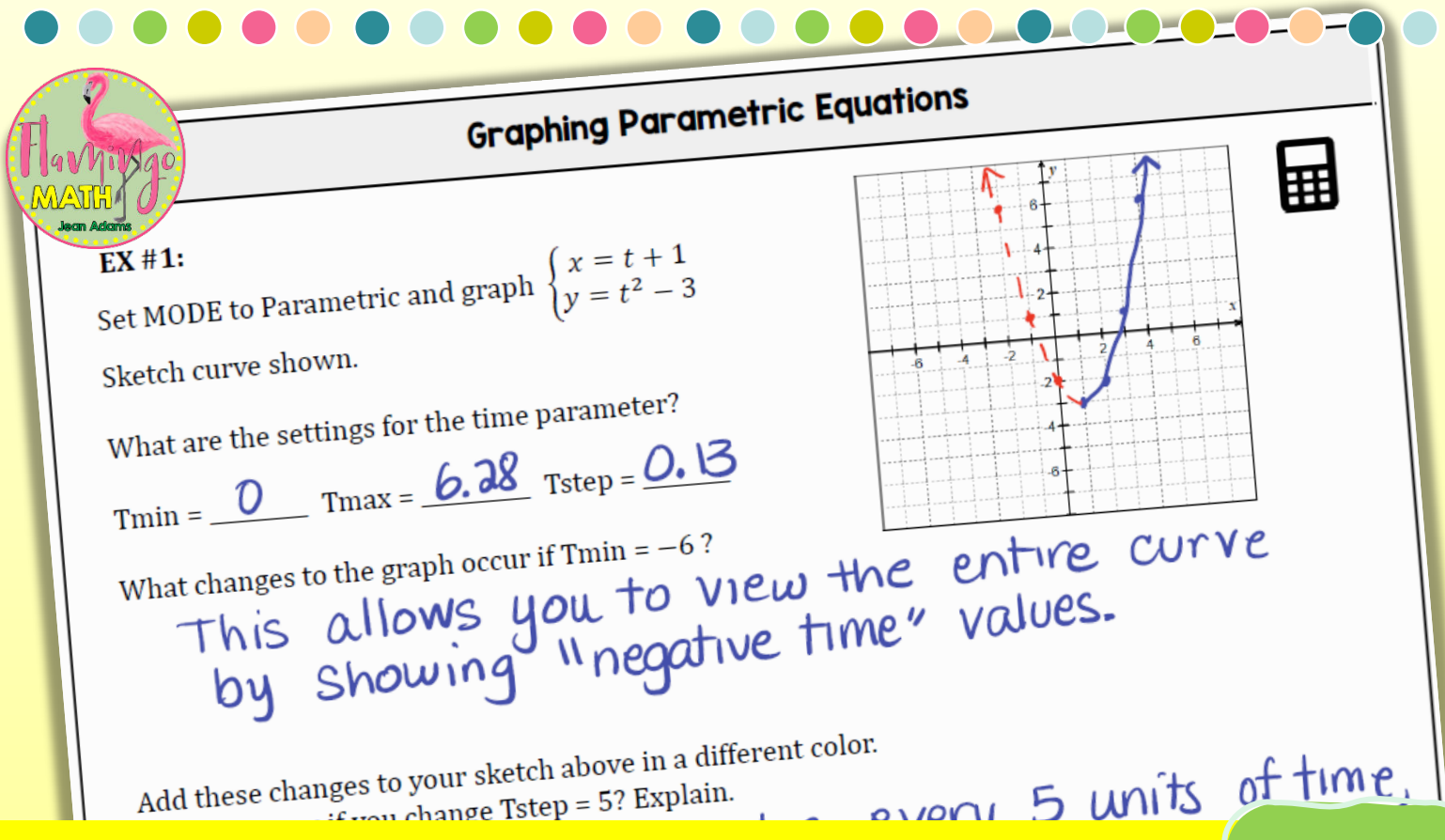
Precalculus Honors

Parametric Equation Applications



Completely Editable 8.6

Lesson 8.6: Parametric Applications

In algebra we explore how things are connected or related through similarities and differences. We have discussed functions and relations to find input and output values. However, there are some situations that need a different approach.

For example, let's consider when a hurricane is heading toward the southern United States. The closer the storm approaches the coast the more gas and bottled water is sold. Suppose for every 100 miles the storm moves that a convenience store sells x gallons of gas and x^2 bottles of water. Algebraically, that relationship would be written as follows:

$$bottles\ of\ water = (gallons\ of\ gas)^2$$

Even though this is correct, it is misleading. The equation implies that gas is changing the demand for bottled water, when really it is the proximity of the storm, in miles, to a region that changed both items. It would make better sense to write two separate equations where we considered either miles from shore or time as the changing parameter. We could write two separate equations to model the situation.

$$Gallons\ of\ gas = miles - 100$$
$$Bottled\ water = (miles - 100)^2$$

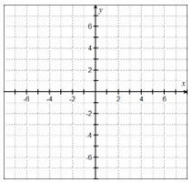
In this way you can see that miles from shore is the cause for changes in the sale of gas and water due to the impending storm. Parametric equations help us look deeper at hidden parameters that affect change.

Graphing Parametric Equations

EX #1:

Set MODE to Parametric and graph $\begin{cases} x = t + 1 \\ y = t^2 - 3 \end{cases}$

Sketch curve shown.



What are the settings for the time parameter?

Tmin = _____ Tmax = _____ Tstep = _____

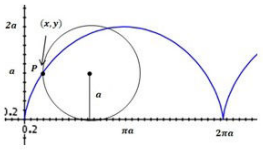
What changes to the graph occur if Tmin = −6 ?

Add these changes to your sketch above in a different color.
What happens if you change Tstep = 5? Explain.

The Cycloid

Consider a circle with radius a that rolls along a horizontal line. As the circle rolls along the line, a point P on the circle will trace out a curve called a **cycloid**. Cycloids are used to solve problems that involve the "shortest time." When the radius of the circle is a, the parametric equations of the cycloid are

$$x = a(t - sint\ t) \text{ and } y = a(1 - \cos t)$$



EX #2: Use a graphing utility in parametric mode to graph the plane curve represent by the given parametric equation. Use the viewing rectangle $[0,60,5] \times [0,8,1]$ with $0 \le t \le 6\pi$, and a tstep=0.13

$$x = 2(t - \sin t)$$
$$y = 2(1 - \cos t)$$

EX #3: A baseball is hit straight up from a height of 5 ft with an initial velocity of 80 ft/sec.

A. Write an equation that models the height of the ball as a function of time, t.

B. Use Parametric Mode to simulate the pop-up.

Settings: $0 \le t \le 5.1$	Equations:
Window: $[0, 7]$ by $[0, 120]$	

C. Use Parametric Mode to graph height against time.

D. How high is the ball after 4 sec?

D. What is the maximum height of the ball? How many seconds does it take to reach its maximum height?

Hitting a Baseball

EX #4: Sammy hits a ball when it is 4 ft above the ground with an initial velocity of 120 ft/sec. The ball leaves the bat at a 30° angle with the horizontal and heads toward a 30-ft fence 350 feet from home plate.

A. Does the ball clear the fence?



B. How do you know?

C. If so, by how much does it clear the fence? If not, could the ball be caught?

Hitting Golf Balls

EX #5: Denise hits golf balls off the tee with an initial velocity of 80 yds/sec with her driver. How far down the fairway does the ball hit the ground if it comes off the club making the specified angle with the horizontal?

A. 20°

B. 25°

NOTES

8.6 **Parametric Equation Applications Homework**

Name _____

Date _____ Period _____

Problems 1 – 7, solve.

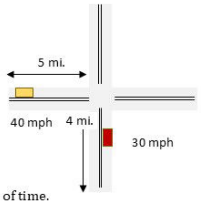
1. Hayden throws a ball straight up with an initial velocity of 50 feet per second from a height of 5 feet.

   A. Write parametric equations that describe the motion of the ball as a function of time.

   B. How long is the ball in the air?

   C. When is the ball at its maximum height? Find the maximum height.

2. Two elderly couples are out for a Sunday drive heading east at 40 mph and a student on a scooter is heading north at 30 mph toward the same intersection. See figure at right.



   A. Find parametric equations that represent the motion of the two vehicles.

   B. Find a formula for the distance between the cars as a function of time.

   C. What is the minimum distance between the two vehicles? When are they the closest?

3. Suppose you throw a softball off a cliff 900 feet high with an initial speed of 90 feet per second at an angle of 45° to the horizontal.

   A. Write parametric equations that describe the position of the ball as a function of time.

   B. How long is the ball in the air?

   C. When is the ball at its maximum height? Find the maximum height of the ball.

   D. Find the distance the ball traveled.

4. Jennifer shot an arrow from a long bow. The arrow leaves the bow at an angle of 15° with the horizontal and at an initial speed of 75 feet per second, from a height of 5 feet above the ground.

   A. Assuming the ground is level, find the distance the arrow travels before it hits the ground. (Ignore air resistance.)

   B. How long was the arrow in the air?

5. Casey is launching a potato from his potato cannon from a height of 3.5 feet above the ground. The initial speed is 140 feet per second at an angle of 25° with the horizontal.

   A. Write parametric equations that describe the motion of the potato.

   B. What is the maximum height the potato reaches?

   C. When does the potato hit the ground?

   D. Find the distance the potato traveled.

6. Brandon throws a 16-pound shot from a height of 5 feet with an initial speed of 40 feet per second at an angle of 45°. Assuming the ground is level, how far did Brandon's shot go?

Practice Sets

Lesson Warm-Ups

regard for air resistance) for position (x, y) at time t seconds are: $x(t) = 75t$
$y(t) = 85t - 16t^2$

Find the position of the ball at $t = 3$ sec.

3. At what two times is the ball 100 feet above the ground? Find x at the times?

4. A 25-ft. high fence is at $x = 400$ ft. According to this parametric function, will the ball go over the fence, hit the fence, or hit the ground before reaching the fence?

Name _____ Date _____ Period _____

Lesson 8.6: Parametric Equation Applications **Warm-Up # 2**

1. Suppose a ball is hit with a horizontal velocity of 70 ft/sec and an initial upward velocity of 90 ft/sec. The parametric equations (without regard for air resistance) for position (x, y) at time t seconds are: $x(t) = 70t$
$y(t) = 90t - 16t^2$

Find the position of the ball at $t = 3$ sec.

2. When is the ball at a horizontal distance of $x = 150$ ft? How high is the ball at that time?

3. At what two times is the ball 100 feet above the ground? Find x at the times?

4. A 25-ft. high fence is at $x = 400$ ft. According to this parametric function, will the ball go over the fence, hit the fence, or hit the ground before reaching the fence?

Form A

_____

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tal velocity of
city of 90
without regard
) at time t

1. Find the position of the ball at time t = 3 sec.

2. When is the ball at a horizontal distance of x = 200 ft.? How high is the ball at that time?

3. At what two times is the ball 100 feet above the ground? Find x at the times.

4. A 25-foot high fence is at $x = 400$ ft. According

Grade:

8.6 Parametric Applications **Form B**

Name _____

Date _____ Period _____

Suppose a ball is hit with a horizontal velocity of 65 ft/sec and an initial upward velocity of 85 ft/sec. The parametric equations (without regard for air resistance) for position (x, y) at time t seconds are
$x(t) = 65\ t$
$y(t) = 85t - 16t^2$

1. Find the position of the ball at time $t = 3$ sec.

2. When is the ball at a horizontal distance of $x = 84$ ft.? How high is the ball at that time?

3. At what two times is the ball 100 feet above the ground? Find x at the times.

4. A 25-foot high fence is at $x = 350$ ft. According to

Grade:

Daily Lesson Quiz