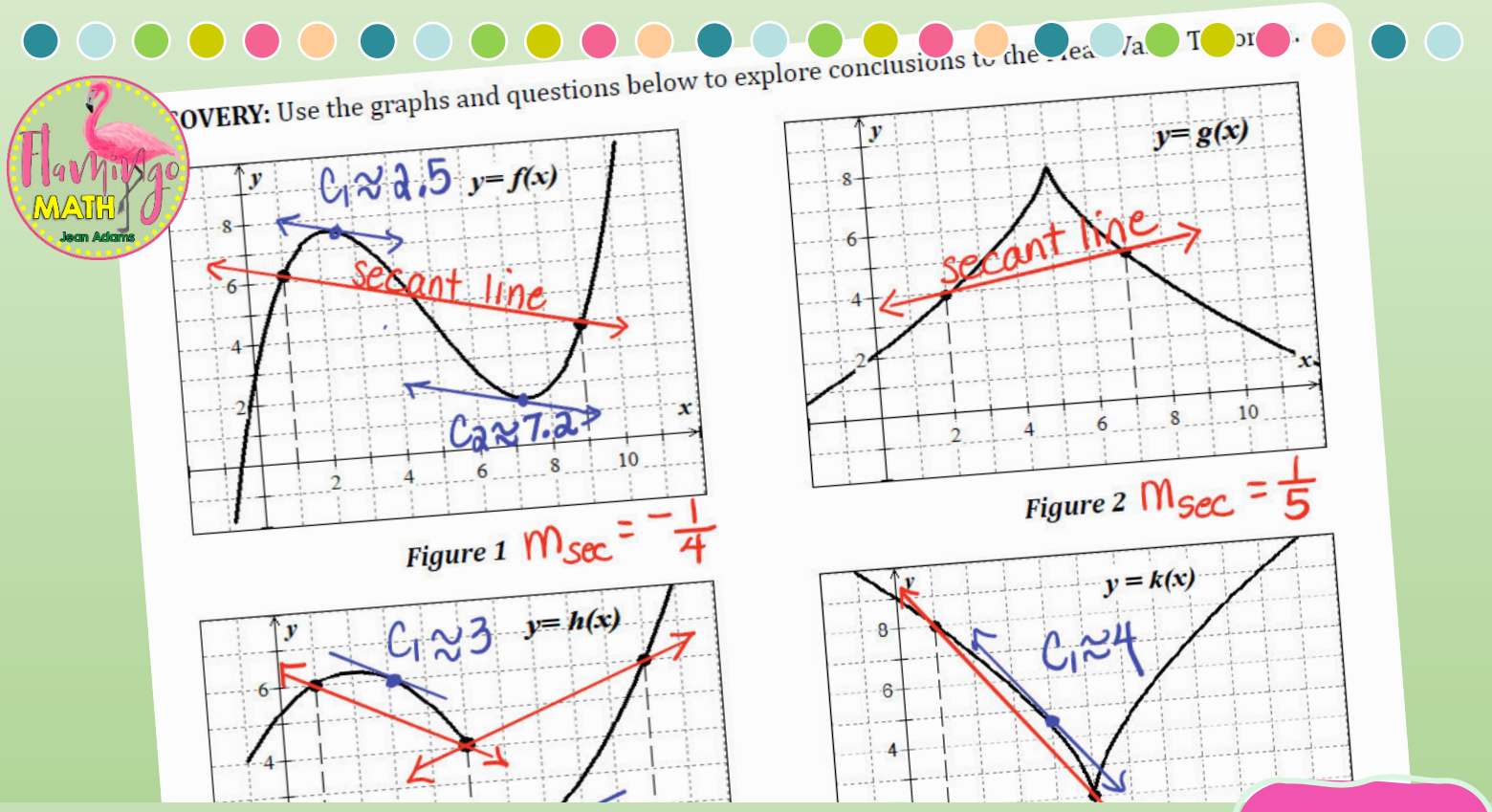
AP® Calculus

Mean Value Theorem and Rolle's Theorem



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5.1

Lesson I: Mean Value Theorem

Topic 5.1: Using the Mean Value Theorem

DISCOVERY: Use the graphs and questions below to explore conclusions to the Mean Value Theorem.

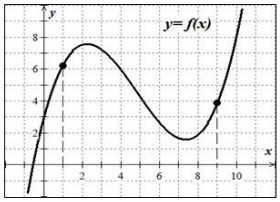


Figure 1

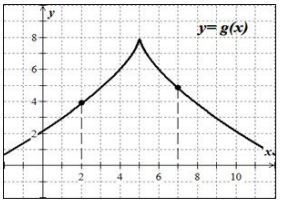
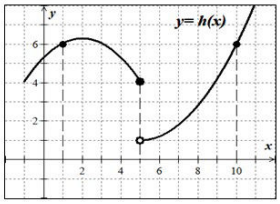


Figure 2



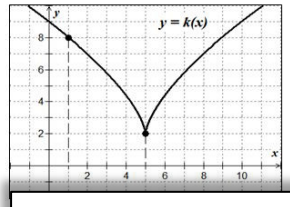


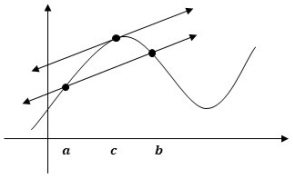
Figure 3

1. Draw the secant lines for each figure on the given interval
2. Are there any points on the graph of each function where to the secant line you've drawn? Draw a tangent line(s) be possible. Let this point of tangency be called $x = c$, for ea
3. For each figure, estimate the value of $x = c$ where the tan why it is not possible.
4. Which functions are continuous on the given intervals?
5. Which functions are differentiable on the given intervals?
6. Which are not differentiable? Explain why.

Mean Value Theorem

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

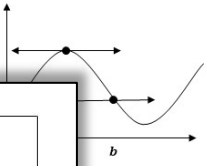


VERBALLY SAYS : Instantaneous rate of change equals average rate of change.

GRAPHICALLY SAYS: Tangent line is parallel to the secant line.

Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$ then there exists at least one number c in (a, b) where $f'(c) = 0$.



least one number between a and b so that the

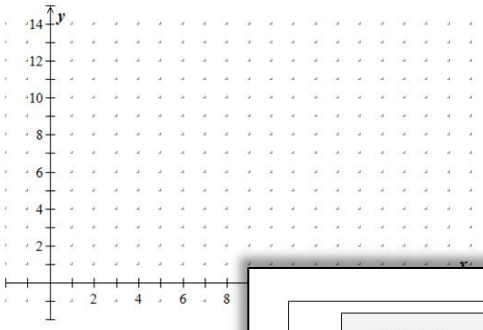
and the MVT guarantee at least one value
e OPEN interval.

the MVT and Rolle's Theorem?

t on a given interval?

EX. #1: Use the table of values below to sketch the differentiable continuous function on the grid.

x	0	2	4	6	7	9	11	13	15	17
$g(x)$	1	4	8	12	13	11	8	6	8	14



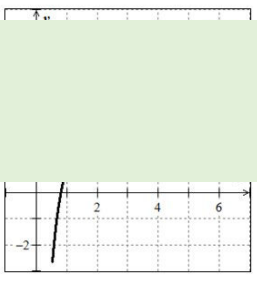
A. Find the average rate of change of $g(x)$ on the interval $[2, 9]$.

C. Does Rolle's Theorem apply on the interval $[4,9]$? Explain why or why not.

EX#2: Let f be the function given by $f(x) = x^2 + 2x$ conclusion of Rolle's Theorem for f on $[-3, 1]$.

EX #3: A sports car speedometer registers 50 miles per passes a patrol car stationed at a mileage marke later, the car passes another patrolman at a mar five miles from the first. The driver notes that th speedometer registers 55 miles per hour. Can t patrol use the Mean Value Theorem to prove tha exceeding the speed limit of 70 miles per hour a markers?

EX #4: Let $h(x) = 5 - \frac{4}{x}$, find all values of c in the open



M.V.T. allows us to identify exactly where graphs rise or fall:

Functions with positive / negative derivatives are increasing / decreasing functions.

EX#5: Find all values between the x-intercepts for which Rolle's Theorem holds for the roots of $f(x) = 4x^2 - x^4$. Verify with your calculator.

EX #6: Determine whether $g(x) = \sin 2x + 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, \pi]$. If so, find all numbers c in (a, b) such that $g(b) - g(a) = g'(c)(b - a)$

NOTES

5.1 **The Mean Value Theorem Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, Determine whether Rolle's Theorem can be applied to the function on the closed interval. If Rolle's Theorem can be applied, find all values of $x = c$ that satisfy the theorem, or explain why not.

1. $f(x) = x^2 - 3x - 10$ $[-2, 5]$

2. $f(x) = x^{2/3} - 2$ $[-1, 1]$

3. $f(x) = \sin x$ $[0, 2\pi]$

4. $f(x) = x - 2\ln x$ $[1, 3]$

Problems 5 – 10, Determine whether the Mean Value Theorem can be applied to the function on the closed interval. If the Mean Value Theorem can be applied, find all values of $x = c$ that satisfy the theorem, or explain why not.

5. $f(x) = x^3 - x^2 - 2x$ $[-1, 1]$

6. $f(x) = \sqrt{3 - x}$ $[-6, 3]$

7. $f(x) = \frac{x+3}{x}$ $\left[\frac{1}{2}, 3\right]$

8. $f(x) = |2x - 1|$ $[-2, 3]$

9. $f(x) = x^3$ $[-1, 0]$

10. $f(x) = x\ln x$ $[1, 2]$

11. Mr. O. ran the Disney Marathon in 2 hours 18 minutes. Remember a marathon is 26.2 miles. Show that at least twice he had to be running at a rate of 11 miles per hour.

12. Cecil's *ABC Cars* sells used automobiles. His sales, for the year, S, are a function of time, in months, given by $S(t) = 240\left(6 - \frac{8}{t+2}\right)$. Find the average rate of cars sold over the entire year for $0 \le t \le 11$. During what month does the average rate of car sales equal the rate of change of the car sales?

13. Contaminated water is being pumped from a retention pond. $R(t)$ is the amount of water that is left in the pond at time t, measured in hours.

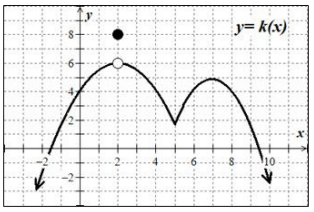
t, *hours*	0	3	6	9	12	15	18	21	24
$R(t)$ *gallons*	20,000	17,000	16,000	14,500	13,000	10,000	9,000	7,000	5,600

A. Using correct units of measure, find the average rate of change of $R(t)$ from $t = 3$ hours to $t = 21$ hours.

B. Estimate the rate that water is being pumped at $t = 4$ hours. Use the correct units. Explain what this value means in the context of the problem.

C. Is there a time on the interval $9 < t < 12$ at which the equation $R'(t) = -500$ gal/hr? If so, write a justification

Problems 14- 17, Given the function $y = k(x)$ shown below, discuss whether Rolle's Theorem or the Mean Value Theorem can be applied on the stated intervals. Justify your reason.



14. $[5, 8]$ **Mean Value Theorem**

15. $[0, 4]$ **Rolle's Theorem**

16. $[3, 6]$ **Mean Value Theorem**

Practice Sets

5.1 Mean Value Theorem **Form A**

Name _____

Date _____Period _____

1. Can Rolle's Theorem be applied to $y = (x-2)^{\frac{2}{3}}$ on the interval $[0, 4]$? Justify.

2. Find the value of c in the interval (a, b) such that $f'(c) = 0$ for $f(x) = (x-3)(x+1)^2$ on $[-1, 3]$.

3. Determine whether the Mean Value Theorem can be applied to $f(x) = \sqrt{(x+3)}$ on $[-3, 6]$. Find all values of c in $(-3,6)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$, if possible.

Grade:

5.1 Mean Value Theorem **Form B**

Name _____

Date _____Period _____

1. Can Rolle's Theorem be applied to $y = |x+1| - 2$ on the interval $[-3, 0]$? Justify.

2. Find the value of c in the interval (a, b) such that $f'(c) = 0$ for $f(x) = -x^2 + 3x$ on $[0, 3]$.

3. Determine whether the Mean Value Theorem can be applied to $f(x) = \sqrt{2-x}$ on $[-7, 2]$. Find all va
$f'(c) = \frac{f($

Grade:

5.1 Mean Value Theorem **Form C**

Name _____

Date _____Period _____

1. Can Rolle's Theorem be applied to $y = 3 - |x-2|$ on the interval $[0, 4]$? Justify.

2. Find the value of c in the interval (a, b) such that $f'(c) = 0$ for $f(x) = (x-1)(x+1)^2$ on $[-2, 2]$.

3. Determine whether the Mean Value Theorem can be applied to $f(x) = \sqrt{x-2}$ on $[2, 11]$. Find all values of c in $(2,11)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$, if possible.

Grade:

5.1 Mean Value Theorem **Form D**

Name _____

Date _____Period _____

1. Can Rolle's Theorem be applied to $y = x^{\frac{2}{3}} + 1$ on the interval $[-8, 8]$? Justify.

2. Find the value of c in the interval (a, b) such that $f'(c) = 0$ for $f(x) = (x-1)(x+2)^2$ on $[-3, 2]$.

3. Determine whether the Mean Value Theorem can be applied to $f(x) = \sqrt{5-x}$ on $[-4, 5]$. Find all values of c in $(-4, 5)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$, if possible.

Grade:

Daily Lesson Quiz