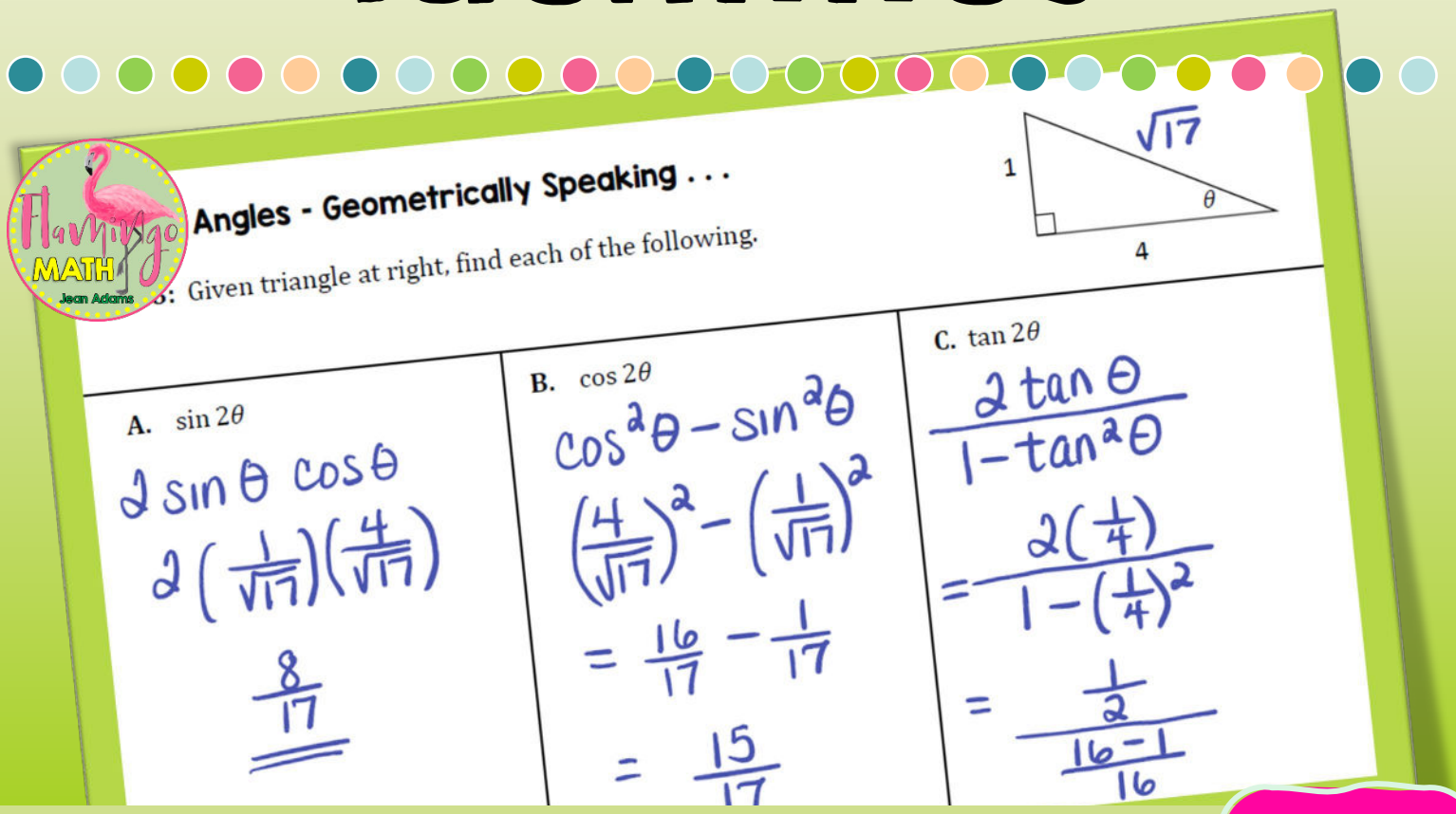
Precalculus Honors

Multiple Angle Identities



Completely Editable

5.4

Lesson 4: Multiple Angles Identities

We see four more categories for identities . . .

Double Angle Identities

$\sin 2u = 2\sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$

$= 2\cos^2 u - 1$

$= 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

Half-Angle Identities

$\sin\dfrac{u}{2} = \pm\sqrt{\dfrac{1-\cos u}{2}}$

$\tan\dfrac{u}{2} = \pm\sqrt{\dfrac{1-\cos u}{1+\cos u}}$

$= \dfrac{1-\cos u}{\sin u}$

$\cos\dfrac{u}{2} = \pm\sqrt{\dfrac{1+\cos u}{2}}$

$= \dfrac{\sin u}{1+\cos u}$

Power-Reducing Formulas

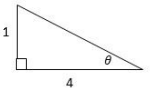
$\sin^2 u = \dfrac{1-\cos 2u}{2}$ $\quad$ $\cos^2 u = \dfrac{1+\cos 2u}{2}$ $\quad$ $\tan^2 u = \dfrac{1-\cos 2u}{1+\cos 2u}$

Proving an Identity

EX #1: $\sin(2u) = 2\sin u \cdot \cos u$

EX #2: $\cos^4\theta - \sin^4\theta = \cos 2\theta$

Double Angles - Geometrically Speaking . . .



EX #3: Given triangle at right, find each of the following.

A. $\sin 2\theta$

B. $\cos 2\theta$

C. $\tan 2\theta$

Half-Angle Identities

EX #4: Given $\frac{u}{2} = 165°$ Find each of the following.

A. $\sin\frac{u}{2}$

B. $\cos\frac{u}{2}$

C. $\tan\frac{u}{2}$

Reducing a Power of 4

EX #5: Rewrite $\cos^4\theta$ in terms of trigonometric functions with no power greater than 1.

$\cos^4\theta = (\cos^2\theta)^2$

Solving Algebraically

EX #6: Solve $\cos 2x = \frac{1}{2}$ for $0 \le x \le 2\pi$

EX #7: Solve. $\sin 2x = 0$ for $0 \le x \le 2\pi$

EX. #8: Solve and find the general solution. $\cos 2x + \cos x = 0$

NOTES

5.4 **Multiple Angle Identities Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, Use the information given about the angle θ, $0 \le \theta \le 2\pi$, to find the exact value of

A) $\sin 2\theta$ **B)** $\cos 2\theta$ **C)** $\sin \frac{\theta}{2}$ **D)** $\cos \frac{\theta}{2}$

1. $\sin\theta = \frac{3}{5};\ 0 < \theta < \frac{\pi}{2}$

2. $\cos\theta = \frac{3}{5};\ 0 < \theta < \frac{\pi}{2}$

3. $\tan\theta = \frac{4}{3};\ \pi < \theta < \frac{3\pi}{2}$

4. $\sec\theta = 3;\ \sin\theta > 0$

Problems 5 – 8, Use the Half-Angle Formulas to find the exact value of each trigonometric function.

5. $\cos 22.5°$

6. $\sin 195°$

7. $\tan \frac{7\pi}{8}$

8. $\cos\left(\frac{-3\pi}{8}\right)$

Problems 9 – 12, Write the expression as one involving only sin θ and cos θ.

9. $\sin 2\theta + \cos\theta$

10. $\sin 2\theta + \cos 2\theta$

11. $\sin 3\theta$

Problems 13 – 16, Prove the identity.

13. $\sin 4x = 2\sin 2x \cos 2x$

14. $\cos^4\theta - \sin^4\theta = \cos 2\theta$

15. $2\cot 2x = \cot x - \tan x$

16. $\cos 4\theta = 1 - 8\sin^2\theta\cos^2\theta$

Problems 17 – 20, Find all solutions to the equation in the interval $[0, 2\pi)$

17. $\sin 2x = 2\sin x$

18. $\cos 2x = \sin x$

Practice Sets

Lesson Warm-Ups

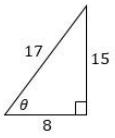


3. Write $2\cos^2\left(\frac{\pi}{12}\right) - 1$ as a double angle. Then, find the exact value of the expression.

Name _____ Date _____ Period _____

Lesson 5.4: Multiple Angle Identities **Warm-Up # 2**

1. Find $\cos\left(\frac{\theta}{2}\right)$



2. If θ lies in quadrant II, find $\sin(2\theta)$.

3. Write $2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$ as a double angle. Then, find the exact value of the expression.

5.4 Using Identities **Form A**

Name _____

Date _____ Period _____

1. Given $\sec x = -\sqrt{10}$ and $\tan x = -3$ find $\sin x$

2. Simplify: $\frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$

3. Verify: $\csc\theta - \csc\theta\cos^2\theta = \sin\theta$

Grade:

5.4 Using Identities **Form B**

Name _____

Date _____ Period _____

1. Given $\sec x = -\frac{\sqrt{13}}{3}$ and $\tan x = -\frac{2}{3}$ find $\sin x$

2. Simplify: $\frac{1-\sin^2 x}{\sin^2 x}$

3. Verify: $\cos\alpha + \sin\alpha\tan\alpha = \sec\alpha$

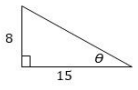
Grade:

5.4 Multiple Angle Identities **Form A**

Name _____

Date _____ Period _____

For questions 1–2, use the figure at right.



1. Find $\sin\frac{\theta}{2}$

2. If θ lies in Quadrant II, find $\cos 2\theta$.

3. Write $2\cos^2\left(\frac{\pi}{8}\right) - 1$ as a double angle. Then find the exact value of the expression.

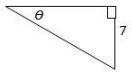
Grade:

5.4 Multiple Angle Identities **Form B**

Name _____

Date _____ Period _____

For questions 1–2, use the figure at right.



1. Find $\sin\frac{\theta}{2}$

2. If θ lies in Quadrant II, find $\sin 2\theta$.

3. Write $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$ as a double angle. Then find the exact value of the expression.

Grade:

Daily Lesson Quiz