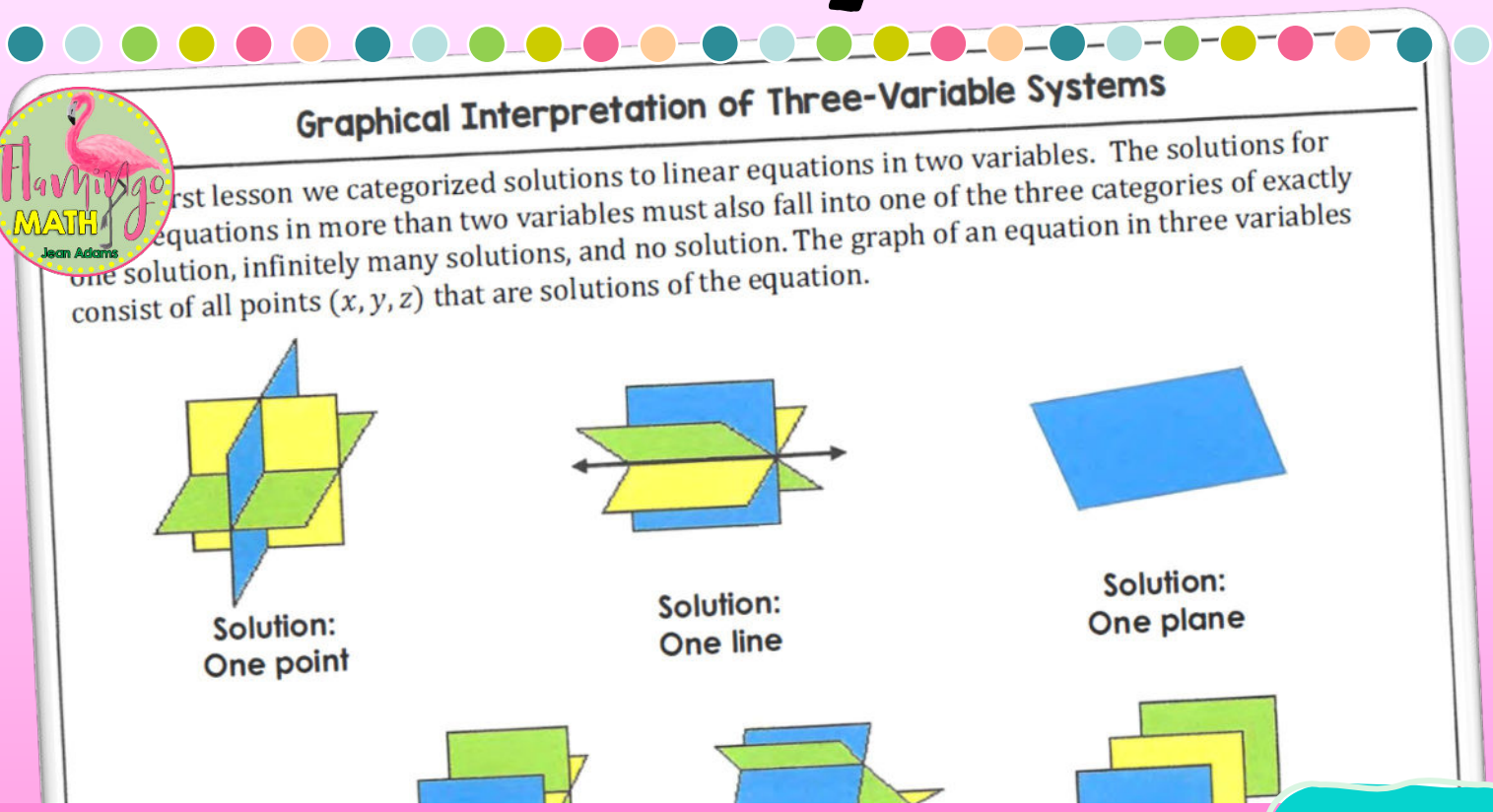
Precalculus Honors

Multivariable Linear Systems



Completely Editable

7.2

Lesson 7.2: Multivariable Linear Systems

When using the elimination method to solve a system of linear equations, our goal is to rewrite the system in a form where we can use back-substitution. This is known as writing the system in Row-Echelon Form.

EX #1: Solve the system in row echelon form using back-substitution. $\begin{cases} 2x + 3y + 4z = 9 \\ y + 3z = 7 \\ z = 2 \end{cases}$

Elementary Row Operations

To be able to solve a system of equations that is not in row-echelon form we can perform elementary row operations on the system. This procedure is called **Gaussian Elimination**.

1. Interchange the position of two equations.
2. Multiply one of the equations by a nonzero constant.
3. Add a multiple of one equation to another equation.

EX #2: Solve the system of linear equations.

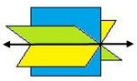
$$\begin{cases} x - 2y + 3z = 7 \\ -x + y + 6z = 3 \\ 2x \quad\quad - 3z = 1 \end{cases}$$

Graphical Interpretation of Three-Variable Systems

In our first lesson we categorized solutions to linear equations in two variables. The solutions for linear equations in more than two variables must also fall into one of the three categories of exactly one solution, infinitely many solutions, and no solution. The graph of an equation in three variables consist of all points (x, y, z) that are solutions of the equation.



Solution: One point



Solution: One line

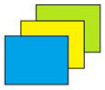


Solution: One plane

No Solutions:







Identify an Inconsistent System of Equations

EX #3: Solve. $\begin{cases} x - y + 2z = 2 \\ x + 2y - z = 5 \\ 2x - y + 3z = 1 \end{cases}$

Solving Systems with Three Variables

EX #4: Solve. $\begin{cases} x - y + z = 6 \\ 2x - 3y + 4z = 19 \\ 4x + y - 2z = -3 \end{cases}$

Solving a Dependent System of Equations

EX #5: Solve. $\begin{cases} x - 2y - 7z = 1 \\ 2x - 3y - 9z = 4 \\ 4x - 5y - 13z = 10 \end{cases}$

Real-World Applications

EX #6: An airplane can fly 2000 miles in 4 hours with a tailwind. The return trip, into the wind took the pilot 5 hours. Find the speed of the plane in still air and the speed of the wind.

EX #7: A safety research firm has collected data for the braking distance of a new car model. The data in the table below describes the relationship between the number of feet the car travels once the brakes are applied, d, and the number of seconds the car is in motion after the brakes are applied t.

Time t, seconds the car is in motion after brakes are applied	$d(t)$, feet car travels after applying the brakes
1	52
2	82
3	108

A. Use the data to find a quadratic function $d(t) = at^2 + bt + c$, that models the data.
B. Use your function from part (A) to find the value for $d(t)$ when $t = 6$.
C. Explain the meaning of your findings.

NOTES

7.2 **Multivariable Linear Systems Homework**

Name _____

Date _____ Period _____

Problems 1-2, use back-substitution to solve the system of linear equations.

1. $\begin{cases} 2x + y - 3z = 16 \\ y - 2z = 8 \\ z = -2 \end{cases}$

2. $\begin{cases} 4x - 3y - 2z = -14 \\ 6y - 5z = -21 \\ z = 1 \end{cases}$

Problems 3 - 5, perform the row operation and write the equivalent system.

3. Add Equation 1 to Equation 2. Explain what this accomplished.

$\begin{cases} x - 2y + 3z = 1 \\ -x + 3y - 5z = 0 \\ 2x - 3z = 7 \end{cases}$

4. Add Equation 1 to Equation 3. Explain what this accomplished.

$\begin{cases} x - 2y + 3z = 1 \\ -x + 3y - 5z = 0 \\ 2x - 3z = 7 \end{cases}$

5. Use your work from questions 3 and 4 to perform row operations that will solve the system of equations. Explain your steps and state your solution.

Problems 6-9, solve the system of linear equations.

6. $\begin{cases} 3x - 2y + z = -5 \\ x + 3y - 2z = 14 \\ x + y - z = 6 \end{cases}$

7. $\begin{cases} x + y - 2z = 8 \\ 3x - 2y + 4z = 1 \\ 2x - 3y + 6z = 8 \end{cases}$

8. $\begin{cases} x + 2y + 4z = 11 \\ 2x + y - z = -2 \\ x - 2y - 8z = -21 \end{cases}$

9. $\begin{cases} 2x + y - z = -3 \\ 4x - 2y - z = 7 \\ -2x + 2y - 3z = -10 \end{cases}$

Free Response Questions

10. Vertical Motion. The height of an object moving in a vertical line with constant acceleration is given by the position equation $s(t) = \frac{1}{2}at^2 + v_0t + s_0$. The height of the object s is measured in feet, t is measured in seconds, a is acceleration, and v_0 is the initial velocity, in feet per second at $t = 0$ and s_0 is the initial height. When $t = 1, s = 60$, at $t = 2, s = 56$, and *when* $t = 3$ $s = 20$.

A. Write a system of equations to obtain the values of a, v_0, and s_0.

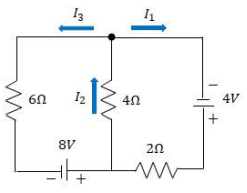
B. Solve the system of equations to find a position function that models the behavior of the object.

C. What is the maximum height the object obtains?

Free Response Questions

11. Kirchhoff's Rules An electrical network applies Kirchhoff's Laws are shown below. The currents I_1, I_2, and I_3 are the solution of the system

$\begin{cases} I_1 - I_2 + I_3 = 0 \\ 4 - 2I_1 - 4I_2 = 0 \\ 8 - 4I_2 - 6I_3 = 0 \end{cases}$

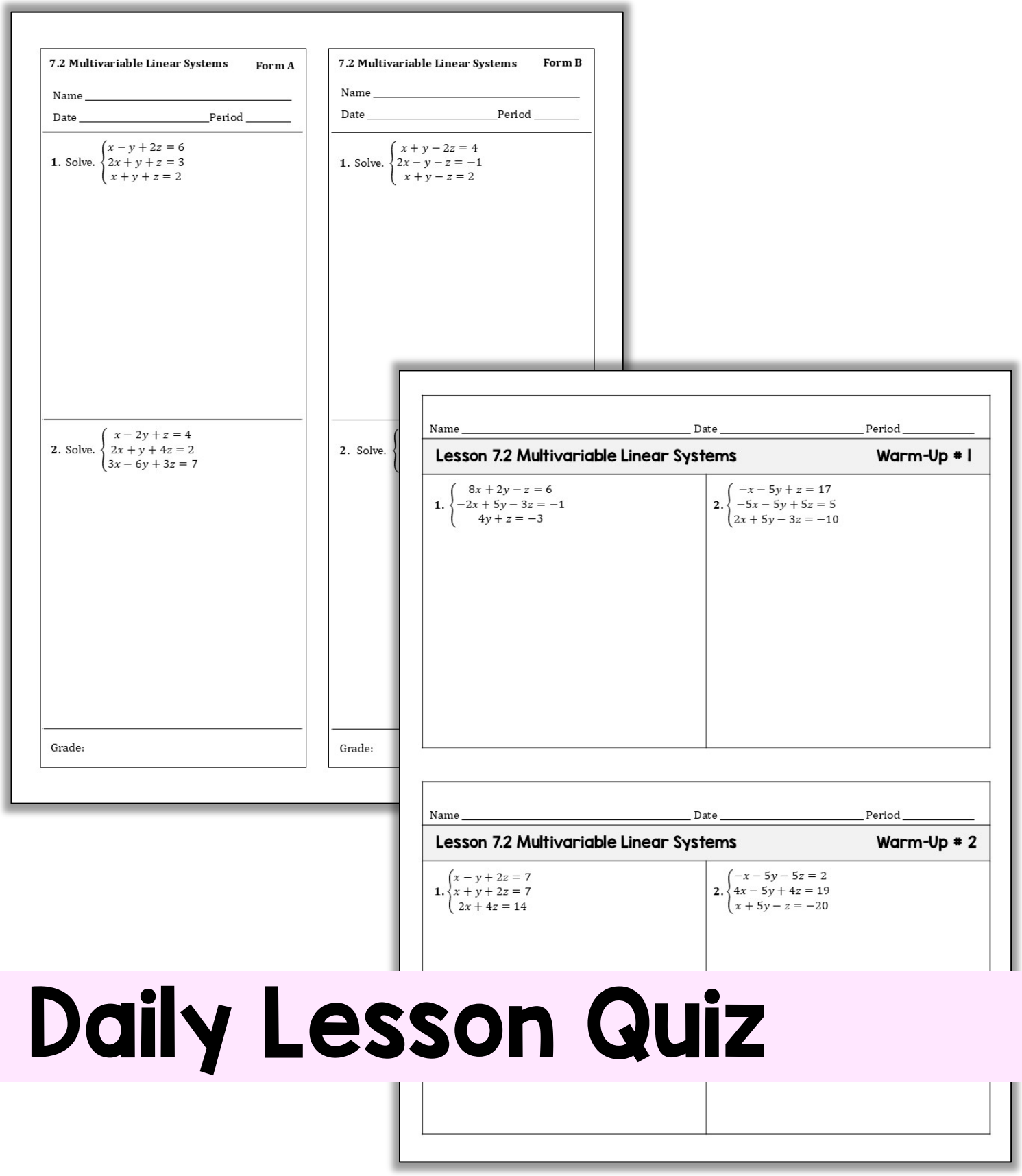


12. Economics The Hicks-Hansen model or IS-LM model is a macroeconomic tool that shows the relationship between interest rates and assets market. The IS curve represents all combinations of income Y and interest rates r that preserve equilibrium in the market for goods in the economy. The LM curve represents all combinations of income Y and interest rates r that maintain equilibrium in the market for money in the economy. Suppose that the equilibrium level of income (in billions of dollars) and interest rates satisfy the system of equation

$\begin{cases} 0.05Y - 1000r = 15 \end{cases}$

Practice Sets

Lesson Warm-Ups



7.2 Multivariable Linear Systems Form A

Name _____

Date _____ Period _____

1. Solve. $\begin{cases} x - y + 2z = 6 \\ 2x + y + z = 3 \\ x + y + z = 2 \end{cases}$

2. Solve. $\begin{cases} x - 2y + z = 4 \\ 2x + y + 4z = 2 \\ 3x - 6y + 3z = 7 \end{cases}$

Grade:

7.2 Multivariable Linear Systems Form B

Name _____

Date _____ Period _____

1. Solve. $\begin{cases} x + y - 2z = 4 \\ 2x - y - z = -1 \\ x + y - z = 2 \end{cases}$

2. Solve.

Grade:

Name _____ Date _____ Period _____

Lesson 7.2 Multivariable Linear Systems Warm-Up # 1

1. $\begin{cases} 8x + 2y - z = 6 \\ -2x + 5y - 3z = -1 \\ 4y + z = -3 \end{cases}$

2. $\begin{cases} -x - 5y + z = 17 \\ -5x - 5y + 5z = 5 \\ 2x + 5y - 3z = -10 \end{cases}$

Name _____ Date _____ Period _____

Lesson 7.2 Multivariable Linear Systems Warm-Up # 2

1. $\begin{cases} x - y + 2z = 7 \\ x + y + 2z = 7 \\ 2x + 4z = 14 \end{cases}$

2. $\begin{cases} -x - 5y - 5z = 2 \\ 4x - 5y + 4z = 19 \\ x + 5y - z = -20 \end{cases}$

Daily Lesson Quiz