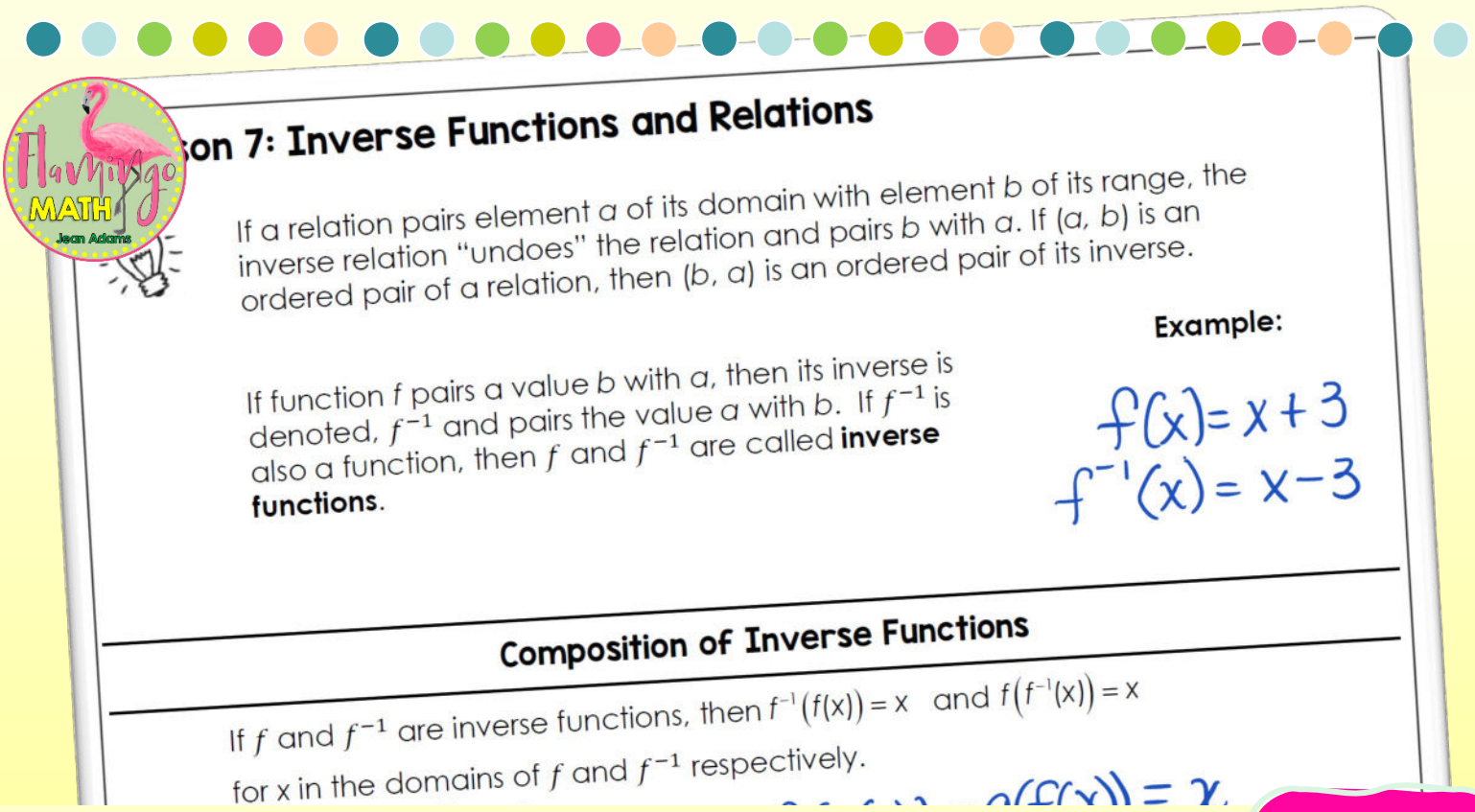
Algebra 2 Honors

Inverse Functions and Relations



on 7: Inverse Functions and Relations

If a relation pairs element a of its domain with element b of its range, the inverse relation "undoes" the relation and pairs b with a. If (a, b) is an ordered pair of a relation, then (b, a) is an ordered pair of its inverse.

If function f pairs a value b with a, then its inverse is denoted, f^{-1} and pairs the value a with b. If f^{-1} is also a function, then f and f^{-1} are called **inverse functions**.

Example:

$f(x) = x + 3$

$f^{-1}(x) = x - 3$

Composition of Inverse Functions

If f and f^{-1} are inverse functions, then $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

for x in the domains of f and f^{-1} respectively.

Completely Editable 6.7

Lesson 6.7: Inverse Functions and Relations

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Composition of Inverse Functions

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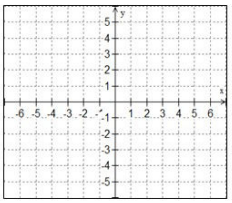
Finding the Inverse of a Relation

EX #1: Find the inverse of the relation, then graph.

Switch the x and y values to get the inverse.

Relation S	
x	y
–2	–2
–1	1
0	2
1	1
2	–2

Inverse of Relation S	
x	y



Is S a function? Is the inverse of S a function? How do you know?

Finding an Equation for the Inverse

EX #2: What is the inverse of the relation?

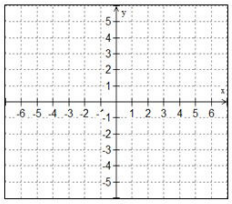
A. $y = 2x + 3$

B. $y = x^2 + 1$

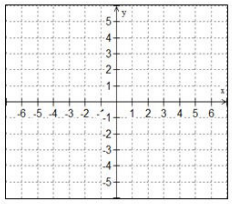
Graphing a Relation and Its Inverse

EX #3: Graph the function and its inverse.

A. Graph $y = 2x + 3$ and its inverse.



B. Graph $y = x^2 + 1$ and its inverse.



Finding an Inverse

EX #4: Let $f(x) = \sqrt{x+2}$

A. What are the domain and range of $f(x)$?

B. Find $f^{-1}(x)$

C. What are the domain and range of $f^{-1}(x)$?

D. Is $f^{-1}(x)$ a function? Explain.

One-to-One Functions

A function f is a **one-to-one function** if, for elements a and b in the domain of f, $a \neq b$ **implies** $f(a) \neq f(b)$.

That is, different values of the domain correspond to different values of the range.

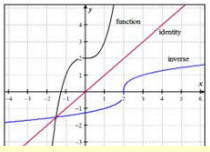
If the graph of a function f is known, you can use the **horizontal-line test** to determine whether f is one-to one.

Horizontal Line Test

If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

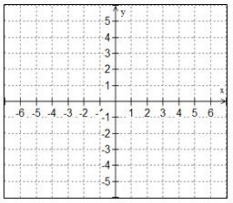
Properties of Inverse Functions

- ❑ Graphs of inverse functions are reflections in the line _____.
- ❑ Given a function $f(x)$, the notation for the inverse function is _____.
- ❑ The domain of a function is equal to the _____ of the inverse function.



Inverses with Restricted Domains

EX #6: Find the inverse of $f(x) = x^2 - 4x - 1$. Then, graph the function and its inverse. If necessary, restrict the domain of the inverse so that it is a function.



Finding the Inverse of a Formula

EX #7: To convert degrees Celsius to Fahrenheit, we use the formula: $F(x) = \frac{9}{5}x + 32$

A. Find the inverse function $F^{-1}(x)$.

B. Explain the meaning of the inverse function in context to the formula.

NOTES

Name _____ Date _____

6.7

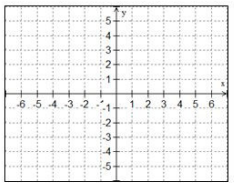
Inverse Functions Practice

Problems 1 – 2, find the inverse of each relation. Graph the given relation and its inverse.

1.

x	y
0	−3
1	−1
2	1
3	3

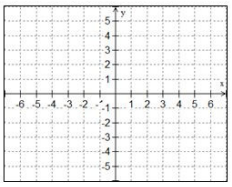
x	y



2.

x	y
−6	3
−1	2
2	1
3	0

x	y



Problems 3 – 6, find the inverse of each function. Is the inverse a function?

3. $y = 2x - 5$

4. $y = 3 - 2x^2$

5. $y = \sqrt{x - 2}$

6. $y = (x - 1)^2$

Problems 7 – 10, for each function f, find f^{-1}. State the domain and range of f and f^{-1}.

7. $f(x) = -\frac{2}{3}x + 4$

8. $f(x) = \sqrt{x + 1}$

9. $f(x) = x^2 + 5$

10. $f(x) = \sqrt{3x} + 5$

Problems 11 – 12, Let $f(x) = 5x + 8$. Find each value. Hint: Rewrite $f(x)$ as y and switch x and y.

11. $(f \circ f^{-1})(4)$

12. $(f^{-1} \circ f)(-3)$

Problems 13 – 16, solve.

13. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$

A. Find the inverse of the formula. Is the inverse a function?

B. Use the inverse to find the radius of a sphere that has a volume of 28,000 ft^3.

14. The function $d = \frac{r^2}{24}$ is a model for the distance d in feet that a car with locked brakes skids when coming to a complete stop from a speed of r mi/hr. Find the inverse of the function. Use the inverse to estimate the speed of a car that made skid marks 136 feet long.

15. Mr. Swenson used the formula $y = 140x + 175$ to predict a student's score on a state End-of-Course exam using the student's 11-grade GPA number x.

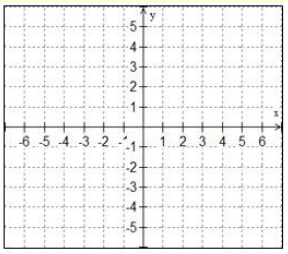
A. Find the inverse of the formula.

Practice Sets

Lesson Warm-Ups

$y = 2x - 3$	
x	y
−1	
0	
1	
2	
3	
4	

Inverse of y	
x	y



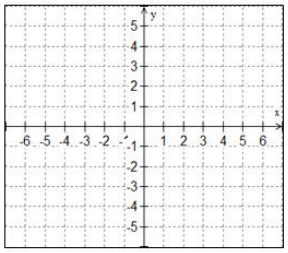
Name _____ Date _____ Period _____

Lesson 6.7: Inverse Relations and Functions **Warm-Up #2**

Complete the table for the function below. Then, find the inverse table and graph the function and its inverse.

$y = \frac{1}{2}x - 3$	
x	y
−4	
−2	
0	
2	
4	
6	

Inverse of y	
x	y



Form A

Name _____

Period _____

2. What is the inverse of $f(x)$?

3. Domain of $f^{-1}(x)$:

   Range of $f^{-1}(x)$:

4. Is $f^{-1}(x)$ a function? Explain.

5. Graph both f and f^{-1} on the grid below.

6.7 Inverse Relations **Form B**

Name _____

Date _____ Period _____

Questions 1 – 5: $f(x) = \sqrt{x+1}$

1. Domain of $f(x)$:

   Range of $f(x)$:

2. What is the inverse of $f(x)$?

3. Domain of $f^{-1}(x)$:

   Range of $f^{-1}(x)$:

4. Is $f^{-1}(x)$ a function? Explain.

5. Graph both f and f^{-1} on the grid below.

Daily Lesson Quiz