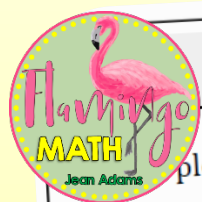


Precalculus Honors

Inverse Trigonometric Functions



The Inverse Cosine Function

plot $y = \cos x$ on the interval $[-2\pi, 2\pi]$.

Is this function one-to-one? no

We can restrict the domain so the graph will pass the horizontal line test. Here is a little help:

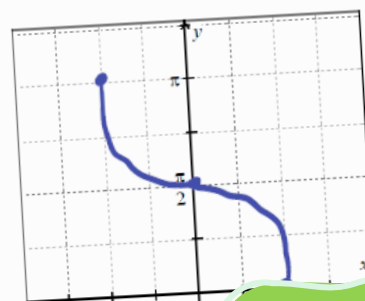
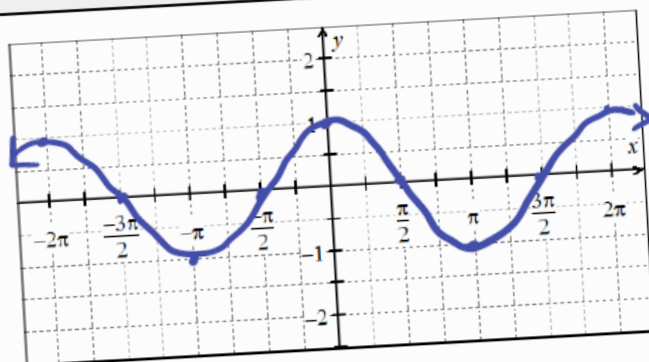
To do this, first find the range of $y = \cos x$ $[1, 1]$

What domain value will give you a range value of -1 ? π

What domain value will give you a range value of 1 ? 0

Use the last two answers to restrict your domain for $y = \cos x$.
This domain will become the range of the inverse cosine function.

$f(x) = \cos^{-1} x$ where -1 $\leq x \leq$ 1



Completely Editable 4.8

Lesson 8: Inverse Trigonometric Functions

Inverse Properties

If a function is one-to-one (passes the horizontal line test) then its inverse relation is also a function. For sine, cosine, and tangent, we will restrict the domains for each function to discover these properties.

Property:	For the Domain:
$\sin(\sin^{-1}x) = x$	$[-1, 1]$
$\sin^{-1}(\sin x) = x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos(\cos^{-1}x) = x$	$[-1, 1]$
$\cos^{-1}(\cos x) = x$	$[0, \pi]$
$\tan(\tan^{-1}x) = x$	All reals
$\tan^{-1}(\tan x) = x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Remember that a function will have an inverse if it is one-to-one.

The Inverse Sine Function

First, plot $y = \sin x$ on the interval $[-2\pi, 2\pi]$.

Is this function one-to-one? _____

We can restrict the domain so the graph will pass the horizontal line test. Here is a little help:

Find the range of $y = \sin x$. _____

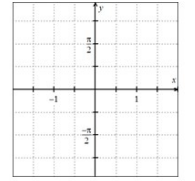
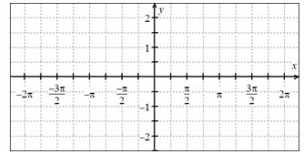
What domain value will give you a range value of -1 ? _____

What domain value will give you a range value of 1 ? _____

We will use the last two answers to restrict our domain for $y = \sin x$. This domain will become the range of our inverse sine function.

In summary, $f(x) = \sin^{-1}x$ where _____ $\leq x \leq$ _____
and _____ $\leq y \leq$ _____.

Now plot $y = \sin^{-1}x$ on the graph at right. You can use your calculator to help.



The Inverse Cosine Function

First, plot $y = \cos x$ on the interval $[-2\pi, 2\pi]$.

Is this function one-to-one? _____

We can restrict the domain so the graph will pass the horizontal line test. Here is a little help:

To do this, first find the range of $y = \cos x$. _____

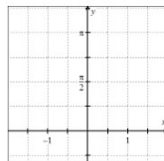
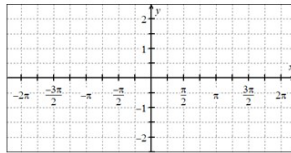
What domain value will give you a range value of -1 ? _____

What domain value will give you a range value of 1 ? _____

Use the last two answers to restrict your domain for $y = \cos x$. This domain will become the range of the inverse cosine function.

In summary, $f(x) = \cos^{-1}x$ where _____ $\leq x \leq$ _____
and _____ $\leq y \leq$ _____.

Now plot $y = \cos^{-1}x$ on the graph at right. You can use your calculator to help.



The Inverse Tangent Function

First, plot $y = \tan x$ on the interval $[-\pi, \pi]$.

Is this function one-to-one? _____

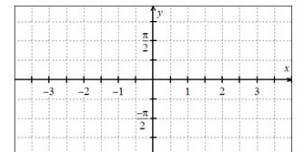
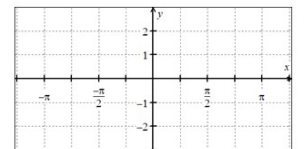
We can restrict the domain to only one _____ so the graph will pass the horizontal line test.

Use the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for the domain of $y = \tan x$.

$y = \tan^{-1}x$ where _____ $< x <$ _____

and _____ $< y <$ _____.

Now plot $y = \tan^{-1}x$ on the graph at right. You can use your calculator to help.



Evaluating with a Calculator

EX #4: Use a calculator in radian mode to approximate the expression. Round your answer to two decimal places.

A. $\sin^{-1}(-0.74) =$

B. $\cos^{-1}(1.5) =$

C. $\sin^{-1}(\sin 2.5\pi) =$

D. $\tan(\tan^{-1}(-4)) =$

Evaluate Inverse Trigonometric Functions without a Calculator

EX #5: Find the exact value of the expression without a calculator.

A. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) =$

B. $\tan^{-1}(-\sqrt{3}) =$

C. $\cos^{-1}(\cos(-1.1)) =$

Exact Values Graphically

EX #6: Find the exact value: $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$



Applications of Inverse Trigonometric Functions

EX #8: Calculating a viewing angle.

The bottom of a 20-foot replay screen at Dodger Stadium is 45 ft. above the playing field. As you move away from the wall, the angle formed by the screen at your eye changes. There is a distance from the wall at which the angle is the greatest. What is that distance?

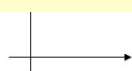
A. Draw a picture to show θ is the greatest angle.

B. Set up relationships for angles where $\theta = \theta_1 - \theta_2$.

C. Write an equation of the relationship.

D. Graph the equation in degree mode with window settings $[0, 400] \times [0, 16]$.

NOTES



4.8

Inverse Trigonometric Functions Homework

Name _____

Date _____ Period _____

Problems 1-6, Find the value of each expression.

1. $\sin^{-1}(-1)$

2. $\cos^{-1}(0)$

3. $\tan^{-1}0$

4. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

5. $\tan^{-1}(-\sqrt{3})$

6. $\cos^{-1}\left(-\frac{1}{2}\right)$

Problems 7-12, Use a calculator to find the value of each expression rounded to two decimal places.

7. $\cos^{-1}(0.75)$

8. $\tan^{-1}4$

9. $\sin^{-1}\left(\frac{3}{8}\right)$

10. $\cos^{-1}\left(\frac{5}{8}\right)$

11. $\tan^{-1}(-0.4)$

12. $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

Problems 13-18, Find the exact value of each expression, do not use a calculator.

13. $\sin[\sin^{-1}(0.35)]$

14. $\tan[\tan^{-1}(8.2)]$

15. $\cos^{-1}\left[\cos\left(\frac{2\pi}{5}\right)\right]$

16. $\sin^{-1}\left[\sin\left(-\frac{\pi}{5}\right)\right]$

17. $\cos^{-1}\left[\cos\left(-\frac{3\pi}{7}\right)\right]$

18. $\tan^{-1}\left[\tan\left(\frac{2\pi}{5}\right)\right]$

Problems 19-24, True or False. Do not use a calculator. Give a reason for your answer:

19. $\sin^{-1}\left[\sin\left(-\frac{\pi}{10}\right)\right] = -\frac{\pi}{10}$

20. $\cos[\cos^{-1}(2)] = 2$

21. $\cos^{-1}\left[\cos\left(-\frac{\pi}{4}\right)\right] = -\frac{\pi}{4}$

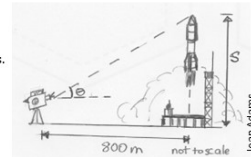
22. $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$

23. $\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right] = \frac{2\pi}{3}$

24. $\tan\left[\tan^{-1}\left(-\frac{1}{2}\right)\right] = -\frac{1}{2}$

Problems 25-26, Solve.

25. A remote camera at ground level is filming the lift-off of an Atlas rocket at a point 800 meters from the launch pad. Let θ be the angle of elevation to the rocket and let s be the height of the rocket. Find θ when $s = 1500$ meters.



26. A motorboat is being pulled into a seawall dock by an electric pulley at a dock that is 8 feet above the deck of the boat. Let θ be the angle of elevation from the boat to the pulley and let s be the length of the rope from the pulley to the boat. Find θ when $s = 30$ feet.



Free-Response Question

- A. Find the inverse function $f^{-1}(x)$ for $f(x) = 3 \sin x - 1$; $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

- B. Find the range of f and the domain and range of f^{-1} , state your answer in interval notation.

- C. Find the exact solution of the equation $3 \sin^{-1} x = \pi$

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 4.8 - Inverse Trigonometric Functions Warm-Up # I	
1. Evaluate: $\arccos\left(-\frac{1}{2}\right)$	2. Evaluate: $\cos^{-1}\left(\cos\frac{-\sqrt{2}}{2}\right)$
3. Evaluate: $\sin\left(\arccos\frac{12}{13}\right)$	4. Use a calculator; round to 3 decimal places. $\tan^{-1}(3.2)$

Name _____ Date _____	
Lesson 4.8 - Inverse Trigonometric Functions	
1. Evaluate: $\arcsin(1)$	2. Evaluate: $\sin^{-1}\left(\sin\frac{\pi}{2}\right)$
3. Evaluate: $\cos\left(\arcsin\frac{12}{13}\right)$	4. Use a calculator; round to 3 decimal places. $\cos^{-1}(0.4)$

4.8 Inverse Trig Functions Form A Name _____ Date _____ Period _____ 1. Evaluate: $\arctan(1)$ 2. Evaluate: $\cos^{-1}\left(\cos\frac{\pi}{2}\right)$ 3. Evaluate: $\sin\left(\arccos\frac{3}{5}\right)$	4.8 Inverse Trig Functions Form B Name _____ Date _____ Period _____ 1. Evaluate: $\arcsin\left(-\frac{1}{2}\right)$ 2. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{2}\right)$ 3. Evaluate: $\cos\left(\arcsin\frac{3}{5}\right)$
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Daily Lesson Quiz

Grade: _____	Grade: _____
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