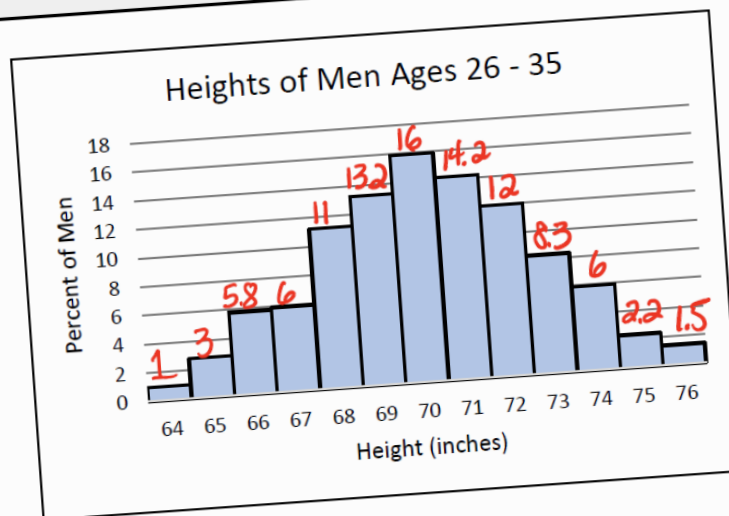


## Algebra 2 Honors

# Normal Distributions



### Estimating Probabilities Using a Normal Curve



EX #1: In a survey of American men, the heights are distributed normally about the mean, as shown in the graph above.

# Completely Editable

13.5

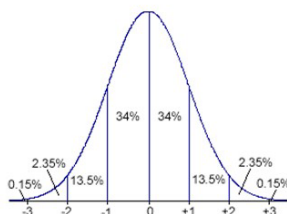
## Lesson 13.5: Normal Distributions

Normal curves are used in many situations to estimate probabilities. The pattern the data form is a bell-shaped curve called a normal curve.

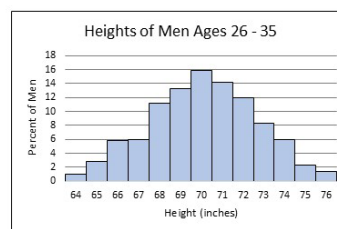
|                              |                                                                                                                                              |
|------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Normal Distribution</b>   | Normal curves are used in many situations to estimate probabilities. The pattern the data form is a bell-shaped curve called a normal curve. |
| <b>Standard Normal Curve</b> | A normal distribution centered on the y-axis. The mean of the standard normal curve is 0. The standard deviation is 1.                       |

### Characteristics of the Normal Curve

1. Maximum value of the curve occurs at the mean, usually symbolized as  $\mu$ , (Greek letter mu).
2. The standard deviation of  $\sigma$  (sigma).
3. Nearly all of the area under the curve (99.7%) is within three standard deviations from the mean,  $\mu + 3\sigma$ .
4. The total area under a normal curve is always 1.
5. The area under the curve between two x-values corresponds to the probability that x is between those values.



## Estimating Probabilities Using a Normal Curve



**EX #1:** In a survey of American men, the heights are distributed normally about the mean, as shown in the graph above.

1. About what percent of men aged 26 to 35 are 68 – 70 inches tall?
2. About what percent of the men in the survey are less than 70 inches tall?
3. Suppose this survey was the data on 1500 men. About how many men would you expect to be 68 – 70 inches tall?
4. The mean of the data is 70, and the standard deviation is 2.5. About what percent of the men are within one standard deviation of the mean in height?

## Sketching a Normal Curve

**EX #2:** The table below shows the lengths, in feet, of 40 great white sharks from Scuba News Magazine.

|      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|
| 18.2 | 16.4 | 13.2 | 19.1 | 16.2 | 12.3 | 15.8 | 16.7 | 18.6 | 17.8 |
| 14.3 | 22.8 | 16.2 | 16.6 | 15.7 | 9.4  | 12.6 | 13.6 | 13.8 | 18.3 |
| 17.8 | 14.6 | 15.7 | 18.2 | 13.2 | 12.2 | 15.8 | 19.7 | 13.6 | 15.2 |
| 14.9 | 18.7 | 15.3 | 17.6 | 13.5 | 16.8 | 16.1 | 12.1 | 14.7 | 13.2 |

1. Enter these data items into your calculator and make a histogram.
2. Calculate one-variable statistics.

```

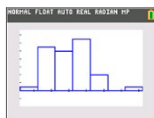
NORMAL FLOAT AUTO REAL RADIAN DP
DISTANCE BETWEEN FEET SHARKS ON AXIS
MIND04
Xmin=8
Xmax=24
Xsc1=2.1666666666667
Ymin=-4.21
Ymax=16.8
Ysc1=1.8
Xres=1
aY=0.86060606060606
TraceStep=0.12121212121212

```

```

NORMAL FLOAT AUTO REAL RADIAN DP
1-Var State
Σ=15.6623
Σx²=10071.07
Sxx=2.37459855
σx=2.542212373
n=40
minX=9.4
Q1=13.6

```



3. Sketch a normal curve showing the frequency of the shark lengths. Label the x-axis values at one, two, and three standard deviations from the mean.

## The 68 - 95 - 99.7 Rule

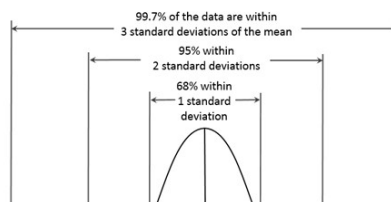
When a data set is normally distributed, about 68% of the data fall within one standard deviation of the mean. About 95% of the data fall within two standard deviations of the mean. In order to find the values that are two standard deviations away from the mean, we would find the values that have a z-score of  $\pm 2$ .

### Sometimes called The Empirical Rule . . .

#### The 68 - 95 - 99.7 Rule

In a Normal distribution with mean  $\mu$  and standard deviation  $\sigma$ :

- ☐ Approximately **68%** of the observations fall within  $\sigma$  of the mean.
- ☐ Approximately **95%** of the observations fall within  $2\sigma$  of the mean.
- ☐ Approximately **99.7%** of the observations fall within  $3\sigma$  of the mean.



# NOTES

### Using Standard Normal Values

If a random variable  $x$  is normally distributed with mean  $\mu$  and standard deviation  $\sigma$ , then the standard normal value of  $x$  is

$$z = \frac{x - \mu}{\sigma}$$

The table shows the approximate area under the curve for all values less than  $z$  for selected values of  $z$ .

| $z$  | -2.5 | -2   | -1.5 | -1   | -0.5 | 0   | 0.5  | 1    | 1.5  | 2    | 2.5  |
|------|------|------|------|------|------|-----|------|------|------|------|------|
| Area | 0.01 | 0.02 | 0.07 | 0.16 | 0.31 | 0.5 | 0.69 | 0.84 | 0.93 | 0.98 | 0.99 |

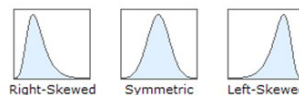
**EX #3:** Scores on a test are normally distributed with a mean of 76 and a standard deviation of 8.

A. Estimate the probability that a randomly selected student scored less than 88%.

B. Estimate the probability that a randomly selected student scored between 72 and 80.

### Is It Normal or Skewed?

While many data sets can be modeled using a normal curve, not all data is normally distributed. Sometimes the "tail" is longer on one side than the other, resulting in a skewed distribution.



**EX #4:** The heights of 24 children entering a theme park are shown below. The mean of all the children at the park is 49 inches and the standard deviation is 6 inches.

A. Does the data appear to be normally distributed? Explain your reasoning.

|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 51 | 46 | 48 | 41 | 51 | 49 | 46 | 45 |
| 52 | 49 | 47 | 45 | 52 | 51 | 51 | 54 |
| 48 | 51 | 53 | 51 | 48 | 53 | 52 | 46 |

B. Use your calculator to draw a histogram and calculate one-variable statistics.



### Real-World Connection

**EX #5:** In a college algebra course with 286 students in a lecture hall, the final exam scores have a mean of 67.5 and a standard deviation of 7.4. The grades on the exams are all whole numbers, and the grade pattern follows a normal curve.

A. Sketch a normal curve, label the x-axis values at one, two, and three standard deviations from the mean.



B. Find the number of students who receive grades from one to two standard deviations above the mean.

C. How many students earned a grade below 60%?

# NOTES

Name \_\_\_\_\_ Date \_\_\_\_\_

13.5

## Normal Distributions Practice

**Problems 1 – 4.** A set of data with a mean of 48 and a standard deviation of 6.5 is normally distributed. Find each value, given its distance from the mean.

- |                                        |                                         |
|----------------------------------------|-----------------------------------------|
| 1. +1 standard deviation from the mean | 2. +2 standard deviations from the mean |
| 3. -1 standard deviation from the mean | 4. +3 standard deviations from the mean |



**Problems 5 – 6.** Sketch a normal curve for each distribution. Label the x-axis values at one, two, and three standard deviations from the mean.

5. mean 80; standard deviation 12

6. mean 65; standard deviation 5.5

**Problems 7 – 10.** A set of data has a normal distribution with a mean of 6.2 and a standard deviation of 1.5. Find the percent of data within each interval.

7. between 4.7 and 6.2

8. between 7.7 and 9.2

9. less than 4.7

10. greater than 9.2

**Problems 11 – 14.** Scores on the Algebra 2 Common Final Exam in your school are normally distributed with a mean of 74 and a standard deviation of 10. Your school tested 225 students.

11. How many students scored above 94?

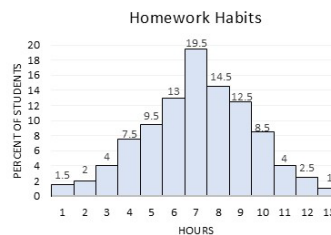
12. How many students scored below 64?

13. How many students score within one standard deviation of the mean?

14. How many students score between two and three standard deviations of the mean?

### Problems 15 – 18.

The bar graph at right gives the hours of homework per week of 800 students surveyed in a high school.



15. About what percent of students have between 5 and 9 hours of homework per week?

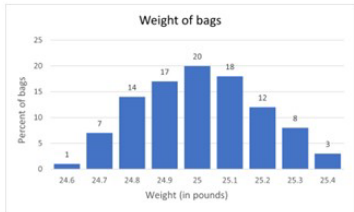
16. Find the number of students that have between 5 and 9 hours of homework per week.

17. The mean of the data is 7 hours and the standard deviation is two. About what percent of students have between 3 and 11 hours of homework per week?

18. How many students in the survey have an amount within 1 standard deviation of the mean?

# Practice Sets

# Lesson Warm-Ups



2. About what percent of bags of rice are between 24.9 and 25.1 pounds?

3. About how many total bags of rice would you expect to weigh more than 25.2 pounds?

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

## Lesson I3.5 Normal Distributions

## Warm-Up #2

The actual hours spent exercising each week for a survey of 1800 high school students are normally distributed. The mean time spent exercising is 5 hours, with a standard deviation of 0.2 hour.



1. What percent of students are below the mean?

2. About what percent of students are between 4 and 6 hours of exercise?

3. About how many students would you expect to exercise for 8 or 9 hours each week?

Form A

Period \_\_\_\_\_

64 and a normally thin +3 mean?

## 13.4 Binomial Distributions

Form B

Name \_\_\_\_\_

Date \_\_\_\_\_ Period \_\_\_\_\_

1. A set of data with a mean of 63 and a standard deviation of 5.4 is normally distributed. What value is within  $-2$  standard deviations of the mean?

A survey of employees for the XYZ Corporation found that the average commute time to work was 21 minutes. The standard deviation was 5 minutes. Sketch a normal curve showing one, two and three standard deviations from the mean.

2.

3. XY Corp. has 42 employees. How many employees commute between 16 and 26 minutes in travel time?

A survey of employees for the ABC Corporation found that the average commute time to work was 18 minutes. The standard deviation was 5 minutes. Sketch a normal curve showing one, two and three standard deviations from the mean.

2.

3. ABC Corp. has 34 employees. How many employees commute between 13 and 23 minutes in travel time?

# Daily Lesson Quiz

Grade: \_\_\_\_\_

Grade: \_\_\_\_\_