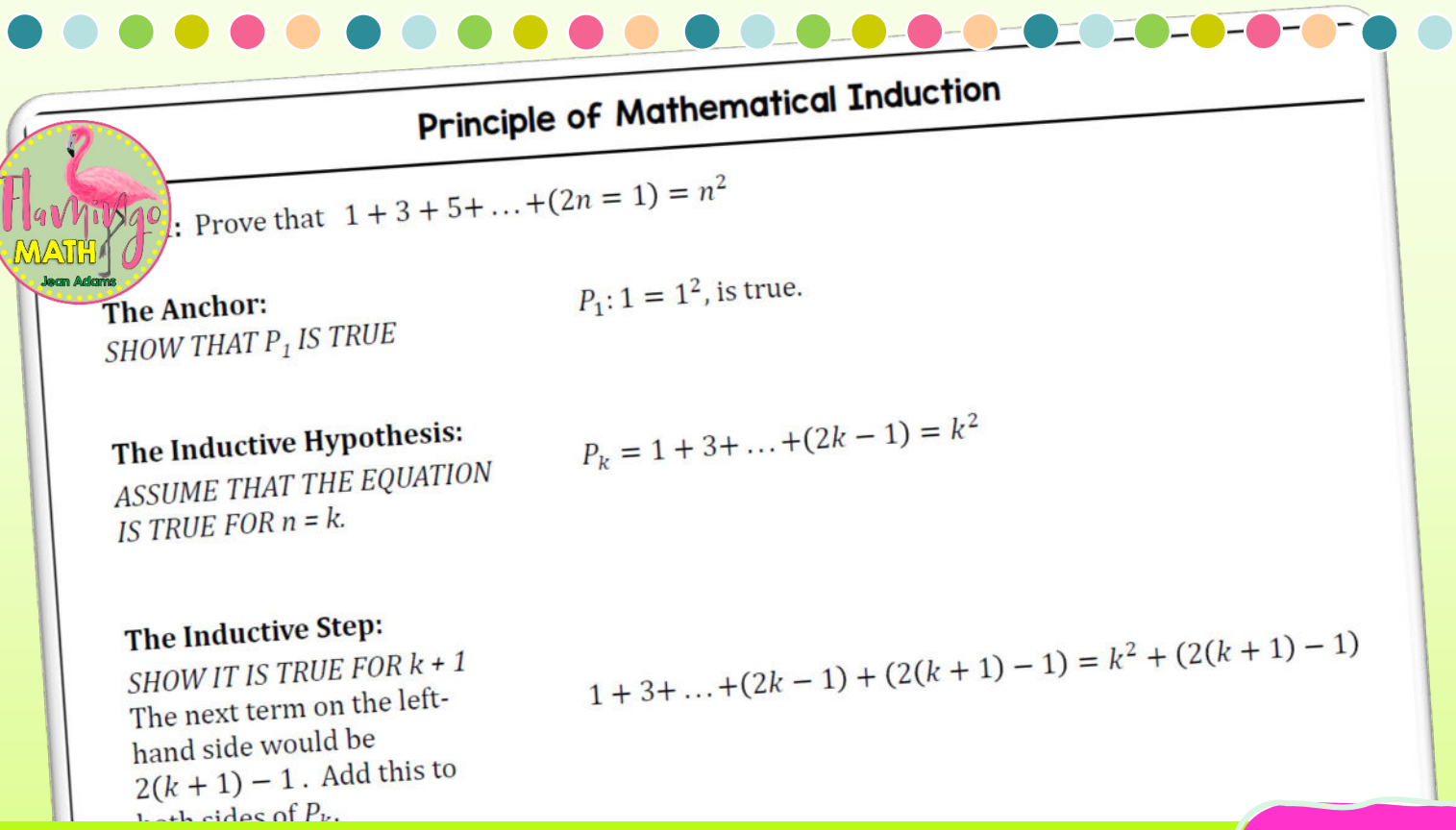
Precalculus Honors

Mathematical Induction



Completely Editable 9.4

Lesson 9.4: Mathematical Induction

In this last section we will learn how to use a principle of proof called *mathematical induction* to prove statements about the positive integers. **Induction** is reasoning from *particular examples* to draw conclusions about general principles, while **deduction** is reasoning from the *general principles* to draw conclusions about specific cases.

Mathematical Induction is a way of proving that if a hypothesis is true for one natural number it is true for the next consecutive natural number and therefore for all numbers.

The Principle of Mathematical Induction

Suppose for each positive integer n there is associated a statement P_n, then all the statements P_n are true, provided the following two conditions are satisfied"

I. P_1 is true.
II. If the statement is true for some natural number k, it is also true for the next natural number $k+1$.

Then the statement is true for all natural numbers.

The Tower of Hanoi Problem

The Tower of Hanoi Solution: The minimum number of moves required to move a stack of n washers in a Tower of Hanoi game is (2^n-1).

The Anchor:

The Inductive Hypothesis:

The Inductive Step:

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Principle of Mathematical Induction

EX #1: Prove that $1+3+5+\ldots+(2n=1)=n^2$

The Anchor:
SHOW THAT P_1 IS TRUE

P_1: $1=1^2$, is true.

The Inductive Hypothesis:
ASSUME THAT THE EQUATION IS TRUE FOR n = k.

$P_k = 1+3+\ldots+(2k-1)=k^2$

The Inductive Step:
SHOW IT IS TRUE FOR k + 1
The next term on the left-hand side would be $2(k+1)-1$. Add this to both sides of P_k.

$$1+3+\ldots+(2k-1)+(2(k+1)-1)=k^2+(2(k+1)-1)$$

Simplify the right side:

$$1+3+\ldots+(2k-1)+(2(k+1)-1)=k^2+2k+2-1$$
$$1+3+\ldots+(2k-1)+(2(k+1)-1)=k^2+2k+1$$
$$1+3+\ldots+(2k-1)+(2(k+1)-1)=(k+1)^2$$

This is exactly the statement, P_{k+1}. So, the equation is true for $n=k+1$. Therefore, P_n is true for all positive integers, by mathematical induction.

Finding the k + 1 Term

EX #2: Find P_{k+1} term for $P_k = 3k^2$

EX #3: Find P_{k+1} term for $P_k = \frac{5}{k(k+1)}$

EX #4: Find P_{k+1} term for $P_k = \frac{k^2(2k+1)}{3}$

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EX #5: Using Mathematical Induction

Prove that $1^2+2^2+3^2+\ldots+n^2=\frac{[n(n+1)(2n+1)]}{6}$

Let P_n be the statement:

The Anchor:

The Inductive Hypothesis:

The Inductive Step:

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Proving Divisibility

EX #6: Prove that 4^n-1 is evenly divisible by 3 for all positive integers n.

EX #7: Use mathematical induction to prove the formula $1+2+3+\ldots+n=\frac{n(n+1)}{2}$ for every positive integer n.

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NOTES

9.4 **Mathematical Induction Homework**

Name _____

Date _____ Period _____

Use the principle of mathematical induction to show that the given statement is true for all natural numbers, n.

1. P_n is true for $n = 1$
2. Spse P_k is true
3. Then, P_{k+1}
4. $\therefore P_n$ is true for all integers, n.

1. $1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$

2. $3 + 5 + 7 + \ldots + (2n + 1) = n(n + 2)$

3. $3 + 4 + 5 + \ldots + (n + 2) = \frac{n}{2}(n + 5)$

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4. $1 + 4 + 7 + \ldots + (3n - 2) = \frac{n}{2}(3n - 1)$

5. $2 + 5 + 8 + \ldots + (3n - 1) = \frac{n}{2}(3n + 1)$

6. $-2 - 3 - 4 - \ldots - (n + 1) = -\frac{1}{2}n(n + 3)$

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7. $1 + 2 + 2^2 + \ldots + 2^{n-1} = 2^n - 1$

8. $1 + 3 + 3^2 + \ldots + 3^{n-1} = \frac{1}{2}(3^n - 1)$

9. $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

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Practice Sets

Lesson Warm-Ups

3. Prove the statement is true for $k + 1$.	**4.** Use the given statements to find the sum of the first 20 terms.

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Name _____ Date _____ Period _____

Lesson 9.4: Mathematical Induction **Warm-Up # 2**

1. Show the statement is true for $n = 1$. Given: $3 + 5 + 7 + \cdots + (2n + 1) = n(n + 2)$	**2.** Assume the statement is true for k. Write the P_k statement.
3. Prove the statement is true for $k + 1$.	**4.** Use the given statements to find the sum of the first 20 terms.

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Form A

eriod _____

$= n^2$

$r\ n = 1$

2. Assume the statement is true for k.
Write the P_k statement.

3. Prove the statement is true for $k + 1$

Grade:

9.4 Mathematical Induction **Form B**

Name _____

Date _____ Period _____

Given: $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

1. Show the statement is true for $n = 1$

2. Assume the statement is true for k.
Write the P_k statement.

3. Prove the statement is true for $k + 1$

Grade:

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Daily Lesson Quiz