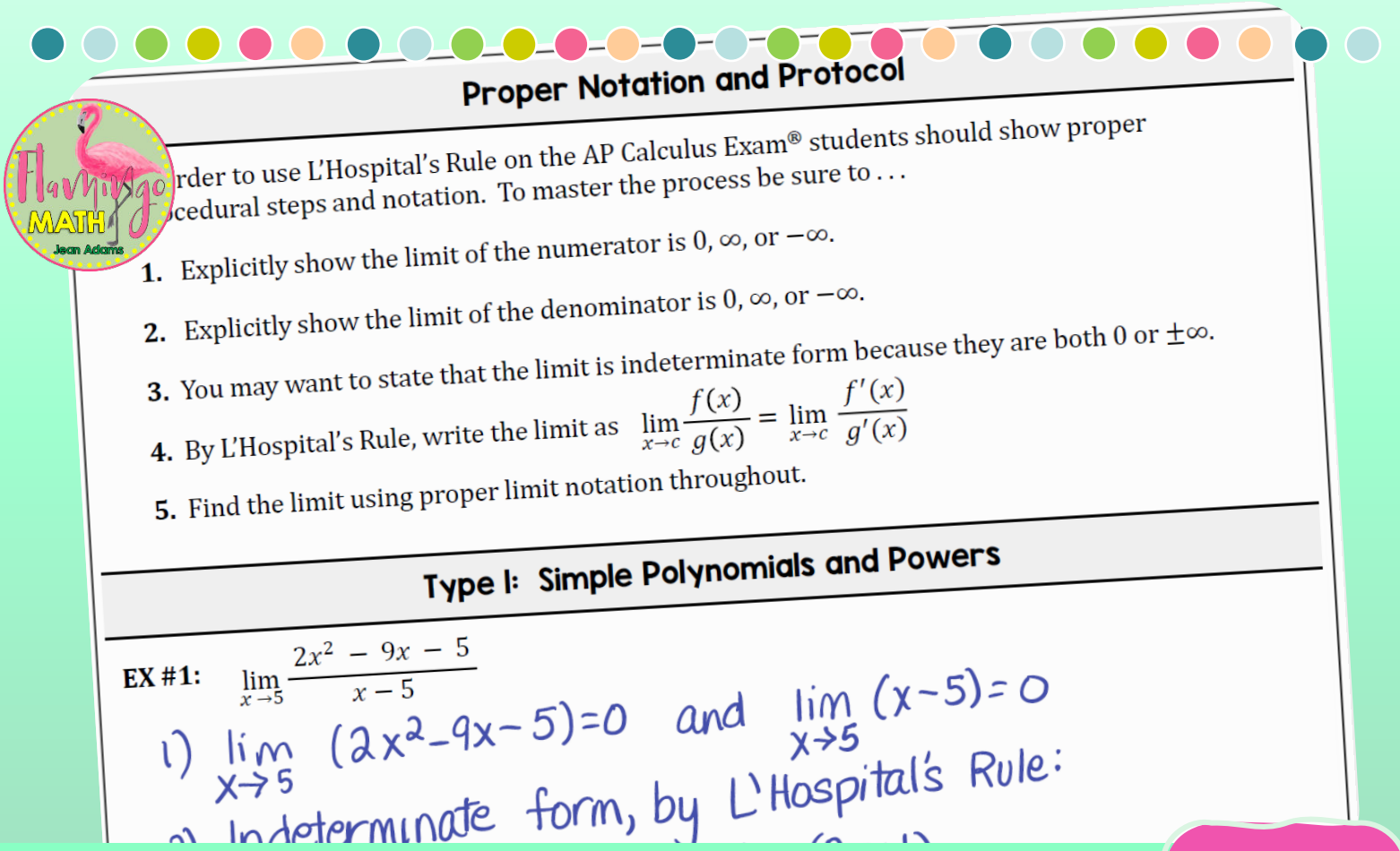
AP® Calculus

L'Hospital's Rule



Proper Notation and Protocol

rder to use L'Hospital's Rule on the AP Calculus Exam® students should show proper cedural steps and notation. To master the process be sure to . . .

1. Explicitly show the limit of the numerator is 0, ∞, or −∞.
2. Explicitly show the limit of the denominator is 0, ∞, or −∞.
3. You may want to state that the limit is indeterminate form because they are both 0 or ±∞.
4. By L'Hospital's Rule, write the limit as $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$
5. Find the limit using proper limit notation throughout.

Type I: Simple Polynomials and Powers

EX #1: $\lim_{x\to 5}\frac{2x^2-9x-5}{x-5}$

1) $\lim_{x\to 5}(2x^2-9x-5)=0$ and $\lim_{x\to 5}(x-5)=0$

2) Indeterminate form, by L'Hospital's Rule:

Completely Editable

4.6

Lesson 6: L'Hospital's Rule

Topic 4.7: Using L'Hospital's Rule for Determining Limits of Indeterminate Forms

L'Hospital's Rule is a great tool for finding limits that are often difficult to evaluate. It can also help with determining limits at infinity for "asymptotic behavior." We will use it for curve sketching in the next section, as well.

When evaluating limits by direct substitution, there are seven expressions known as **indeterminate form**. These expressions are

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 1^{\infty}, 0^{0}, \infty - \infty, \infty^{0}$$

Consider the limit of a quotient: $\lim_{x \to c} \frac{f(x)}{g(x)}$.

L'Hospital's Rule states that *when $f(x)/g(x)$ has an indeterminate form of the type $0/0$ or ∞/∞ at $x = c$, then we can replace $f(x)/g(x)$ by the quotient of the derivatives $f'(x)/g'(x)$.*

L'Hospital's Rule

Assume $f(x)$ and $g(x)$ are differentiable on an open interval (a, b) containing c. Also assume that $g'(x) \neq 0$ for all x in (a, b); except possibly at c itself. Then, if

$\lim f(x) = 0$ and $\lim g(x) = 0$; then

Proper Notation and Protocol

In order to use L'Hospital's Rule on the AP Calculus Exam® students should show proper procedural steps and notation. To master the process be sure to . . .

1. Explicitly show the limit of the numerator is 0, ∞, or −∞.
2. Explicitly show the limit of the denominator is 0, ∞, or −∞.
3. You may want to state that the limit is indeterminate form because they are both 0 or ±∞.
4. By L'Hospital's Rule, write the limit as $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$
5. Find the limit using proper limit notation throughout.

Type I: Simple Polynomials and Powers

EX #1: $\lim_{x \to 5} \frac{2x^2 - 9x - 5}{x - 5}$

Type 2: Repeated Use of L'Hospital's Rule

EX #3: $\lim_{x \to -\infty} \frac{x^2}{e^{-x}}$

EX #4: $\lim_{x \to -2} \frac{x^3 + x^2 - 8x - 12}{x^3 + 8x^2 + 20x + 16}$

Type 3: Trigonometry and Transcendental Functions

EX #5: $\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{x^2}$

EX #6: $\lim_{x \to 1} \frac{\ln x - x + 1}{e^x - e}$

EX #8: $\lim_{x \to \infty} \frac{4x^2 - 3x + 2}{e^{2x} + \ln x}$

Type 5: AP Style Applications of L'Hospital's Rule

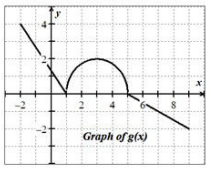
EX #9: The table below gives values of a twice-differentiable function $y = f(x)$. Use the table to find the limit, if it exists. Justify your answer.

$$\lim_{x \to -2} \frac{f(3x + 2)}{x^2 - 4}$$

x	-4	-1	2
$f(x)$	0	3	0
$f'(x)$	8	-2	6

EX #10: Use the graph of $y = g(x)$, shown below to find the following limit, if possible. Use proper notation to justify your findings.

$$\lim_{x \to 3} \frac{g(x)}{x - 3}$$



EX #11: The differential equation for the curve $y = h(x)$ is given by $\frac{dy}{dx} = \frac{3x}{y + 2}$. Given that h is twice-differentiable and $h(3) = -1$, find:

$$\lim_{x \to 3} \frac{h(x) + x - 2}{\ln(7 - 2x)}$$

NOTES

4.6 **L'Hospital's Rule Homework**

Name _____

Date _____ Period _____

Problems 1-18, Use L'Hospital's Rule to evaluate the limit, if it exists, or state that L'Hospital's Rule does not apply.

1. $\lim_{x\to -3} \frac{x^2-9}{6-x-x^2}$

2. $\lim_{x\to 2} \frac{3x^2-5x-2}{x-3}$

3. $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x^3-4x+3}$

4. $\lim_{x\to 0} \frac{\sin 5x}{2x}$

5. $\lim_{x\to \infty} \frac{5x-x^2}{3x^2+2x-4}$

6.

7. $\lim_{x\to 0} \frac{\sin 3x}{\sin 5x}$

8.

9. $\lim_{x\to 0} \frac{x^2}{1-\cos x}$

10. $\lim_{x\to \infty} \frac{e^{2x}-x-1}{x^2}$

11. $\lim_{x\to 0} \frac{\tan x}{x}$

12. $\lim_{x\to 0} \frac{e^x-2}{\sin x}$

13.

14. $\lim_{x\to 0} \frac{x^3}{\sin x - x}$

15. $\lim_{x\to 1} \frac{\tan^{-1} x - \frac{\pi}{4}}{x-1}$

16. $\lim_{x\to 1} \frac{\ln x^2}{x^2-1}$

17. $\lim_{x\to \infty} \frac{x}{\sqrt{x^2+1}}$

18. $\lim_{x\to \infty} \frac{\ln x}{\sqrt{x}}$

Questions 19 – 22, Multiple Choice Practice

19. Find $\lim_{x\to 1^+} \left(\frac{x}{\ln x}\right)$

A. 0 B. 1

C. e D. $+\infty$

20. What is $\lim_{x\to 0} \frac{e^x-1}{\tan x}$

A. -1 B. 0

C. 1 D. The limi

21. If $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} g(x) = +\infty$, and $f'(x) =$

A. 0 B. 1

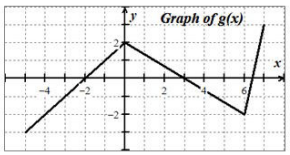
C. e D. The limit

22. Evaluate.

Problems 23-25, For each of the following, find the limit, if it exists, and justify your reasoning.

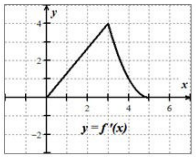
23. Use the graph of $g(x)$, shown at right to find the limit, if it exists

$\lim_{x\to -2} \frac{g(x+5)}{x^2+3x+2}$



24. The function $f(x)$ is differentiable and the graph of $f'(x)$ is shown at right. Given that $f(3) = 0$, find the following. Justify your reasoning.

$\lim_{x\to 3} \frac{x^2-9}{f(x)}$



Practice Sets

4.6 L'Hospital's Rule **Form A**

Name _____

Date _____Period _____

Find each limit below. Use L'Hospital's Rule, if necessary.

1. $\lim_{x\to 0} \frac{3\sin(2x)}{5x}$

2. $\lim_{x\to \infty} \frac{e^x}{x^2}$

3. $\lim_{x\to \infty} \frac{3x+13}{12-4x}$

4. $\lim_{x\to 1} \frac{\ln(x)+x-1}{2-2x^2}$

Grade:

4.6 L'Hospital's Rule **Form B**

Name _____

Date _____Period _____

Find each limit below. Use L'Hospital's Rule, if necessary.

1. $\lim_{x\to 0} \frac{5\tan(3x)}{2x}$

2. $\lim_{x\to \infty} \frac{x^3}{e^x}$

3. $\lim_{x\to -\infty} \frac{1-6x}{3+5x}$

4.6 L'Hospital's Rule **Form C**

Name _____

Date _____Period _____

Find each limit below. Use L'Hospital's Rule, if necessary.

1. $\lim_{x\to 0} \frac{4\sin(2x)}{3x}$

2. $\lim_{x\to -\infty} \frac{e^x}{x^3}$

3. $\lim_{x\to \infty} \frac{2x^2+15}{3x^2-9}$

4. $\lim_{x\to 1} \frac{x^2+5x-6}{\ln(x)}$

Grade:

4.6 L'Hospital's Rule **Form D**

Name _____

Date _____Period _____

Find each limit below. Use L'Hospital's Rule, if necessary.

1. $\lim_{x\to 0} \frac{6\tan(3x)}{4x}$

2. $\lim_{x\to \infty} \frac{x^3}{e^x}$

3. $\lim_{x\to -\infty} \frac{3x^3+7}{11-5x^3}$

4. $\lim_{x\to 1} \frac{x^2+3x-4}{\ln(x)}$

Grade:

Daily Lesson Quiz