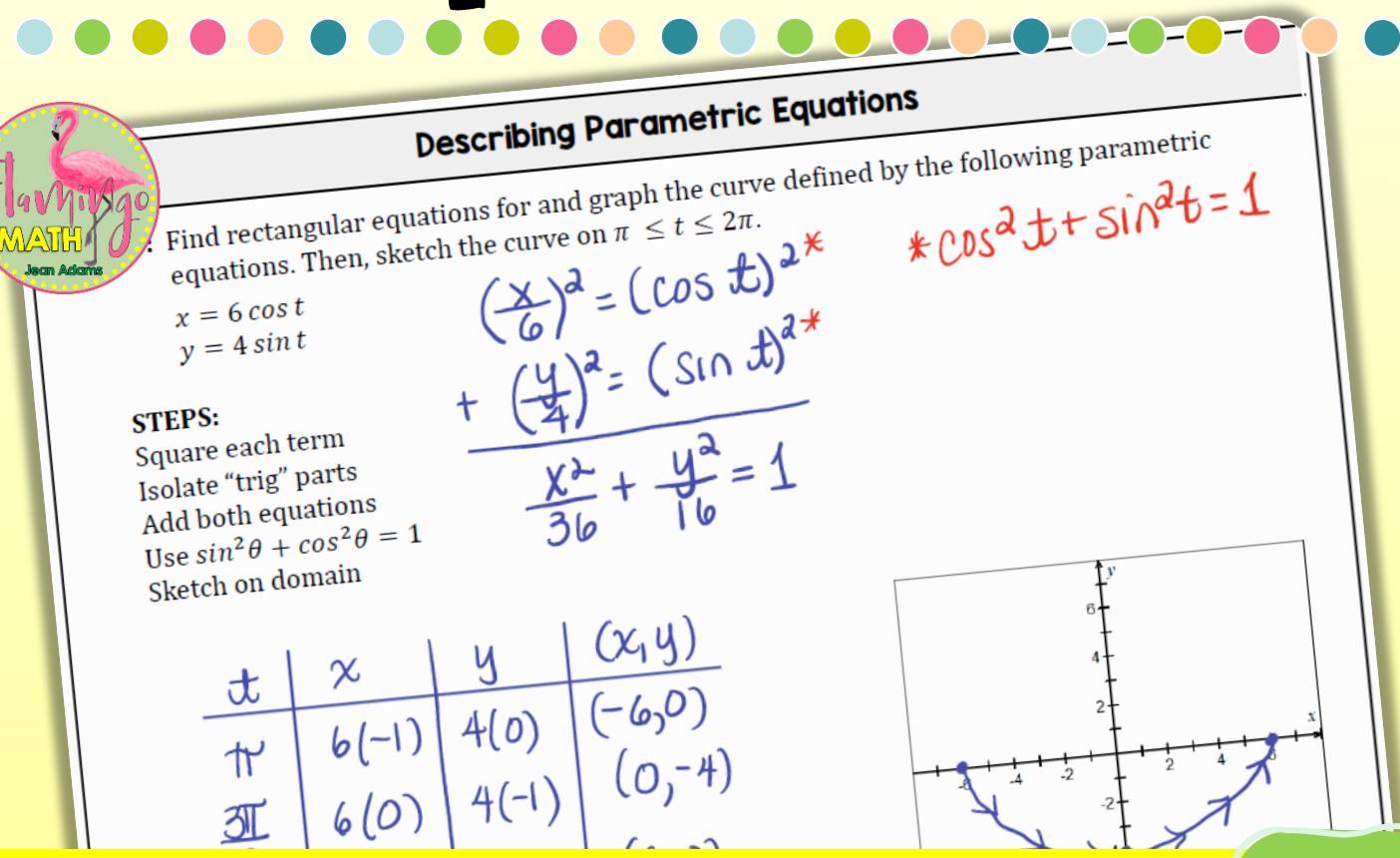
Precalculus Honors

Parametric Equations



Describing Parametric Equations

Find rectangular equations for and graph the curve defined by the following parametric equations. Then, sketch the curve on $\pi \leq t \leq 2\pi$.

$x = 6\cos t$
$y = 4\sin t$

$\left(\frac{x}{6}\right)^2 = (\cos t)^2$ *
$+ \left(\frac{y}{4}\right)^2 = (\sin t)^2$ *

$\frac{x^2}{36} + \frac{y^2}{16} = 1$

* $\cos^2 t + \sin^2 t = 1$

STEPS:
Square each term
Isolate "trig" parts
Add both equations
Use $\sin^2\theta + \cos^2\theta = 1$
Sketch on domain

t	x	y	(x, y)
π	$6(-1)$	$4(0)$	$(-6, 0)$
$\frac{3\pi}{2}$	$6(0)$	$4(-1)$	$(0, -4)$

Completely Editable 8.5

Lesson 8.5: Parametric Equations

In the 1960's ***Home Run Derby*** was a favorite tv game show for many Americans. Baseball stars like Willie Mays and Mickey Mantle would compete in a head-to-head home-run-hitting contest at Wrigley Field in LA. In just a matter of seconds, a baseball would travel a horizontal distance of more than 500 feet. In this section, we will explore ways to describe motion along a curve. We will use parametric equations and a third variable for time, that can reveal both the horizontal and vertical distance at any time for that home-run baseball.

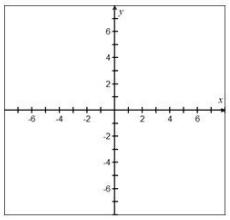
Definition of a Plane Curve

Let $x = f(t)$ and $y = g(t)$, where f and g are two functions whose common domain is some interval I. The collection of points defined by $(x, y) = (f(t), g(t))$ is called a ***plane curve***. The equations $x = f(t)$ and $y = g(t)$ where t is in I, are called ***parametric equations*** of the curve.
The variable t is called a ***parameter***.

EX #1: Graph the following for given domain: $0 \le t \le 3$

$$\begin{cases} x = t - 1 \\ y = t^2 - 2 \end{cases}$$

t	x	y	(x, y)
0			
1			
2			
3			



Steps to Graphing Parametric Curves

1. Select values for t over the given interval.
2. Find the ordered pair corresponding to each value of t.
3. Plot the points.
4. Connect the points with a smooth curve.
5. Draw arrows to show the direction, or ***orientation***, along the curve as *time*, t increases.

Finding the Rectangular Equation of a Parametric Curve

EX #2: Eliminate the parameter: $\begin{cases} x = t - 1 \\ y = t^2 - 2 \end{cases}$

STEP 1: Solve for one variable in terms of t.

STEP 2: Substitute into the other equation

STEP 3: Simplify.

Analyze the function over the real numbers:

Domain:

Range:

Max/Min:

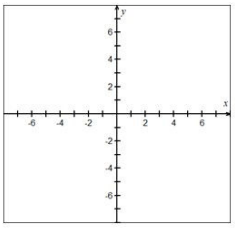
Increases:

Decreases:

EX #3: Graph the plane curve on $-2 \le t \le 2$

$$\begin{cases} x = t^2 + 1 \\ y = 3t \end{cases}$$

t	x	y	(x, y)
-2			
-1			
0			
1			
2			



Eliminate the Parameter

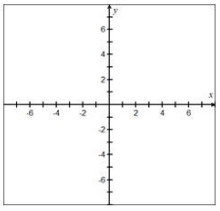
EX #4: Eliminate the parameter, then sketch the curve.

$$x = \sqrt{t}$$
$$y = 2t - 1$$

DOMAIN:

Substitute, solve for y.

Vertex:

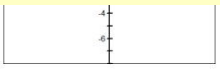


Describing Parametric Equations

EX #5: Find rectangular equations for and graph the curve defined by the following parametric equations. Then, sketch the curve on $\pi \le t \le 2\pi$.

$$x = 6\cos t$$
$$y = 4\sin t$$

STEPS:
Square each term
Isolate "trig" parts
Add both equations
Use $\sin^2\theta + \cos^2\theta = 1$
Sketch on domain



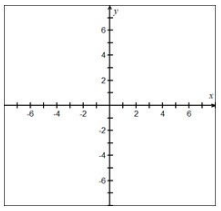
Finding Parametric Equations

EX #6: Find a parametric equation from the given conditions.

GIVEN: Parabola, Vertex $(2, -3)$; opens downward
KNOW: $y = a(x - h)^2 + k$
STEP 1: Substitute given information:

STEP 2: Add time as a third variable:

STEP 3: Equations:



EX #7: Find parametric equations for the ellipse $x^2 + \frac{y^2}{16} = 1$ where the parameter t is time (in seconds) and the motion around the ellipse is counter- clockwise, begins at the point (1,0) and requires 2 seconds to complete one revolution.

NOTES

8.5 **Parametric Equations Homework**

Name _____

Date _____ Period _____

Problems 1 – 6, eliminate the parameter and identify the graph of the parametric curve.

1. $x = 2t - 3, y = 10 - 4t, -2 \le t \le 5$

2. $x = t, y = t^2 - 3$

3. $x = 2t, y = t^3 - 2t + 1$

4. $x = 2t^2 - 1, y = t$

5. $x = 5\cos t, y = 5\sin t$

6. $x = 3\sin t, y = 2\cos t$

Problems 7-10, find two different parametric equations for each rectangular equation.

7. $y = 3x - 2$

8. $y = x^2 + 4$

9. $y = \sqrt{x - 2}$

10. $y = x^3 - x$

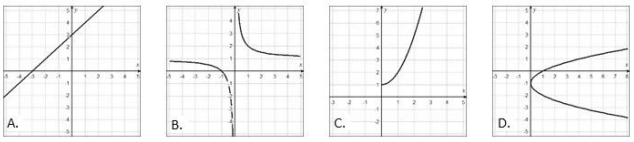
Problems 11 – 14, match the equations to the graph.

11. $x = t^2, y = t - 1$

12. $x = \sqrt{t}, y = t + 1$

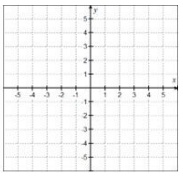
13. $x = t, y = t + 3$

14. $x = \frac{1}{t}, y = t + 1$

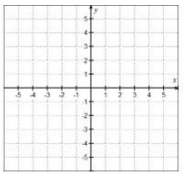


Problems 15 – 18, sketch the curve represented by the parametric equations. Indicate the orientation of the curve.

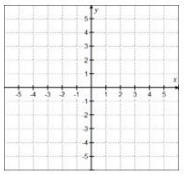
15. $x = 3t - 3; y = 2t + 1$



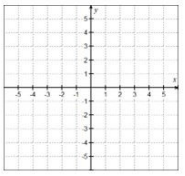
16. $x = t + 2; y = t^2$



17. $x = \frac{t}{2}; y = |t - 4|$



18. $x = t; y = (t + 2)^3$



Problems 19 – 21, find parametric equations for an object that moves along the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ with the motion described.

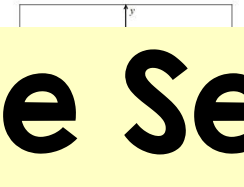
19. The motion begins at $(3, 0)$, is clockwise, and requires 2 seconds for a complete revolution.

20. The motion begins at $(0, 2)$, is counterclockwise, and requires 1 second for a complete revolution.

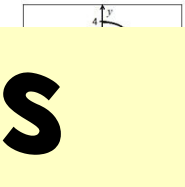
21. The motion begins at $(3, 0)$, is counterclockwise, and requires 3 seconds for a complete revolution.

Problems 22-23, Find parametric equations that define the curve shown.

22.



23.

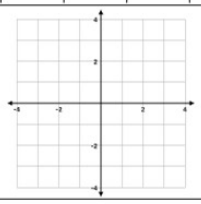


Practice Sets

Lesson Warm-Ups

x					
y					

3. Plot the points from the table you completed in #2.



Name _____ Date _____ Period _____

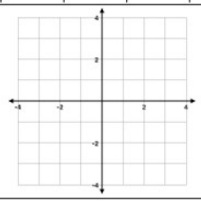
Lesson 8.5: Parametric Equations **Warm-Up # 2**

1. Eliminate the parameter t to find the resulting rectangular equation. $x = t - 4$ and $y = t^2 + 2$

2. Complete the table of values for the domain $-2 \le t \le 2$

t	-2	-1	0	1	2
x					
y					

3. Plot the points from the table you completed in #2.



Form A

_____

_____iod _____

nd the

2. Complete the table of values for the domain: $-2 \le t \le 2$

$$\begin{cases} x = t^2 - 1 \\ y = 2t \end{cases}$$

t	x	y	(x, y)
-2			
-1			
0			
1			
2			

3. Plot the points from the table above.



Grade:

8.5 Parametric Equations **Form B**

Name _____

Date _____ Period _____

1. Eliminate the parameter t to find the resulting rectangular equation.
$x = t - 3$ and $y = t^2 + 1$

2. Complete the table of values for the domain: $-2 \le t \le 2$

$$\begin{cases} x = t^2 - 2 \\ y = 2t \end{cases}$$

t	x	y	(x, y)
-2			
-1			
0			
1			
2			

3. Plot the points from the table above.



Grade:

Daily Lesson Quiz