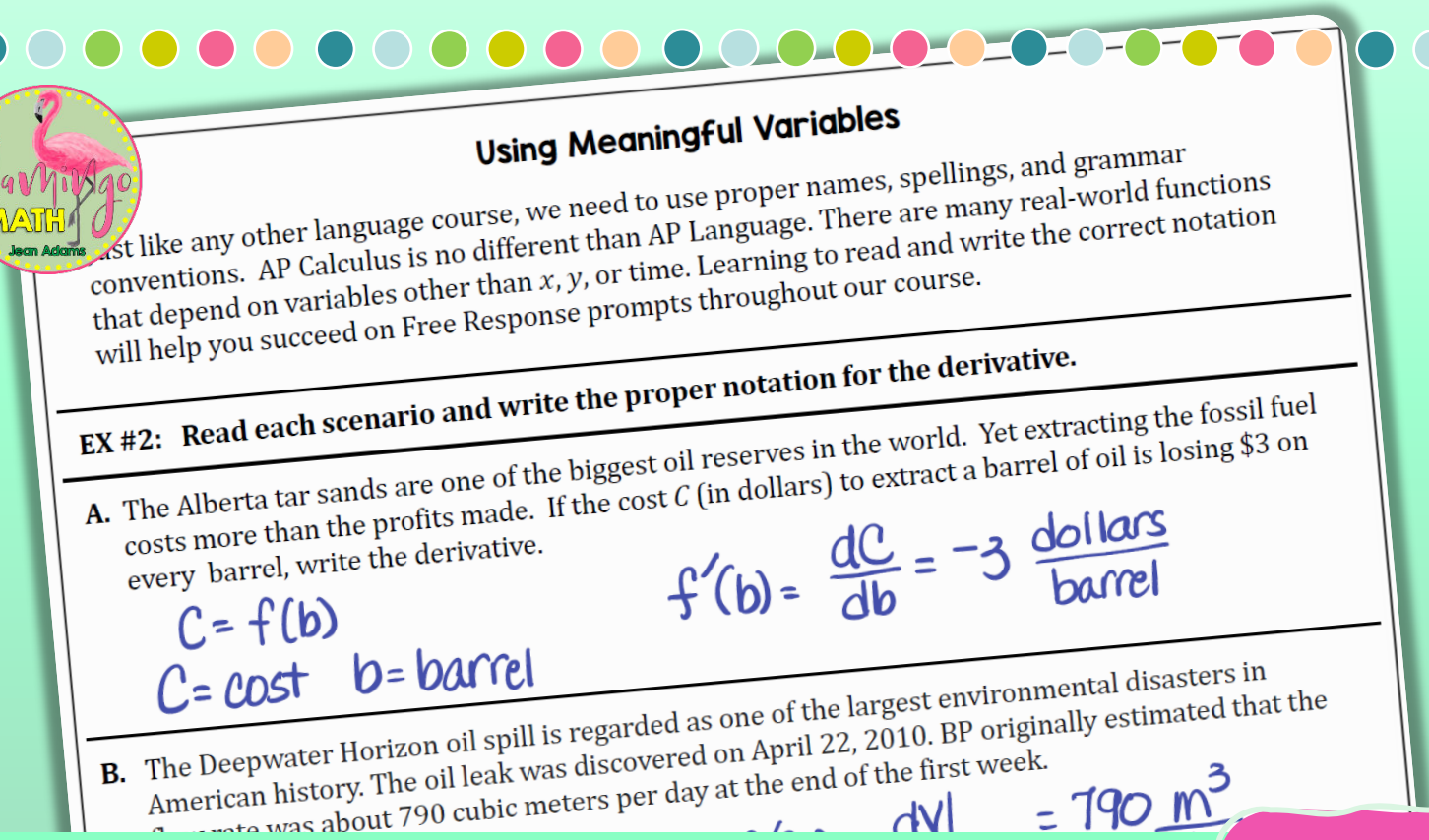
AP® Calculus

Interpretations of the Derivative



Using Meaningful Variables

...st like any other language course, we need to use proper names, spellings, and grammar conventions. AP Calculus is no different than AP Language. There are many real-world functions that depend on variables other than x, y, or time. Learning to read and write the correct notation will help you succeed on Free Response prompts throughout our course.

EX #2: Read each scenario and write the proper notation for the derivative.

A. The Alberta tar sands are one of the biggest oil reserves in the world. Yet extracting the fossil fuel costs more than the profits made. If the cost C (in dollars) to extract a barrel of oil is losing $3 on every barrel, write the derivative.

$C = f(b)$

C = cost b = barrel

$f'(b) = \frac{dC}{db} = -3 \frac{\text{dollars}}{\text{barrel}}$

B. The Deepwater Horizon oil spill is regarded as one of the largest environmental disasters in American history. The oil leak was discovered on April 22, 2010. BP originally estimated that the ...rate was about 790 cubic meters per day at the end of the first week.

$\frac{dV}{} = 790\ m^3$

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4.1

Lesson I: Interpretation of the Derivative In Context

Topic 4.1: Interpreting the Meaning of the Derivative in Context

In our last two units we have defined the derivative graphically, algebraically, and numerically as a slope of the tangent line and the instantaneous rate of change at a point. In this lesson, we want to interpret the meaning of the derivative in a verbal manner, as it relates to the context of real-world problems.

A Look at Some Mathematical History

When Newton was developing his ideas of the infinitesimal calculus, namely when given a **fluent** (*we call function*) to find its **fluxion** (*we call derivative*), his thinking was concentrated on velocity. But, today, we know the derivative's importance reaches into so many other areas involving rates of change.

Let's look at the Leibniz notation one more time in a different way. He stated that the variable y depended on x, and if $y = f(x)$, then the derivative could be expressed as dy/dx. If we allow ourselves to think of "d" as representing *"a small difference in some process,"* we are reminded that the derivative is simply the limit of ratios in the familiar form

$$\frac{dy}{dx} = f'(x) = \frac{\Delta \text{ in } y\text{–values}}{\Delta \text{ in } x\text{–values}}.$$

Similarly, we can read $\frac{d}{dx}$ as a single symbol meaning *"the derivative with respect to x of …."* which brings us to this new place of understanding the notation $\frac{d}{dx}(y)$ or $\frac{d}{dx}[f(x)]$, *"the derivative with respect to x of* y or $(f(x))$.

The Leibniz notation can be a little cumbersome when you need to specify the exact x-value where you need to evaluate the derivative. However, the benefit is that you can read so much about the rate of change through this expression. It's very easy to write $f'(3)$ but with Leibniz's notation we will write the derivative format like this:

$$\left.\frac{dy}{dx}\right|_{x=3}$$

EX #1: Sean is cleaning out his saltwater fish tank and needs to drain some of the water in order to adjust the saline levels. If the volume of the tank V, is in gallons, is being drained over time, t, in minutes. What is a meaningful interpretation of the following statement as it relates to Sean's fish tank?

$$\left.\frac{dV}{dt}\right|_{t=3} = -2$$

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B. The Deepwater Horizon oil spill is regarded as one of the largest environmental disasters in American history. The oil leak was discovered on April 22, 2010. BP originally estimated that the flow rate was about 790 cubic meters per day at the end of the first week.

C. The population P of the endangered mountain gorilla has an ongoing study tracking the number of gorillas since 1992. The population increased by 114 gorillas during 2003 of this same study.

EX #3: Suppose a planet is discovered in 2024; and, is named *Newtonia*. If the alien population $P(t)$ is recorded in millions of aliens, and t represents the number of years since the planet's discovery. Explain the meaning of each of the statements below.

A. $P'(3) = 0.5$

B. $P^{-1}(7.5) = 5$

C. $[P^{-1}(7.5)]' = 0.75$

Using Units to Interpret the Derivative

In the next examples, we will show how clues from the units can help you recognize interpretations of the derivative as they relate to real-world scenarios.

EX #4: Suppose you launch a water balloon from a giant slingshot. Let $s(t)$ give the distance, in feet, that the water balloon traveled from its initial starting point as a function of time, t, in seconds. Explain the meaning of the derivative notation.

$$\left.\frac{ds}{dt}\right|_{t=2} = s'(2) = 38\frac{ft.}{sec.}$$

EX #5: The total cost C ,in dollars, of an airplane flight is a function of the number of passengers, n can be calculated by Derivative Airlines using the function $C = f(n)$. What does it mean to say $f'(182) = 58$?

EX #6: Non-potable water is being pumped into a processing center where the depth, in feet, of the water at time t in hours is given as $y = h(t)$. Interpret the following statements using the proper units.

A. $h(7) = 5$

D. $(h^{-1})'(7) = 1.4$

NOTES

4.1 **Interpretations of the Derivative in Context Homework**

Name _____

Date _____ Period _____

1. The temperature, T, in degrees Fahrenheit, of a pizza placed on a serving dish is given by $T = f(t)$, where t is the time in minutes since the pizza was taken out of the oven.

A. Is $f'(t)$ positive or negative? Justify with a reason.

B. What are the units of $f'(5)$?

C. What is the meaning of $f'(5) = -3$ in context to the scenario?

2. The cost, C (in dollars) to produce g gallons of fresh-squeezed orange juice can be expressed as $C = f(g)$. Using the proper units, explain the meaning of the following statements in terms of the orange juice.

A. $f(500) = 2150$

B. $f'(500) = 0.84$

3. You have an inheritance of $100,000 from your grandparents, but you must invest the money in a mutual fund until you are 21 years old, that will pay an annual interest rate of r% compounded continuously. If you are 17 years old now, your balance, in dollars, can be defined as $B = A(r)$. Interpret the following statements related to your inheritance, use the appropriate units.

A. $A(7) \approx 13230$

B. $A'(7) \approx 3307.50$

4. The Apollo 11 Crew left Earth for the moon on July 16, 1969, at 9:32 a.m. in a Saturn V rocket. They experienced forces of about 1.2g at liftoff and reached a maximum force of about 3.9g. The g-force is measured in Newtons defined as $1\text{N} = kg(m/s^2)$. Let $F = p(h)$ represent the force of pressure in Newtons, p on the astronauts at a height h, in meters. Explain what each of the following quantities mean in relation to the astronauts. Give units for the quantities in Newtons.

A. $p(500) = 3.2$

B. $p'(500) = 0.2$

C. $p'(h) = -1$

5. A law firm pays a third-party to pick up confidential documents and deliver them to a shredding service. This cost, C, in dollars, is a function of the weight of paper, w, in pounds, that is shredded for each client's case, such that $C = f(w)$. Describe the meaning of each quantity.

A. $f(65) = 45$

B. $f'(80) = 1.25$

C. $[f^{-1}(200)]' = 2.5$

6. When a patient is prescribed an antibiotic to fight off infection, the dose, D, in milligrams is a function of the patient's weight, in pounds. Let $D = f(w)$ define the dosage. Interpret the meaning of each statement.

A. $f(135) = 120$

B. $f'(135) = 2.5$

C. Estimate the dosage for a person weighing 140 pounds.

7. During a hurricane in Central Florida, the amount of rainfall, R, in centimeters, is a function of time. Let $R = f(t)$ define the amount of rain recorded by meteorologists beginning at midnight. Give an interpretation of the statements below, using the appropriate units.

A. $f(6) = 4.8$

B. $f'(6) = 0.5$

C. $f^{-1}(5) = 14$

D. $(f^{-1})'(5) = 1.5$

8. Nate took an initial dose of his prescription antibiotic medication. The amount of antibiotic, in milligrams, in Nate's bloodstream after t hours is given by $m(t) = 18e^{-0.75t}$. Find the instantaneous rate of change of the remaining amount of medication after 1 hour. Explain the meaning of your findings as it relates to Nate.

9. The population of Eulerville can be modeled by $P = f(t)$ where the population, P, is in thousands

C. $(f^{-1})'(47.5) = 0.3$

Practice Sets

4.1 Interpretations of the Derivative in Context **Form A**

Name _____

Date _____Period _____

1. Jamie drove from Miami to Atlanta. The function D, gives the total distance Jamie has driven in miles, t hours after she left. What is the meaning of $D'(2) = 73$

2. The cost C (in dollars) to produce electric rental scooters is $C = f(s)$. Write an expression that represents the statement, "When 400 scooters have been produced, the cost to produce one more scooter is increasing at a rate of $27.50.

Oil is being drained from a large storage barrel, whose depth, in feet, of the oil at time t hours is $B = h(t)$. **Interpret the following statements, with units.**

3. $h'(8) = -1.2$

4. $h^{-1}(8) = 5$

Grade:

4.1 Interpretations of the Derivative in Context **Form B**

Name _____

Date _____Period _____

1. The cost, in dollars, per person on a Hop-On Hop-Off Bus Tour, with n tourists, can be calculated by Big Bus Tours using the function $T = f(n)$. What is the meaning of $T'(42) = 8.25$

2. The cost C (in dollars) to produce skateboards is $C = f(s)$. Write an expression that represents the statement, "When 235 skateboards have been produced, the cost to produce one more board is decreasing at a rate of $7.85.

Maple syrup is being drained from a large storage vat, whose depth, in inches, of the syrup at time t hours is $S = h(t)$. **Interpret the following statements, with units.**

3. $h'(3) = -18$

4. $(h^{-1}$

Grade:

4.1 Interpretations of the Derivative in Context **Form C**

Name _____

Date _____Period _____

1. Alex drove from Houston to Dallas. The function D, gives the total distance Alex has driven in miles, t hours after he left. What is the meaning of $D'(3.5) = 48$

2. The cost C (in dollars) to remove T tons of Sargasso seaweed from the beaches is $C = f(s)$. Write an expression that represents the statement, "When 5,000 tons of seaweed have been removed from the beach, the cost to remove seaweed is decreasing at a rate of $120.

Water is flowing through a pipe to fill a new swimming pool, whose depth, in feet, of the pool at time t hours is $P = h(t)$. **Interpret the following statements, with units.**

3. $h'(4) = 3.5$

Grade:

4.1 Interpretations of the Derivative in Context **Form D**

Name _____

Date _____Period _____

1. The cost, in dollars, per person for the school awards banquet, with n students, can be calculated by Alma's Italian Restaurant using the function $B = f(n)$. What is the meaning of $B'(125) = 16.50$

2. The amount of medication in the bloodstream for a dose D (in milligrams) of an allergy injection is a function of time in hours, $D = f(t)$. Write an expression that represents the statement, "After 6.5 hours, the medication in a patient's bloodstream is decreasing at the rate of 0.5"

Soda is pumped into large steel containers, whose depth, in inches, of the soda at time t minute is $S = h(t)$. **Interpret the following statements, with units.**

3. $h'(15) = 1.8$

Grade:

Daily Lesson Quiz