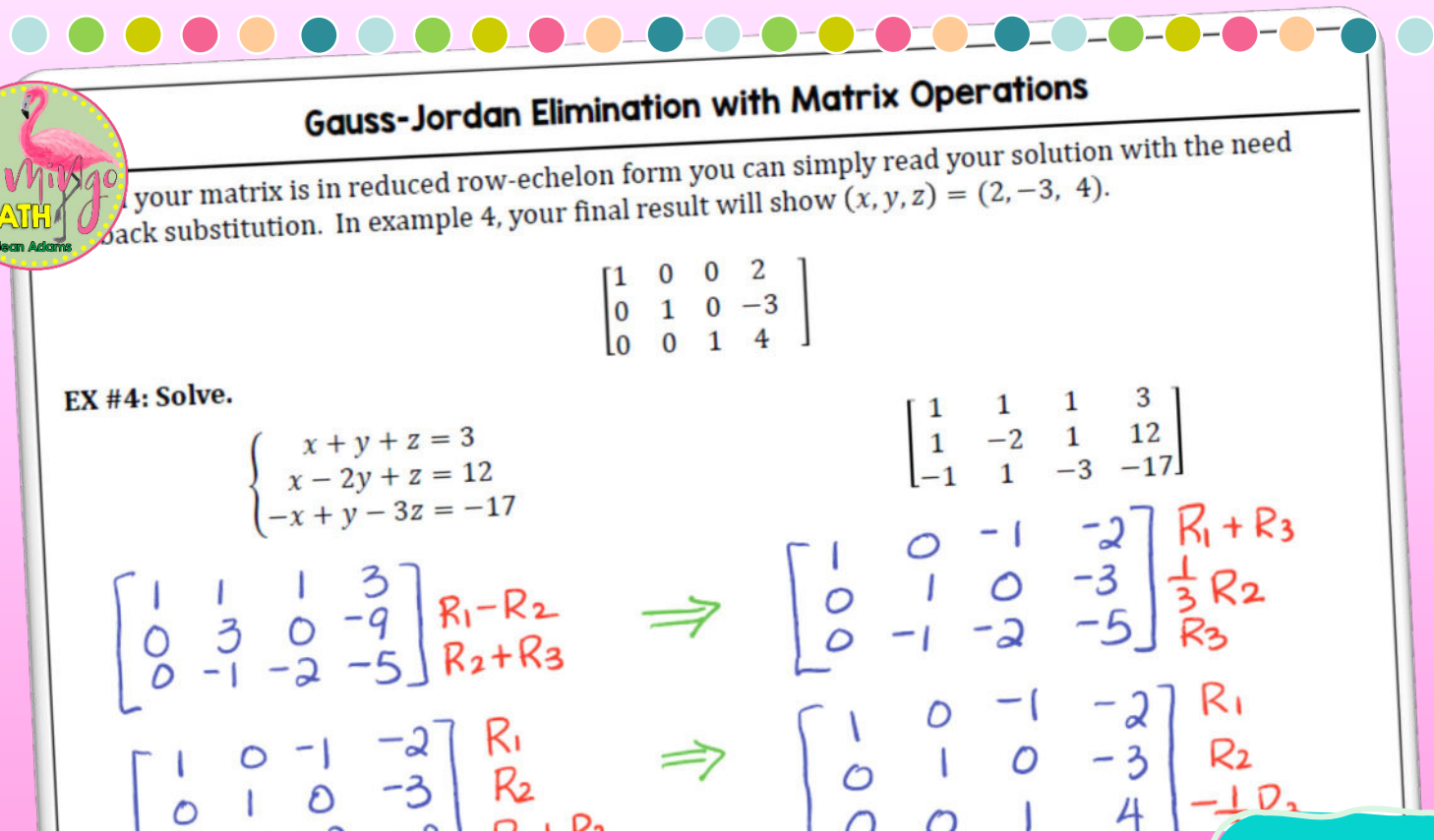
Precalculus Honors

Matrices & Systems of Equations



$\Rightarrow$
$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ R_1 R_2

Completely Editable

7.3

Lesson 7.3: Matrices and Systems of Equations

In this lesson we will explore techniques for solving systems of linear equations by using matrices. You might recall that a matrix is a rectangular array of real numbers. Every item listed in a matrix is called an ***element***. An element is identified by its position in the matrix. For example, a_{21} is the entry in the second row and the first column. A matrix having **m** rows and **n** columns is said to be of **order** $\boldsymbol{m \times n}$ Each subscript a_{ij} identifies the location of the element in the matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Order of Matrices

EX #1: Determine the order of each matrix.

A. $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$ **B.** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \end{bmatrix}$ **C.** $\begin{bmatrix} 0 & 1 \\ 3 & -2 \\ 1 & 0 \end{bmatrix}$

Writing an Augmented Matrix

When given a system of equations written in standard form with the constant term on the right, a corresponding matrix can be written by dropping the variables. This is known as an **augmented matrix**. If the constant terms are omitted, then the matrix is called a **coefficient matrix**.

System of Equations	Augmented Matrix	Coefficient Matrix
$\begin{cases} x+y+z=3 \\ x-2y+z=12 \\ -x+y-3z=-17 \end{cases}$	$\left[\begin{array}{ccc\|c} 1 & 1 & 1 & 3 \\ 1 & -2 & 1 & 12 \\ -1 & 1 & -3 & -17 \end{array}\right]$	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & -3 \end{bmatrix}$

When writing an augmented matrix, be sure to align the variables in the equations and use zero for any missing coefficients of variables.

EX #2: Write the augmented matrix for the system of linear equations.

$$\begin{cases} x+y+3z=-1 \\ y-2z=0 \\ x+3z=1 \end{cases}$$

Elementary Row Operations

In our last lesson, we used elementary row operations to write the system in row-echelon form. The term ***echelon*** refers to the stair-step pattern that is formed by the nonzero elements in system. These same row operations can be applied to an augmented matrix. It is important to track your work by noting your arithmetic steps.

Elementary Row Operations:

1. Interchange the position of two equations.
2. Multiply one of the equations by a nonzero constant.
3. Add a multiple of one equation to another equation.

EX #3: Perform the row operations as stated.

Original Matrix:

$$\begin{bmatrix} 1 & 1 & 3 & -1 \\ 0 & 1 & -2 & 0 \\ 1 & 0 & 3 & 1 \end{bmatrix}$$

A. New Row 1: $R_1 - R_2$

B. New Row 3: $R_1 - R_3$

C. New Row 3: $\frac{1}{2}R_3$

D. New Row 1: $R_1 - 5R_3$

E. New Row 2: $R_2 + 2R_3$

Reduced Row-Echelon Form

A matrix in *row-echelon form* is in ***reduced row-echelon form*** if every column that has a leading 1 has zeros in every position above and below its leading 1. Is this the case in EX #3, Part E? If so, then you have found the solution to your system.

Gauss-Jordan Elimination with Matrix Operations

When your matrix is in reduced row-echelon form you can simply read your solution with the need for back substitution. In example 4, your final result will show $(x, y, z) = (2, -3, 4)$.

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

EX #4: Solve.

$$\begin{cases} x+y+z=3 \\ x-2y+z=12 \\ -x+y-3z=-17 \end{cases} \qquad \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 3 \\ 1 & -2 & 1 & 12 \\ -1 & 1 & -3 & -17 \end{array}\right]$$

Solving a System with Infinite Solutions

EX #5: Solve. $\begin{cases} 3x+7y \quad\quad = 1 \\ 2x+4y-2z=0 \end{cases}$

Solving a System with No Solution

EX #6: Solve. $\begin{cases} x+2y=0 \\ x+y=4 \\ 3x-2y=6 \end{cases}$

Point-Fitting Data

EX #7: Use a system of equations to find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ that satisfies the equation. Solve the system using matrices.

$f(-2) = 4$

$f(-1) = 11$

$f(1) = 1$

$f(3) = -1$

NOTES

7.3 **Matrices and Systems of Equations Homework**

Name _____

Date _____ Period _____

Problems 1 – 3, write the augmented matrix to represent each system.

1. $\begin{cases} 3x + y = 11 \\ 2x - y = 14 \end{cases}$

2. $\begin{cases} x - 4y + z = 9 \\ 3y + 4z = 10 \\ 2x + 3z = 6 \end{cases}$

3. $\begin{cases} 2x - 5y - 3z = -5 \\ x - 3y + 3z = -5 \\ 3x + 2y - 4z = -6 \end{cases}$

Problems 4 – 7, perform each row operation on the given augmented matrix.

4. $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 1 \\ 2 & -5 & 6 & 2 \\ -3 & 3 & 4 & -7 \end{array}\right]$

A.) $R_2 = -2r_1 + r_2$

5. $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 1 & -2 & -1 & 3 \\ 2 & -3 & -3 & 4 \end{array}\right]$

A.) $R_3 = r_3 + r_2$

Problems 8 – 9, the reduced row echelon form of a linear system is given. Write the system of equations corresponding to the given matrix. Determine whether the system is consistent. If it is consistent, give the solution.

8. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -2 \end{array}\right]$

9. $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Problems 10 – 15, solve each system of equations using matrix row operations. If the system has no solution, say that it is inconsistent.

10. $\begin{cases} 3x + 2y = -16 \\ 7x + 9y = -33 \end{cases}$

12. $\begin{cases} x - 2y + 3z = 11 \\ 2x + y + z = 2 \\ -3x + 2y - 2z = -14 \end{cases}$

13. $\begin{cases} 2x + y = 10 \\ -2y + 4z = 16 \\ 3x - 2z = 22 \end{cases}$

15. $\begin{cases} x + y + z = \frac{5}{2} \\ 2x - y + z = \frac{3}{2} \\ 3x + 2y = 3 \end{cases}$

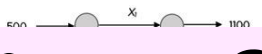
Problems 16 – 18, solve each system using a GRAPHING CALCULATOR.

Problems 19-23, write and solve a matrix for each problem.

19. At a spring carnival, children can ride a carousel for $1.50, adults pay $3.00 and senior citizens pay $2.00. On the first day of spring break, 1200 tickets were sold for a total of $2235.00. There were 380 more children riding than all of the other riders. Find the number of each ticket sold.

20. Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ that contains the following coordinates $(-2,4), (0,-2), (-1,3), (1,1)$

21. Water is being pumped through a network of PVC pipes in cubic gallons per hour, as shown in the diagram below. Find the network flow pattern when $x_2 = 800$ and $x_3 = 200$.



22. Currents in an electrical network are given by the solution of the system below where I_1, I_2, and I_3 are measured in amperes. Solve the system using matrices.

$\begin{cases} I_1 - I_2 + I_3 = 0 \\ 2I_1 + 2I_2 = 8 \\ 2I_2 + 4I_3 = 10 \end{cases}$

23. The drama department collected receipts for a production of *Arabian Nights* totaling $5500. The ticket prices were $5 for students, $7 for adults, and $4 for children under 12. Twice as many children attended as adults, and the number of adults was 20 less than one-half of the students who attended. Use matrices to find the number of tickets sold to adults, students, and children.

Practice Sets

Lesson Warm-Ups

$$\begin{cases} x - y + z = 6 \\ 2x + y - 2z = -3 \end{cases}$$

1. Write the augmented matrix.

2. Replace Row 2 with this matrix operation.
$R_2 = r_1 - r_2$

3. Use Row operations to find reduced row echelon form. Solve the system.

Grade:

$$\begin{cases} x - y - z = 0 \\ 2x + y - 2z = -3 \end{cases}$$

1. Write the augmented matrix.

2. Replace Row 1 with this matrix operation.
$R_1 = r_1 + r_2$

3. Use Row operations to find reduced row echelon form. Solve the system.

Grade:

Date _____ Period _____

quations and Matrices **Warm-Up # 1**

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11

2. Write the augmented matrix.

uced row echelon form. Solve the system.

Name _____ Date _____ Period _____

Lesson 7.3 System of Equations and Matrices **Warm-Up # 2**

1. Use the system of equations below:

$$\begin{cases} -5x + 5y + 10z = -10 \\ 3x - y - 2z = -2 \\ 3x - 6y - 6z = -24 \end{cases}$$

2. Write the augmented matrix.

3. Use row operations to find reduced row echelon form. Solve the system.

Daily Lesson Quiz