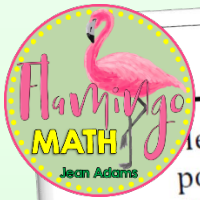


Precalculus Honors

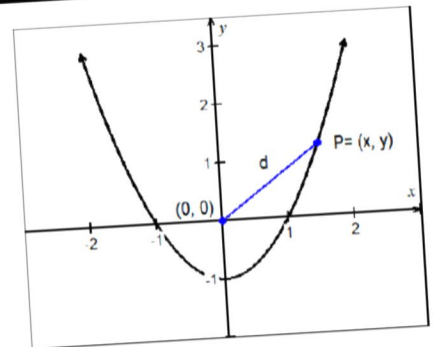
Modeling

with Functions



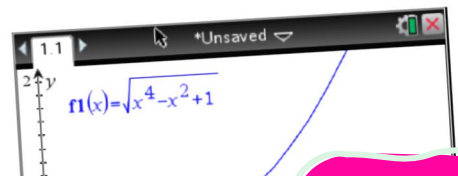
Finding Distance from the Origin

Here is a famous problem from Calculus. Let's explore a point $P(x, y)$ on the graph of $y = x^2 - 1$. Our goal is to find an approximation of a point on the graph that is closest to the origin.



- A. Express the distance d from P to the origin O as a function of x .

- B. Use a graphing utility to graph the function $d(x)$, for $x \geq 0$. Find the value(s) of x where d has a local minimum. This will give the point(s) on the graph of $y = x^2 - 1$ closest



Completely Editable

1.9

I.9 Modeling with Functions

In real-world settings we need to construct functions by translating verbal descriptions into the language of mathematics. We are learning to assign meaningful variables and symbols that will represent both independent and dependent events. Let's explore some situations where we can create a function or rule that represents real-world events.

Writing Functions from Formulas

EX #1: The perimeter of a rectangular garden is 84 feet. Express its area A as a function of the width w of a side.

EX #2: The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$. If the height is twice the radius.
A. Express the volume V as a function of the radius r .

B. Find the volume of a paper Sno-Cone cup whose radius is 2 inches.

Business and Economics

In the business world we are concerned with sales, costs, revenue, consumer demands and profit. Revenue R is defined as the money received for the sale of x units sold at the retail price p . That is $R = px$. The Law of Demand, from economics, tells us that the price and units sold are related. Specifically, as one increases, the other decreases.

EX #3: The demand for tickets to a local waterpark is given by $p = 75 - 0.02x$, where p is the price of a ticket in dollars, and x is the number of people going to the park at that price.

A. What price would provide an attendance of 3000 people?

B. What is the total revenue at this ticket price?

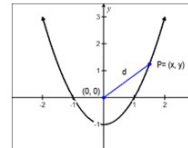
C. Write the revenue function as a function of attendance, x , at the waterpark.

EX #4: A skateboard designer has fixed monthly costs of \$9,202 and variable costs of \$18 for each skateboard produced. The skateboards sell for \$125 each. How many skateboards should the designer produce and sell each month to break even? What is the total revenue at the point where the designer breaks even?

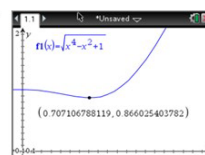
Finding Distance from the Origin

EX #5: Here is a famous problem from Calculus. Let's explore a point $P(x, y)$ on the graph of $y = x^2 - 1$. Our goal is to find an approximation of a point on the graph that is closest to the origin.

A. Express the distance d from P to the origin O as a function of x .

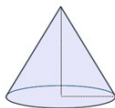


B. Use a graphing utility to graph the function $d(x)$, for $x \geq 0$. Find the value(s) of x where d has a local minimum. This will give the point(s) on the graph of $y = x^2 - 1$ closest to the origin.



Finding a Model and Solving

EX #6: Grain is leaking through a hole in a storage bin at a constant rate of 6 cubic inches per minute. The grain forms a cone-shaped pile on the ground below. As it grows, the height of the cone always remains equal to its radius. If the cone is two-feet-tall now, how tall will it be in one hour?



Variation Problem

EX #7: Hooke's Law states that the force required to stretch (or compress) a spring by some distance is directly proportional to that distance. A force of 275 Newtons stretches a spring 0.2 meter.

A. How far will a force of 620 Newtons stretch the spring?

Writing Regression Equations

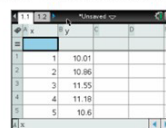
EX #8: Bottled Water Production

The US bottled water producer's revenue, in billions of US dollars, from 2005 to 2012 is shown in the table at right. Use a graphing utility to write a regression equation. The time, in years, is given by t .

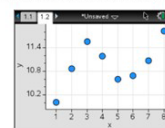
Year	Revenue, in billions USD
2005	\$10.01
2006	\$10.86
2007	\$11.55
2008	\$11.18
2009	\$10.60
2010	\$10.69
2011	\$11.07
2012	\$11.82

Source: Beverage Marketing Corp.

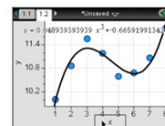
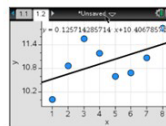
1. Create a list/spreadsheet of the data with $t = 1$ corresponding to 2005.



2. Create a scatter plot of your data and determine the best type of regression model.



3. Below we see linear on the left and cubic on the right.



Linear: _____

5. Determine the model that would "make sense" in this scenario. Explain why.

NOTES

1.9

Modeling with Functions
Practice

Name _____

Date _____ Period _____

Problems #1-4: Write an equation to model each scenario.

- | | |
|---|--|
| 1. The total cost C of an item selling for x dollars if the sales tax rate is 7.25%. | 2. The total cost is \$8200 increased by 6% plus \$9.75 for each item produced. |
| 3. A right circular cylinder with height h and base radius r has a total surface area of 80π square inches. Express the volume of the cylinder as a function of r . | 4. A rectangle with base x and height y has a perimeter of 150 ft. Express the area A of the rectangle as a function of its base x . |

Problems #5-13: Answer each question.

5. $P(x, y)$ is an arbitrary point on the circle $x^2 + y^2 = 1$. Express the distance d from P to the point $A = (3, 0)$ as a function of the x -coordinate of P . State the domain and range of the function.

6. The diameter of the largest particle that can be moved by a stream varies directly as the square of the velocity of the stream. A stream in the Great Smokey Mountains has a velocity of 2 miles per hour can move coarse particles about 0.2 inch in diameter. What velocity is required to carry particles 0.5 inch in diameter?

7. Rachel has 3000 feet of fencing to enclose a rectangular field for her horses. One side of the field lies along the river, so only three sides require fence material.

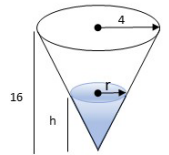
A. Express the area A of the rectangle as function of the width x of the rectangle.

B. What is the domain of A ?

C. Use a graphing utility to find the value of x that produces the largest area.

8. Water is poured into a container in the shape of a right circular cone with radius 4 inches and height 16 inches. Express the volume V of the water in the cone as a function of the height h of the water.

$$V = \frac{1}{3}\pi r^2 h$$



9. A square of side x inches is cut out of each corner of a 9 in. by 12 in. piece of cardboard and the sides are folded up to form an open-topped box.

A. Write the volume V as a function of its side x .

B. State the domain of the function.

C. Graph the function V to find the maximum volume of the box.

10. Mr. O's Physics students recorded a lab experiment for Hooke's Law by measuring the compressed length y (in cm) of a spring when a force of F pounds is applied.

A. Use the data in the table to create a scatter plot. Then find a regression equation. Recall Hooke's Law states that the distance a spring is compressed varies directly as the force on the spring.

Force, F (lbs.)	0	2	4	6	8	10	12
Length, y (cm)	0	1.12	2.26	3.55	4.6	5.82	7.2

B. Use the model to predict the force necessary to compress the spring 9 centimeters.

11. The price p and the quantity sold x of a certain product obey the demand equation $p = -\frac{1}{5}x + 150$ for $0 \leq x \leq 800$

A. Express the Revenue as a function of x . [Recall, $R = xp$]

B. What is the revenue if 300 units are sold?

12. The average hourly earnings, in US dollars, of US production workers for 2005 – 2013 are shown in the table below, where $t = 1$ corresponds to the year 2005.

Year	1	2	3	4	5	6	7	8	9
Average Hourly Earnings	15.91	16.42	17.09	17.75	18.41	18.89	19.31	19.57	19.94

Source: FRED Economic Data, Federal Reserve Bank of St. Louis

A. Find a linear regression equation. Round coefficients to the nearest 0.001.

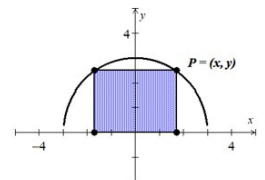
B. Find a quadratic regression equation. Round coefficients to the nearest 0.001.

C. Use both curves to predict the hourly earnings for 2016.

13. A rectangle is inscribed in a semicircle of radius 3. Let $P = (x, y)$ be a vertex point of the rectangle in quadrant I that also lies on the circle.

A. Write the area A of the rectangle as a function of x .

B. What is the domain of A ?



Practice Sets

E. What price should the company charge to maximize revenue?

E. Find the area of the maximum size rectangle.

Lesson Warm-Ups

1.9 Modeling with Functions Form A

Name _____

Date _____ Period _____

1. Write an equation for the area of a triangle whose altitude is 3 more than its base length x .

2. Volume of a Sphere: $V = \frac{4}{3}\pi r^3$. Solve for r .

The demand for the number of widgets sold x , is given by: $p = -\frac{1}{4}x + 125$, where p is the price of the widget, in dollars.

3. What price would provide the sale of 250 widgets?

4. What is the total revenue if 250 widgets are sold?

Grade:

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1.9 Modeling with Functions Form B

Name _____

Date _____ Period _____

1. Write an equation for the area of a rectangle whose length is 9 more than its width x .

2. Volume of a Cone: $V = \frac{1}{3}\pi r^2 h$. Solve for r .

The demand for the number of widgets sold x , is given by: $p = -\frac{1}{5}x + 120$, where p is the price of the widget, in dollars.

3. What price would provide the sale of 300 widgets?

4. What is the total revenue if 300 widgets are sold?

Grade:

Name _____ Date _____ Period _____

Lesson 1.9: Modeling with Functions

Warm-Up # 1

1. Write an equation for the perimeter of a rectangle whose length is 3 more than the width.
2. Area of a circle: $A = \pi r^2$. Solve for r .
3. The demand for the number of widgets sold x , is given by: $p = -\frac{1}{3}x + 120$, where p is the price of the widget, in dollars.
What price would provide the sale of 330 widgets?
4. The demand for the number of widgets sold x , is given by: $p = -\frac{1}{3}x + 120$, where p is the price of the widget, in dollars.
What is the total revenue if 330 widgets are sold?

Name _____ Date _____ Period _____

Lesson 1.9: Modeling with Functions

Warm-Up # 2

1. Write an equation for the perimeter of a rectangle whose length is 5 more than the width.
2. Volume of a cone: $V = \frac{1}{3}\pi r^2 h$. Solve for r .
3. The demand for the number of widgets sold x , is given by: $p = -\frac{1}{4}x + 120$, where p is the price of the widget, in dollars.
What price would provide the sale of 240 widgets?
4. The demand for the number of widgets sold x , is given by: $p = -\frac{1}{4}x + 120$, where p is the price of the widget, in dollars.
What is the total revenue if 240 widgets are sold?

Daily Lesson Quiz