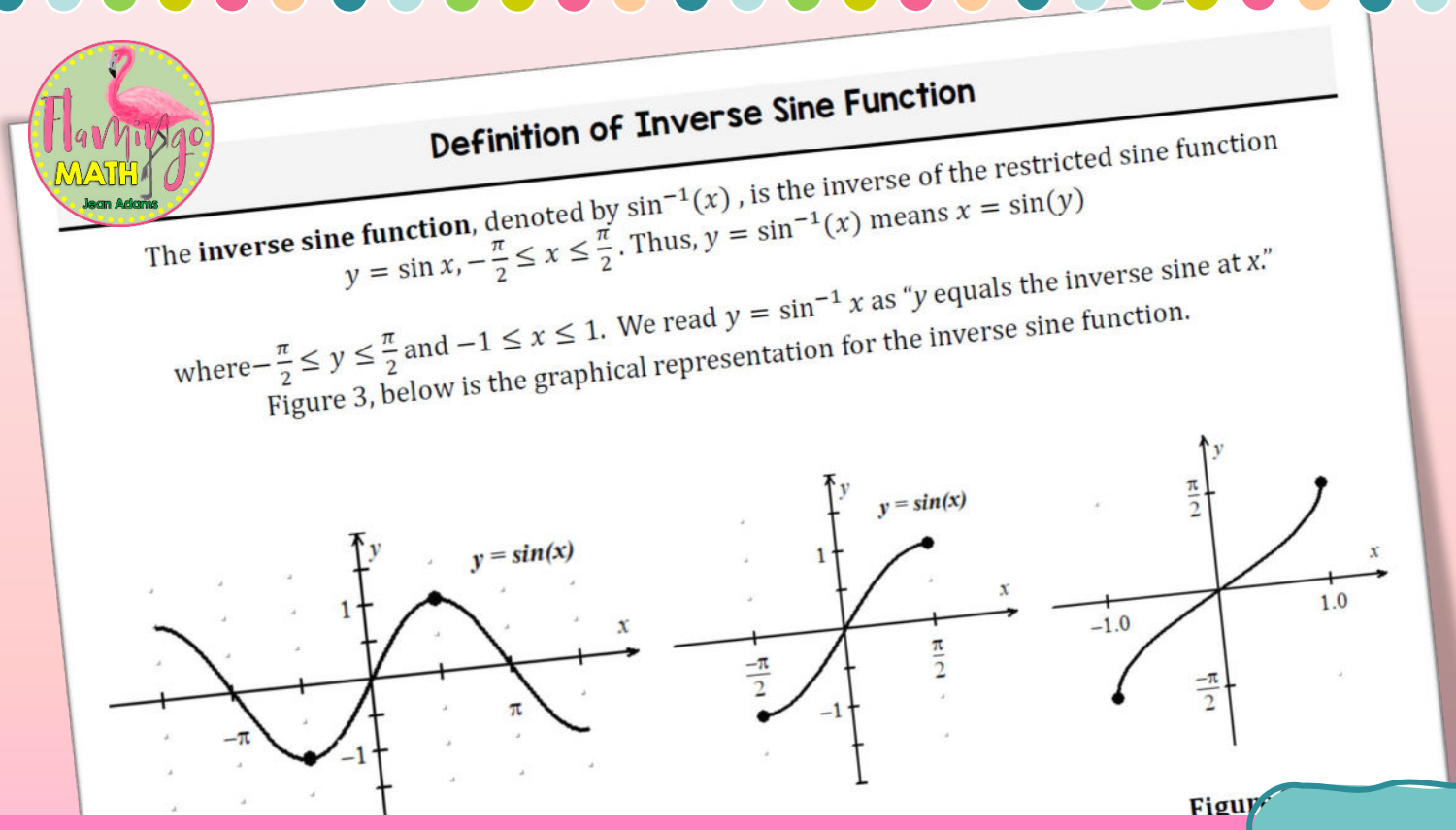
AP® Precalculus

Inverse Trigonometric Functions



Definition of Inverse Sine Function

The **inverse sine function**, denoted by $\sin^{-1}(x)$, is the inverse of the restricted sine function $y = \sin x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Thus, $y = \sin^{-1}(x)$ means $x = \sin(y)$ where $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $-1 \le x \le 1$. We read $y = \sin^{-1} x$ as "y equals the inverse sine at x."

Figure 3, below is the graphical representation for the inverse sine function.

Figur

Completely Editable 3.9

Lesson 9-A: Inverse Trigonometric Functions

Recall in section 2.8 that for a function to have an inverse function, it must be one-to-one. That is, it must pass the *Horizontal Line Test*. In the image below (Fig 1), you can see that the sine function does not pass the test for one-to-one behavior.

However, we can restrict the domain (Fig 2) to the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ which corresponds to the interval between the minimum and maximum values. Now we have a one-to-one function. The inverse of the restricted sine function is called the **inverse sine function**. There are two notations that are commonly used to denote the inverse sine function:

$$y = \sin^{-1} x \text{ and } y = \arcsin x.$$

The notation $y = \sin^{-1} x$ has the same symbol as the inverse function notation $f^{-1}(x)$

Definition of Inverse Sine Function

The **inverse sine function**, denoted by $\sin^{-1}(x)$, is the inverse of the restricted sine function $y = \sin x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Thus, $y = \sin^{-1}(x)$ means $x = \sin(y)$

where $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $-1 \le x \le 1$. We read $y = \sin^{-1} x$ as "y equals the inverse sine at x."

Figure 3, below is the graphical representation for the inverse sine function.

Finding Exact Values of the Inverse Sine Function

EX #1: Remember that $\theta = \sin^{-1} x$ means $x = \sin\theta$, where $-1 \le x \le 1$ and $-\pi/2 \le \theta \le \pi/2$. Complete the table of exact values for the restricted domain.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$									

EX #2: Use the information in your table above to find the exact values of the following.

A. $\sin^{-1} 1$

B. $\sin^{-1}\left(-\frac{1}{2}\right)$

C. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

D. $\sin^{-1}\left(\frac{3}{2}\right)$

Finding Exact Values of the Inverse Cosine Function

EX #3: Remember that $\theta = \cos^{-1} x$ means $x = \cos\theta$, where $-1 \le x \le 1$ and $0 \le \theta \le \pi$. Complete the table of exact values for the restricted domain.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos\theta$									

EX #4: Use the information in your table above to find the exact values of the following.

A. $\cos^{-1}(0)$

B. $\cos^{-1}(2)$

C. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

D. $\cos^{-1}\left(-\frac{1}{2}\right)$

Finding Exact Values of the Inverse Tangent Function

EX #5: Remember that $\theta = \tan^{-1} x$ means $x = \tan\theta$, where $-\infty \le x \le \infty$ and $-\pi/2 \le \theta \le \pi/2$. Complete the table of exact values for the restricted domain.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan\theta$									

EX #6: Use the information in your table above to find the exact values of the following.

A. $\tan^{-1}(1)$

B. $\tan^{-1}(-\sqrt{3})$

C. $\tan^{-1}(0)$

D. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Lesson 9-B: Inverse Trigonometric Functions

If a function is one-to-one (passes the horizontal line test) then its inverse relation is also a function. For sine, cosine, and tangent, we have restricted the domains for each function to discover these properties. Remember our inverse lessons from unit 2, we observed when

$$f(f^{-1}(x)) = x \text{ and } f^{-1}(f(x)) = x.$$

We can apply these properties to the sine, cosine, and tangent and their inverse functions to the following properties.

Inverse Properties

Property:	For Every x in the Interval:
$\sin(\sin^{-1} x) = x$	$[-1, 1]$
$\sin^{-1}(\sin x) = x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos(\cos^{-1} x) = x$	$[-1, 1]$
$\cos^{-1}(\cos x) = x$	$[0, \pi]$
$\tan(\tan^{-1} x) = x$	All reals
$\tan^{-1}(\tan x) = x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

The restrictions on x for inverse properties are a little tricky. Let's consider some examples where you need to find a reference angle.

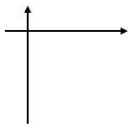
EX #8: Find the exact value, if possible, of the following.

A. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

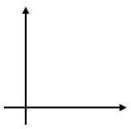
C. $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

Evaluating Composite Trigonometric Expressions

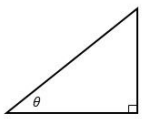
EX #9: Find the exact value: $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$



EX #10: Find the exact value: $\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right)$



EX #11: If $0 < x \le 1$, write $\sin(\cos^{-1} x)$ as an algebraic expression in x.



Application

Guided Notes

3.9 **Inverse Trigonometric Functions Practice Set 1**

Name _____

Date _____ Period _____

Problems 1-6, Find the value of each expression.

1. $\sin^{-1}(-1)$
2. $\cos^{-1}(0)$
3. $\tan^{-1}(0)$
4. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$
5. $\tan^{-1}(-\sqrt{3})$
6. $\cos^{-1}\left(-\frac{1}{2}\right)$
7. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
8. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$
9. $\cos^{-1}(1)$
10.
11. $\tan^{-1}(-1)$
12.

Problems 13–20, Use a calculator to find the value of e

13. $\cos^{-1}(0.75)$
14.
15. $\sin^{-1}\left(\frac{3}{8}\right)$
16.
17. $\tan^{-1}(-0.4)$
18.
19. $\sin^{-1}(0.75)$
20.

3.9 **Inverse Trigonometric Functions Practice Set 2**

Name _____

Date _____ Period _____

Problems 1 – 6, Find the exact value of each expression, do not use a calculator.

1. $\sin[\sin^{-1}(0.35)]$
2. $\tan[\tan^{-1}(8.2)]$
3. $\cos^{-1}\left[\cos\left(\frac{2\pi}{5}\right)\right]$
4. $\sin^{-1}\left[\sin\left(-\frac{\pi}{5}\right)\right]$
5. $\cos^{-1}\left[\cos\left(-\frac{3\pi}{7}\right)\right]$
6. $\tan^{-1}\left[\tan\left(\frac{3\pi}{5}\right)\right]$

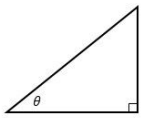
Problems 7 – 12, True or False. Do not use a calculator. Give a reason for your answer.

7. $\cos[\cos^{-1}(2)] = 2$
8.
9. $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$
10.
11. $\tan\left[\tan^{-1}\left(-\frac{1}{2}\right)\right] = -\frac{1}{2}$
12.

Problems 13 – 18, Find the exact value for each composite function.

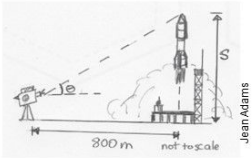
13. $\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$
14. $\cos\left(\sin^{-1}\left(\frac{7}{25}\right)\right)$
15. $\cos\left(\tan^{-1}\sqrt{3}\right)$
16. $\sin(\tan^{-1}(-3))$
17. $\tan\left(\sin^{-1}\left(-\frac{2}{3}\right)\right)$
18. $\tan\left(\cos^{-1}\left(-\frac{8}{17}\right)\right)$

15. Use a right triangle to write the trigonometric expression $\sin(\cos^{-1}(2x))$ as an algebraic expression for $0 \le x \le \frac{1}{2}$.



Problems 16-18, solve.

16. A remote camera at ground level is filming the lift-off of a Falcon rocket at a point 800 meters from the launch pad. Let θ be the angle of elevation to the rocket and let s be the height of the rocket.
Find θ when $s = 1500$ meters.



17. A motorboat is being pulled into a seawall dock by an electric pulley at a dock that is 8 feet above the deck

Free-Response Question

18. Given the function $f(x) = 3\sin x - 1$

A. Find the inverse function $f^{-1}(x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

B. Find the range of f and the domain and range of f^{-1}, state your answer in interval notation.

© 2023 Jean Adams Flamingo Math.com

Practice Sets

3.9 Inverse Trigonometric Functions Form A
Set 2

Name _____

Date _____ Period _____

Find the exact value of each composite function.

1. $\sin\left(\cos^{-1}\left(\frac{3}{4}\right)\right)$

2. $\tan\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)$

3. $\cos(\tan^{-1}(5))$

4. $\sin\left(\cos^{-1}\left(-\frac{2}{5}\right)\right)$

Grade:

3.9 Inverse Trigonometric Functions Form B
Set 2

Name _____

Date _____ Period _____

Find the exact value of each composite function.

1. $\cos\left(\sin^{-1}\left(\frac{3}{4}\right)\right)$

2. $\tan\left(\cos^{-1}\left(-\frac{3}{8}\right)\right)$

3. $\sin(\tan^{-1}(4))$

4. $\cos\left(\text{si}$

Grade:

3.9 Inverse Trigonometric Functions Form A
Set 1

Name _____

Date _____ Period _____

Problems 1 – 4, find the exact value of each expression.

1. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

2. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

3. $\tan^{-1}(\sqrt{3})$

4. $\sin^{-1}(0)$

Problems 5 – 7, use a calculator to find the value of each expression rounded to two decimal places.

5. $\cos^{-1}\left(\frac{5}{7}\right)$

6. $\tan^{-1}(3.14)$

Grade:

3.9 Inverse Trigonometric Functions Form B
Set 1

Name _____

Date _____ Period _____

Problems 1 – 4, find the exact value of each expression.

1. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

2. $\sin^{-1}\left(-\frac{1}{2}\right)$

3. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

4. $\sin^{-1}(1)$

Problems 5 – 7, use a calculator to find the value of each expression rounded to two decimal places.

5. $\tan^{-1}(5.16)$

6. $\sin^{-1}\left(\frac{3}{2}\right)$

Grade:

Daily Lesson Quiz