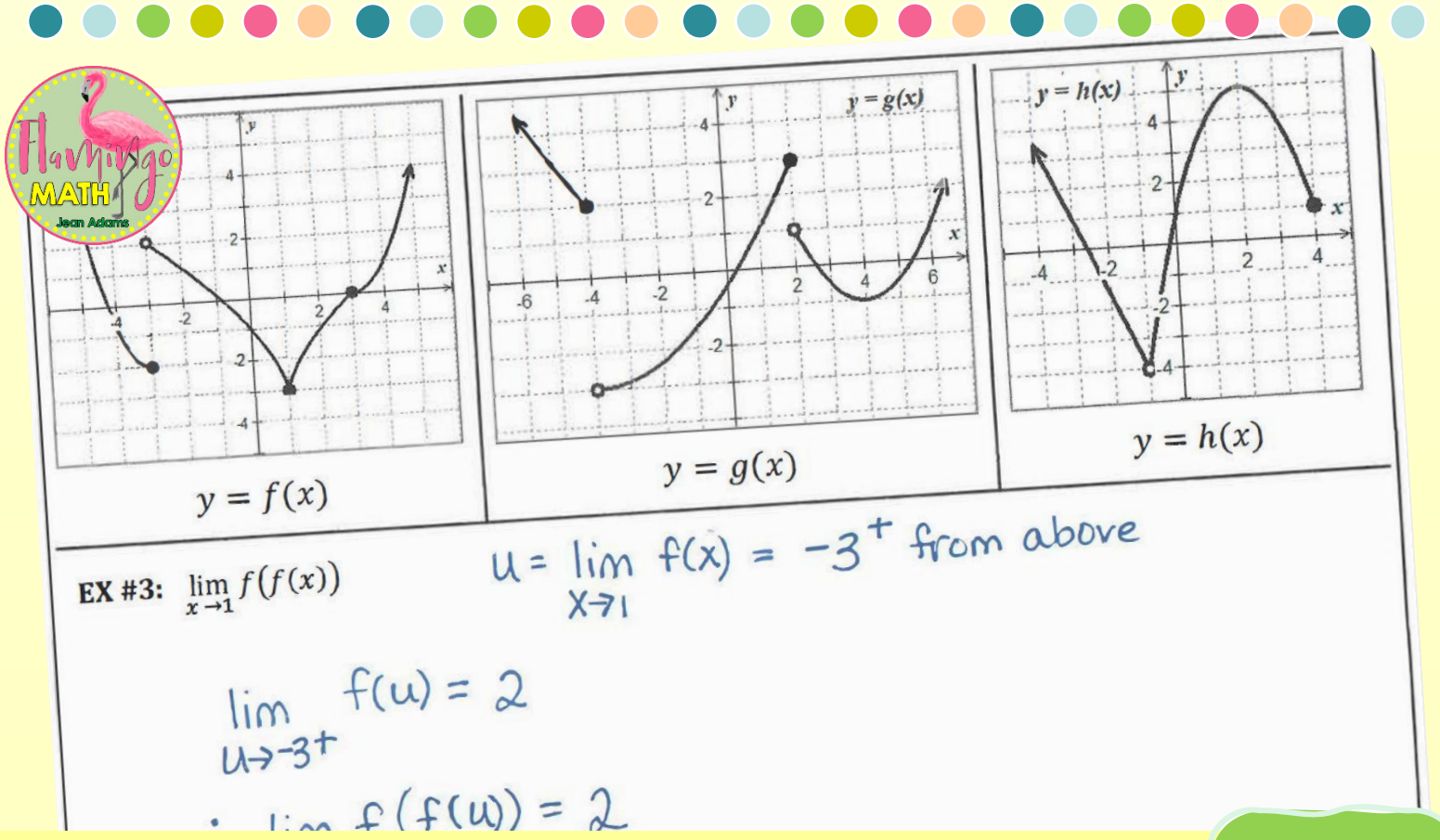
AP® Calculus

Limits of Composite Functions



Completely Editable

1.4

Lesson 4: Limits of Composite Functions

Topic 1.7: Selecting Procedures for Determining Limits
Topic 1.9: Connecting Multiple Representations of Limits

The limit of a composition is the composition of the limits, provided the outside function is continuous at the limit of the inside function. We often see this IF-THEN STATEMENT presented as follows:

$$f \text{ and } g \text{ are functions such that } \lim_{x \to c} g(x) = L \text{ and } \lim_{x \to L} f(x) = f(L) \quad \text{then}$$

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(L)$$

or stated as,

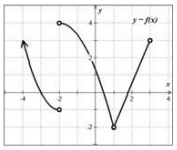
$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) \text{ provided } \lim_{x \to c} g(x) = L \text{ and } f(x) \text{ is continuous at } x = L$$

We should notice this IF-THEN statement requires certain conditions to be met. Problems occur when $f(x)$ is not continuous at $x = L$ or the existence of the limit of $g(x)$ at $x = a$. At issues of this sort, we can't apply the property. You might assume that the limit doesn't exist, but this may or may not be true. So, we will explore some alternative plan to find another approach.

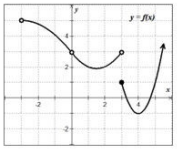
In this case, let's apply the "*if and only if*" definition of a limit:

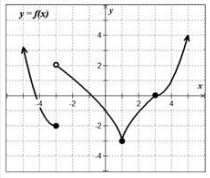
$$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x) \text{ then } \lim_{x \to c} f(x) = L$$

EX #1: $\lim_{x \to 1} f(f(x))$

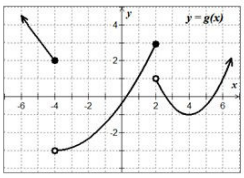


EX #2: $\lim_{x \to 0} f(f(x))$

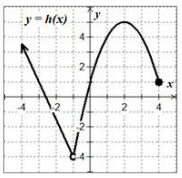




$y = f(x)$



$y = g(x)$

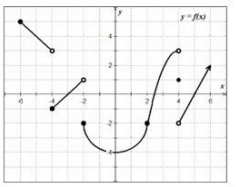


$y = h(x)$

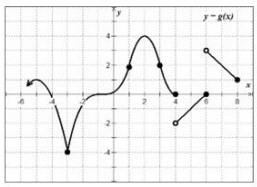
EX #3:

EX #4:

EX #5:



$y = f(x)$



$y = g(x)$

EX #6:

EX #7:

EX #8:

EX #9:

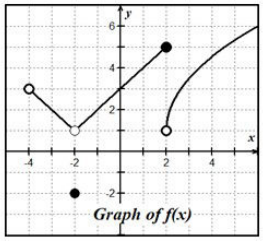
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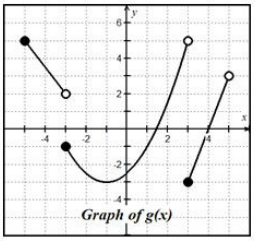
1.4 **Limits of Composite Functions**

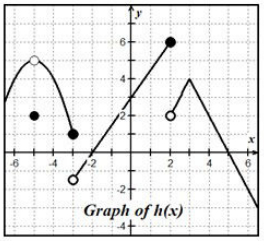
Name _____

Date _____ Period _____

Problems 1 – 8, Use the graphs of $f(x0, g(x)$ and $h(x)$ shown below to answer the following limit questions.







1. $\lim_{x \to -1} [f(x) + g(x)]$

2. $\lim_{x \to 0} [f(x) \cdot h(x)]$

3. $\lim_{x \to -2} f(h(x))$

4. $\lim_{x \to 1} \sqrt{f(x)}$

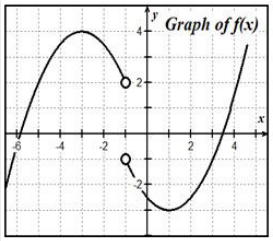
5. $\lim_{x \to 0} h(|x| + 3)$

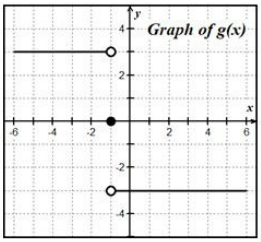
6.

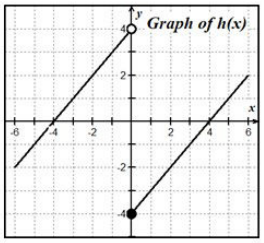
7. $\lim_{x \to -5} g(x + 2)$

8.

Problems 9 – 14, use the graphs of f, g and h below to evaluate the limits, if they exist.







9. $\lim_{x \to 5^+} g(25 - x^2)$

10. $\lim_{x \to -1} (g(x))^2 - 5$

11. $\lim_{x \to 0} \frac{h(x)}{h(-x)}$

12. $\lim_{x \to 0} h(h(x))$

13. $\lim_{z \to -1} f(g(x))$

14. $\lim_{x \to -2} f\left(g(h(x))\right)$

$\begin{cases} 3x, & x = 0 \end{cases}$ $\begin{cases} -3x, & x < 0 \end{cases}$

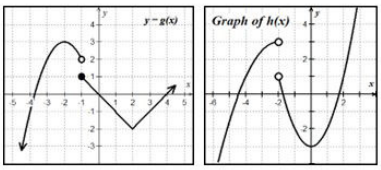
Practice Sets

1.4 Limits of Composite Functions **Form A**

Name _____

Date _____ Period _____

Use the graphs of $g(x)$ and $h(x)$ to answer the questions.



1. $\lim_{x\to 1}[g(x)+h(x)]$

2. $\lim_{x\to -1} g(h(x))$

3. $\lim_{x\to -1^-}\left[\frac{h(-x)}{g(x)}\right]$

4. $\lim_{x\to 2}[x^3+g(x-2)]$

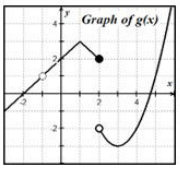
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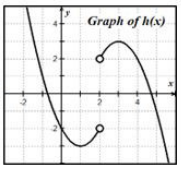
1.4 Limits of Composite Functions **Form B**

Name _____

Date _____ Period _____

Use the graphs of $g(x)$ and $h(x)$ to answer the questions.





1. $\lim_{x\to 3}[g(x)-h(x)]$

2. $\lim_{x\to 3} g(h(x))$

3. $\lim$

4. $\lim$

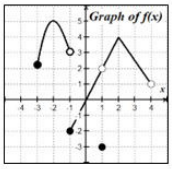
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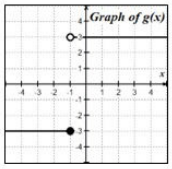
1.4 Limits of Composite Functions **Form C**

Name _____

Date _____ Period _____

Use the graphs of $f(x)$ and $g(x)$ to answer the questions.





1. $\lim_{x\to 2}[f(x)+g(x)]$

2. $\lim_{x\to 1} f(g(x))$

3. $\lim_{x\to -1^-}\left[\frac{g(x-1)}{f(x)}\right]$

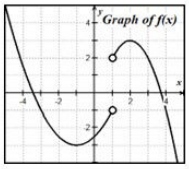
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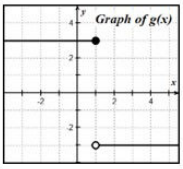
1.4 Limits of Composite Functions **Form D**

Name _____

Date _____ Period _____

Use the graphs of $f(x)$ and $g(x)$ to answer the questions.





1. $\lim_{x\to -1}[f(x)-g(x)]$

2. $\lim_{x\to -2} f(g(x))$

3. $\lim_{x\to 1^-}\left[\frac{g(-x)}{f(x)}\right]$

Grade:

Daily Lesson Quiz