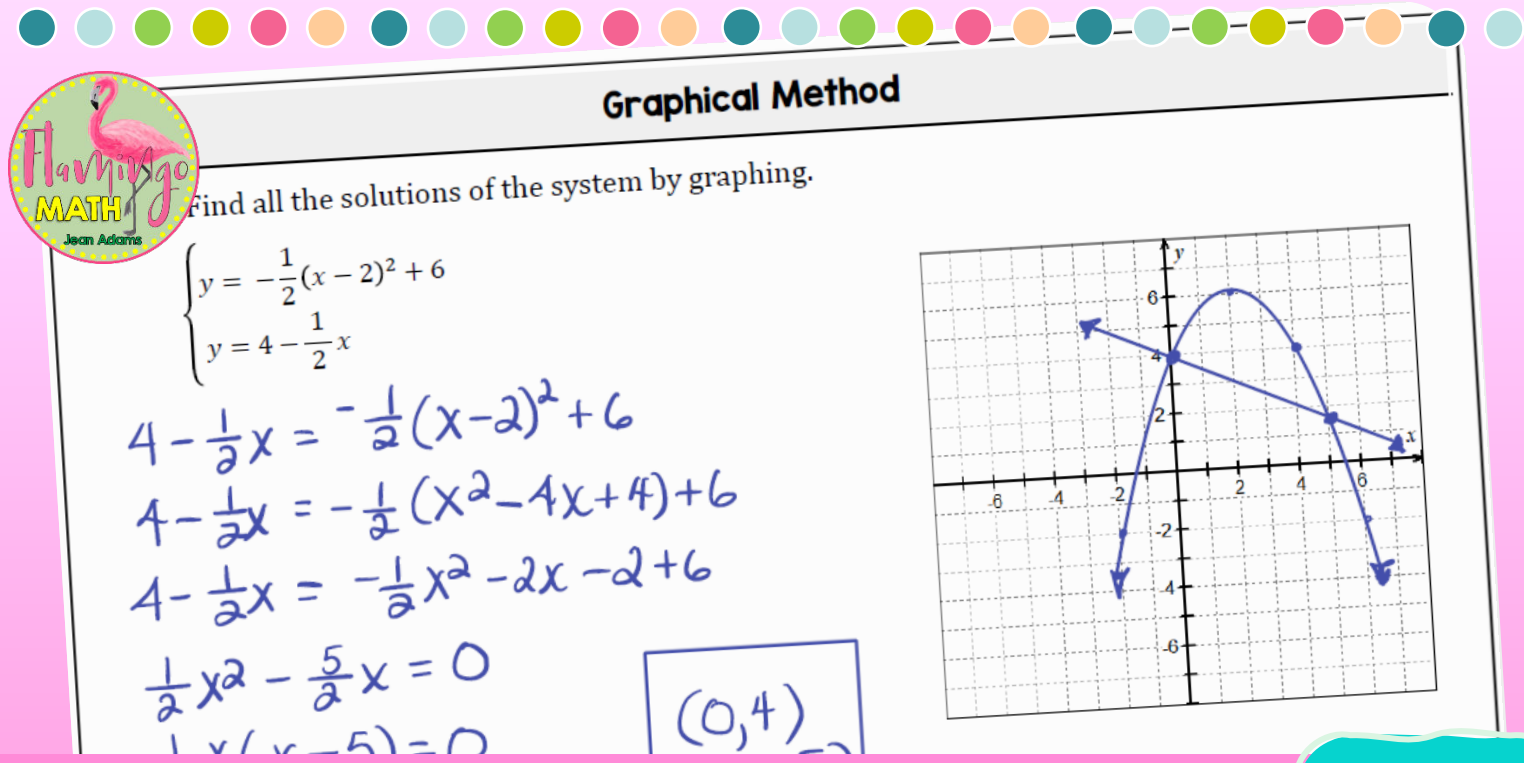
Precalculus Honors

Linear Programming



Completely Editable

7.9

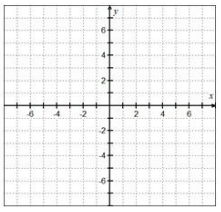
Lesson 7.9: Systems of Inequalities & Linear Programming

In this lesson we will explore systems of inequalities and applications of **optimization.** The strategy of finding minimum and maximum values of a quantity, given a set of constraints, is known as **linear programming.** A linear programming problem consists of a linear **objective function** and a system of linear inequalities called **constraints.**

Graphing Quadratic Inequalities

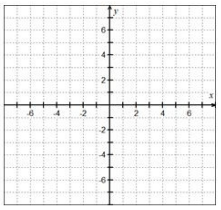
EX #1: Sketch the graph of the solution of the system.

$$\begin{cases} x - y^2 \geq 0 \\ x - y < 2 \end{cases}$$



EX #2: Sketch the graph of the solution of the system.

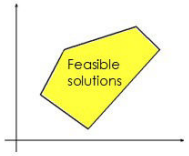
$$\begin{cases} y \geq x^2 - 4x + 3 \\ y < -x^2 + 2x + 3 \end{cases}$$



Feasible Regions

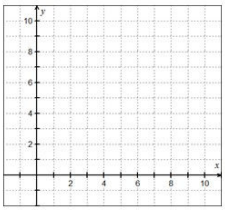
The objective function gives the quantity that is to be maximized or minimized. The constraints determine the set of **feasible solutions.** A **feasible region** is the shaded polygonal area created by the intersection of the graphs of inequalities. It is the location where every constraint is satisfied.

Any region that can be contained within a defined region or space by the vertices of a feasible region is said to be **bounded**.



EX #3: Graph the linear system. Label the vertices and state if the region is bounded or unbounded.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \leq 10 \\ 3x + 2y \leq 18 \end{cases}$$



Vertex Principle of Linear Programming

If there is a **maximum** or a **minimum** value of the linear objective function, it must occur at one or more vertices of the feasible region.

Linear Programming

EX #4: Find the minimum and maximum value, if they exist, of the objective function $f(x, y)$, subject to the constraints.

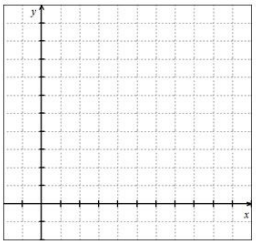
Objective function:

$f(x, y) = 5x + 4y$

Constraints:

$$\begin{cases} 2x + 3y \geq 6 \\ 3x - y \leq 15 \\ y \leq x + 4 \\ 2x + 5y \leq 27 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Vertices	5x+4y	Value



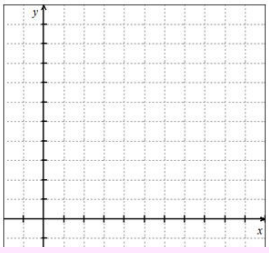
Maximizing Profits

EX #5: An advertising company manufactures promotional products. A desk organizer yields a profit of $2.50 per item, and an insulated water bottle yields a $2.00 profit per item. Demand research shows that the total units produced should be no more than 1200 per month. The number of water bottles should be less than or equal to half the number of desk organizers produced, and the desk organizers should be no more than 600 units plus, three times the number of water bottles. Find the number of each item that should be manufactured in order to maximize the profits.

A. Define variables

B. Write the objective function

C. Write constraints and graph



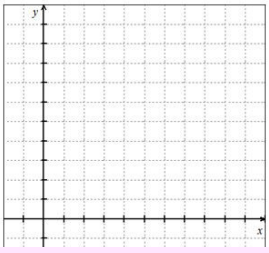
Optimal Cost

EX #6: A weight loss supplement is designed to provide at least 400 calories, 40 units of vitamin C, and 120 units of vitamin D. One scoop of supplement X costs $0.09 and provides 80 calories, 10 units of vitamin C and 20 units of vitamin D. One scoop of supplement Y costs $0.12 provides 80 calories, 5 units of vitamin C and 30 units of vitamin D. How many scoops of each supplement should be consumed each day to obtain an optimal cost and still meet the daily requirements?

A. Define variables

B. Write the objective function

C. Write constraints and graph



D. Find vertices and minimum cost

NOTES

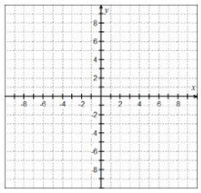
7.9 **Systems of Inequalities & Linear Programming Homework**

Name _____

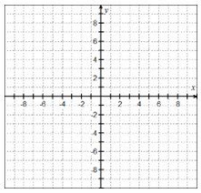
Date _____ Period _____

Problems 1 – 6, graph each system of inequalities.

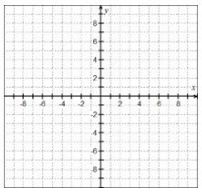
1. $\begin{cases} 4x + 3y < -6 \\ 2x - y \leq -8 \end{cases}$



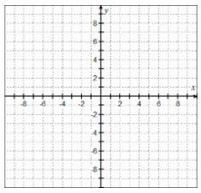
2. $\begin{cases} x^2 + y^2 \leq 25 \\ 3x - 4y \leq 0 \end{cases}$



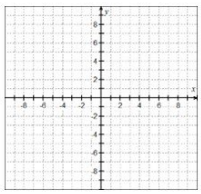
3. $\begin{cases} y < x^2 \\ x^2 + y^2 \leq 25 \end{cases}$



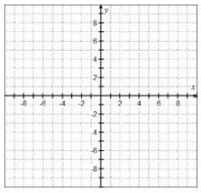
4. $\begin{cases} |x| < 2 \\ 4x + 2y \leq 6 \end{cases}$



5. $\begin{cases} x + 2y < 6 \\ y \geq x^2 - 2 \end{cases}$



6. $\begin{cases} x^2 + y^2 < 25 \\ y \geq |x| - 3 \end{cases}$

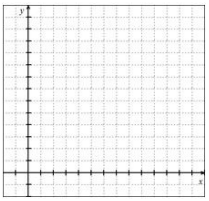


Problems 7 – 8, solve each linear programming problem.

7. Maximize $f(x, y) = 5x + 3y$

Constraints:

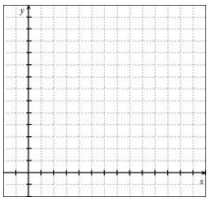
$\begin{cases} x \geq 0 \\ y \geq 3 \\ 2x + 3y \geq 12 \\ 2x + y \leq 12 \end{cases}$



8. Maximize: $f(x, y) = 3x + 5y$

Constraints:

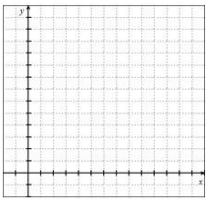
$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 8 \\ y \leq 6 \\ 2x + y \geq 4 \\ x + 2y \leq 16 \end{cases}$



9. Minimize: $f(x, y) = 3x + 4y$

Constraints:

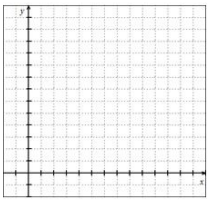
$\begin{cases} x \geq 0 \\ x \leq 8 \\ x - 2y \leq 2 \\ 3x + y \geq 7 \\ x + y \geq 5 \end{cases}$



10. Minimize: $f(x, y) = 2x + 5y$

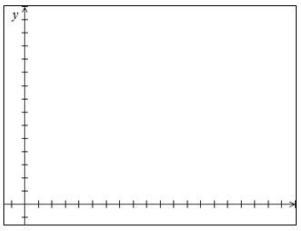
Constraints:

$\begin{cases} x \geq 0 \\ 2y \geq x \\ x + y \geq 3 \\ 2x + y \leq 10 \\ y \leq 6 \end{cases}$

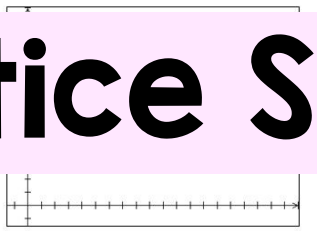


Problems 11 – 14, solve.

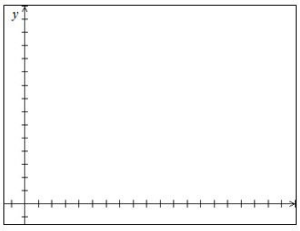
11. Your school store carries two types of t-shirts. The first style has your school logo on the front and costs $3.50 per shirt. The second costs $4.50 per shirt and has the logo on the front and your mascot on the back. You are in charge of ordering from the wholesaler and have a maximum budget of $900. You have storage room for more than 144 shirts. You need to have at least 4 dozen of the single logo shirts and 3 dozen of the double logo style. Write and graph a system of inequalities that describe all the possibilities for stocking the school store with shirts.



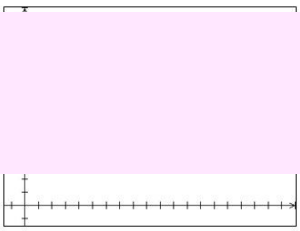
12. Your drama club is promoting the spring play with plans to sell 600 tickets. Adult tickets will sell for $8 and student tickets will cost $5. The club needs to have gross receipts of $3,000. They need to sell at least 225 student tickets and 125 adult tickets. Write a system of inequalities that describes all the possibilities for the number of tickets that can be sold to meet the conditions. Sketch the system.



13. Camp Tenabow is hiring counselors and assistants to fill the staff for summer. The average monthly salary of a counselor is $2400 and the average assistant earns $1100. The camp needs 30 staff members to run smoothly and can accommodate at most 45 employees. State regulations require at least 10 assistants. They may have 3 assistants for every 2 counselors. How many counselors and assistants should the camp hire in order to minimize costs?



14. A furniture manufacturer can make between 30 and 60 tables per day and between 40 and 100 chairs per day. It can make at most 120 units in one day. The profit on a table is $250, and the profit on a chair is $75. How many tables and chairs should they make per day to maximize profit? How much is the maximum profit?

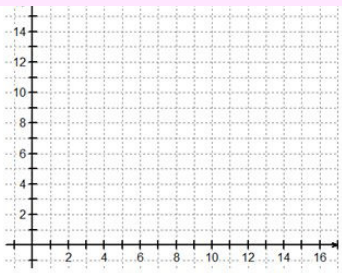


Practice Sets

Lesson Warm-Ups

$x \geq 0$
$y \geq 0$

2. Find the minimum and maximum for the feasible region in #1, if the objective function is $f(x, y) = 5x + 2y$



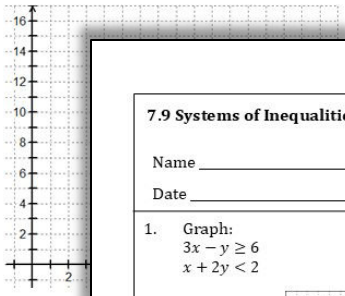
Name _____ Date _____ Period _____

Lesson 7.9 Linear Programming **Warm-Up # 2**

Graph: $\begin{cases} x + 3y \geq 9 \\ 2x + 3y \geq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$

2. Find the minimum and maximum for the feasible region in #1, if the objective function is $f(x, y) = 5x + 2y$

1.

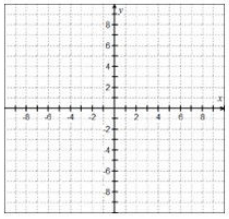


7.9 Systems of Inequalities **Form A**

Name _____

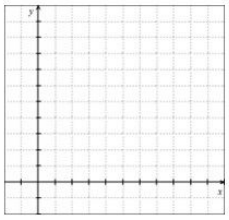
Date _____ Period _____

1. Graph:
$3x - y \geq 6$
$x + 2y < 2$



2. Graph:

$\begin{cases} 2x + y \geq 12 \\ 2x + 3y \geq 24 \\ x \geq 0 \\ y \geq 0 \end{cases}$



3. Find the minimum and maximum for the feasible region above if the objective function is $f(x, y) = 5x + 2y$

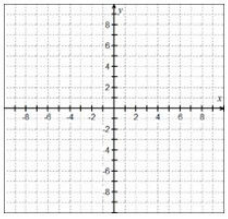
Grade:

7.9 Systems of Inequalities **Form B**

Name _____

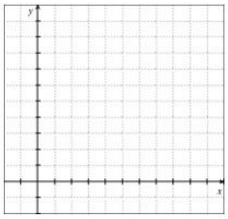
Date _____ Period _____

1. Graph:
$3x + 2y \leq 6$
$2x - 3y < 0$



2. Graph:

$\begin{cases} x + 2y \geq 8 \\ 3x + 2y \geq 16 \\ x \geq 0 \\ y \geq 0 \end{cases}$



3. Find the minimum and maximum for the feasible region above if the objective function is $f(x, y) = 3x + 5y$

Grade:

Daily Lesson Quiz