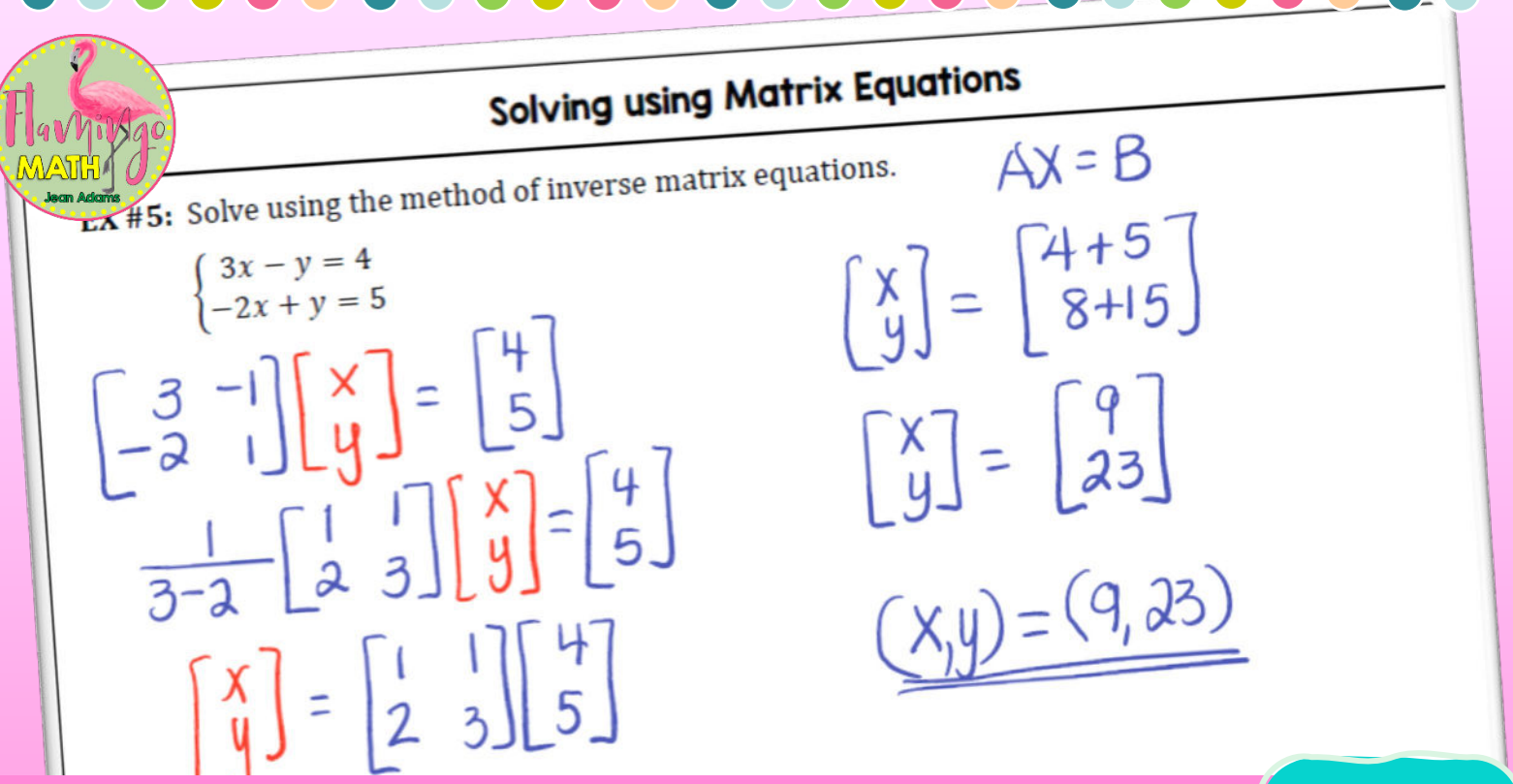
Precalculus Honors

Inverses of Matrices & Matrix Equations



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7.5

Lesson 7.5: Inverses of Matrices and Matrix Equations

We have explored **Gaussian elimination** in a previous lesson. This method is used to transform a system of equations into triangular form. Named after German mathematician Carl Friedrich Gauss (1777-1855).

$$\begin{aligned} x + y - z &= 1 \\ -y + z &= 5 \\ z &= 3 \end{aligned}$$

When using row operations to obtain triangulation of the matrix is called **reduced row echelon form** when the solutions are obtained.
In this lesson we will explore solving systems by using inverse functions.

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

The Inverse of a Matrix

Consider the real number equation $ax = b$. To solve this equation for x, multiply each side of the equation by the multiplicative inverse of a, that is, a^{-1} $(a \neq 0)$. This general idea is similar in matrix operations.

Definition of the Inverse of a Square Matrix

Let A be a square $n \times n$ matrix. If there exists an n × n matrix, A^{-1}, read "A inverse," where $AA^{-1} = A^{-1}A = I$, then A^{-1} is called the inverse of matrix A.

It should be noted that it is not always true that $AB = BA$. If A and B are both square matrices and $AB = I$, it can be shown that $BA = I$.

EX #1: Show that B is the inverse of A by showing $AB = I$ and $BA = I$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

Finding Inverse Matrices

A matrix A is called **invertible** (or **nonsingular**) if the matrix has an inverse, otherwise A is called **singular**. A nonsquare matrix cannot have an inverse. Not all square matrices have inverses, but if a matrix does have an inverse, that inverse is unique.

Also consider, if matrix A is of order $m \times n$ and matrix B is of order $n \times m$ where $m \neq n$, the products AB and BA are different orders and cannot be equal to each other.

Finding an Inverse Matrix

Let A be a square matrix of order n.

1. Write the $n \times 2n$ matrix that consists of the given matrix A on the left and the $n \times n$ identity matrix I on the right to obtain $[A \vdots I]$.
2. If possible, use elementary row operations on the entire matrix. This result will be the matrix $[I \vdots A^{-1}]$. If this isn't possible, then A is not invertible.
3. Check your result that $AA^{-1} = I = A^{-1}A$

EX #2: Find the inverse of matrix A.

$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

Inverse of a 2 x 2 Matrix

Given $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, if $ad - bc \neq 0$, then the inverse matrix

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The number $ad - bc$ is called the **determinant of matrix A**.

EX #3: Find the inverse of the matrix B.

$B = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

Find an Inverse Matrix with a Calculator

EX #4: Find the inverse of matrix A.

$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 0 & 2 & -2 \end{bmatrix}$

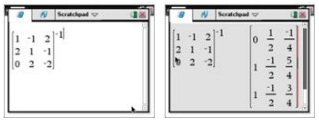
With the TI-Inspire, pick the template and set the matrix size:



Enter the elements in each row and column:



Calculate the inverse:



Now verify that $AA^{-1} = A^{-1}A = I$. Explain.

NOTE: If you receive "error Singular Matrix" this indicates there is no inverse possible.

Solving Systems with Inverse Matrices

$X = A^{-1}B$ Property of Identity Matrix

Solving using Matrix Equations

EX #5: Solve using the method of inverse matrix equations.

$$\begin{cases} 3x - y = 4 \\ -2x + y = 5 \end{cases}$$

Free Response Question

EX #6: The hourly cost of a security company to guard a warehouse and outside fenced area depends on the hourly cost x in dollars of using a security guard and the hourly salary y of using a canine companion. The cost for three security guards and two canine is $285. The hourly cost for a team of four guards with four canine is $480.

A. Set up a matrix equation and

B. Solve the equation.

C. Explain your findings in context to the problem.

NOTES

7.5 **Inverses of Matrices and Matrix Equations Homework**

Name _____

Date _____ Period _____

Problems 1-2, show that B is the inverse of A.

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

2. $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$

Problems 3-7, find the inverse of the matrix, if it exists using the $AX = I$ method.

3. $\begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 3 & 7 \\ 2 & -3 & -1 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 2 \\ 5 & 9 \end{bmatrix}$

6. $\begin{bmatrix} 1 & -2 \\ 5 & -2 \end{bmatrix}$

7. $\begin{bmatrix} 3 & 5 & 4 \\ 1 & 1 & 1 \\ 3 & 6 & 5 \end{bmatrix}$

Problems 8-9, use the matrix feature of a graphing utility to find the inverse of the matrix, if possible.

8. $\begin{bmatrix} 12 & 4 & 8 \\ 5 & 2 & 4 \\ 9 & 4 & 6 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 4 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 3 \end{bmatrix}$

Problems 10-11, use the formula for a 2×2 matrix to find the inverse.

10. $\begin{bmatrix} 5 & -2 \\ 1 & 2 \end{bmatrix}$

11. $\begin{bmatrix} 9 & -24 \\ 12 & 32 \end{bmatrix}$

Problems 12-13, find the value of the constant k such that $B = A^{-1}$.

12. $A = \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 1 & k \end{bmatrix}$

13. $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} k & \frac{1}{7} \\ -\frac{1}{7} & \frac{3}{7} \end{bmatrix}$

Problems 14-15, solve by using matrix equations method.

14. $\begin{cases} 3x + 5y = 3 \\ 15x - 5y = 21 \end{cases}$

15. $\begin{cases} 3x - 2y + z = -5 \\ x + y - z = 6 \\ x + 3y - 2z = 14 \end{cases}$

Problems 16-17, solve using matrix operations and equations

16. A chocolate factory sells a 2-pound box of assorted chocolates for $12.50 that contains three different varieties of chocolate. Dark Chocolate, d, costs $7 per pound, milk chocolate, m, costs $5 per pound and white chocolate, w, costs $4 per pound. The box contains the same amounts of milk chocolate and white chocolate. Write a system of linear equations that represents the scenario. Write a matrix equation that corresponds to your system. Then, solve the system using an inverse matrix.

17. Grains are sold by the bushel at a farmer's market. Casey bought 4 wheat, 3 corn and 4 rye for $31.30, Kevin bought 3 wheat, 2 corn and 4 rye for $26.25. Rachel bought 3 wheat, 4 corn and 3 rye for $29.25. Write a matrix equation to represent the purchases. Then solve the system using an inverse matrix equation

Practice Sets

Lesson Warm-Ups

2. Find the value of the constant k such that $B = A^{-1}$.

$$A = \begin{bmatrix} 8 & -2 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{12} & \frac{1}{6} \\ -\frac{1}{6} & k \end{bmatrix}$$

3. Solve by using matrix equations method.

$$\begin{cases} 3x + 5y = 9 \\ 8x - 5y = -31 \end{cases}$$

Grade:

2. Find the value of the constant k such that $B = A^{-1}$.

$$A = \begin{bmatrix} 9 & 2 \\ 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{3} & k \\ -1 & 3 \end{bmatrix}$$

3. Solve by using matrix equations method.

$$\begin{cases} 5x - 4y = 31 \\ 7x + 4y = 5 \end{cases}$$

Grade:

Name _____ Date _____ Period _____

Lesson 7.5 Inverses of Matrices **Warm-Up # 1**

1. Find the inverse of the matrix.

$$\begin{bmatrix} 5 & -4 \\ 3 & -3 \end{bmatrix}$$

2. Find the value of the constant k such that $B = A^{-1}$.

$$A = \begin{bmatrix} 8 & 3 \\ 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{5} & k \\ -\frac{1}{5} & \frac{4}{5} \end{bmatrix}$$

3. Solve by using matrix equations method.

$$\begin{cases} 9x - 2y = 5 \\ 3x + 7y = 17 \end{cases}$$

Name _____ Date _____ Period _____

Lesson 7.5 Inverses of Matrices **Warm-Up # 2**

1. Find the inverse of the matrix.

$$\begin{bmatrix} 3 & 4 \\ 2 & 6 \end{bmatrix}$$

2. Find the value of the constant k such that $B = A^{-1}$.

3. Solve by using matrix equations method.

$$\begin{cases} 4x + 3y = -16 \\ -x + y = 4 \end{cases}$$

Daily Lesson Quiz