AP® Calculus

Intermediate Value Theorem

The functions f and g are continuous for all real numbers. The table below gives values of the functions at selected values of x. The function h is given by $h(x) = g(f(x)) + 2$ Explain why there must be a value w for $1 < w < 6$ such that $h(w) = 0$.

x	1	2	6	8
$f(x)$	2	9	8	13
$g(x)$	3	−12	5	28

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1.9

Lesson 9: Intermediate Value Theorem

Continuity of a function will prove to be an important characteristic in many theorems for our course. In order for theorems to be properly applied, we need to meet the certain conditions. Namely, the "if" part, called the ***hypothesis***, must be satisfied before we can apply the "then" part, or ***conclusion*** (consequence) of the theorem.

In this lesson, we will explore our first **existence theorem**. Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

Topic 1.16: Working with the Intermediate Value Theorem (IVT)

Intermediate Value Theorem

The Big Idea:

The **IVT** says that a continuous function on an interval cannot skip values.

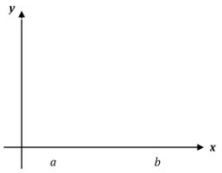
Using If/Then statements:

If a function $y = f(x)$ is continuous on a closed interval $[a, b]$ and $f(a) \neq f(b)$, **Then** $f(x)$ takes on every value between $f(a)$ and $f(b)$ on that interval.

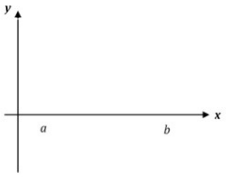
Another Approach:

Said in a different fashion, if we know a y-value, say $y = k$, that resides between the two endpoints, $f(a)$ and $f(b)$, then we are guaranteed at least one x- value, $x = c$, between the endpoints that generates that y-value, such that $f(c) = k$.

EX #1: Graphical representation of the Intermediate Value Theorem and its corollary:

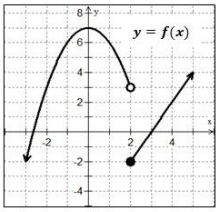


A continuous function on an interval cannot skip values.



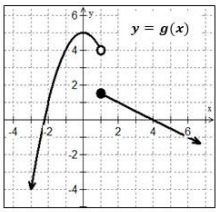
An important outcome of I.V.T. is that it can be helpful in finding zeros of a continuous function on an interval.

EX #2: Does the Intermediate Value Theorem apply on the specified intervals for the piecewise functions shown below? Be sure to explain completely.



A. Is there a value of $x = c$ on the interval $[-2, 5]$ such that $f(c) = 1$?

B. Does I.V.T. guarantee a value of c on the interval $[-2, 5]$ such that $f(c) = 1$? Justify.



C. Is there a value of $x = c$ on the interval $[1, 6]$ such that $g(c) = 3$?

D. Does I.V.T. guarantee a value of c on the interval $[1, 6]$ such that $g(c) = 3$? Justify.

What three conditions are necessary to apply the Intermediate Value Theorem?

1.

2.

3.

EX #3: Apply the IVT, if possible, on $[0, 5]$ so that $f(c) = 1$ for the function $f(x) = x^2 + x - 1$.

EX #4: Use the Intermediate Value Theorem to show that $f(x) = x^3 + 2x - 1$ has a zero in the interval $[0, 1]$.

EX #5: A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity v, measured in feet per second is a continuous function. The table below shows selected values of the function.

t, in seconds	0	15	25	30	35	50	60

EX #6: Let $g(x)$ be a continuous function. Selected values of $y = g(x)$ are given in the table below. For which value of k will the equation $g(x) = 3/4$ have **at least two solutions** on the closed interval $[2, 8]$?

x	2	3	4	5	8
$g(x)$	3	4	k	5	2

(A) 1 **(B)** $\frac{3}{4}$ **(C)** $\frac{9}{16}$ **(D)** $\frac{3}{2}$

EX #7: For the function $f(x) = \begin{cases} (x-3)^2, x = 5 \\ 6, \quad 5 < x \leq 10 \end{cases}$. Find $f(5)$ and $f(10)$. Does IVT guarantee a y-value k on $5 \leq x \leq 10$ such that $f(5) < f(k) < f(10)$? Justify your answer.

EX #8: The functions f and g are continuous for all real numbers. The table below gives values of the functions at selected values of x. The function h is given by $h(x) = g(f(x)) + 2$. Explain why there must be a value w for $1 < w < 6$ such that $h(w) = 0$.

x	1	2	6	8
$f(x)$	2	9	8	13

NOTES

1.9 Intermediate Value Theorem

Name _____

Date _____ Period _____

Problems 1 – 13, Use the Intermediate Value Theorem to complete.

1. In the function $f(x) = x^3 - x - 1$, it can be shown that $f(1) = -1$ and $f(2) = 5$. Complete the table below to find an approximation for a solution of the interval $[1, 2]$.

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$f(x)$											

2. Find the value of c guaranteed by the Intermediate Value Theorem. $f(x) = x^2 + 4x - 13$ on $[0, 4]$ such that $f(c) = 8$

3. Show that $g(x) = 2x^3 - 5x^2 - 10x + 5$ has a root somewhere in the interval $[-1, 2]$.

4. Between which of the following two values does the equation $3x^3 + 5x - 11 = 0$ have a solution?

(A) $[-2, -1]$ **(B)** $[0, 1]$ **(C)** $[-1, 0]$ **(D)** $[1, 2]$

5. Given the function $f(x) = \frac{2x-3}{2x-5}$, determine which interval(s) satisfies the conditions for the Intermediate Value Theorem, such that $f(x) = 0$.

(A) One solution between x = 0 and x = 1

(B) One solution between $x = 1$ and $x = 2$

(C) One solution between $x = 1$ and $x = 2$ and one solution between $x = 2$ and $x = 3$

(D) One solution between $x = 2$ and $x = 3$

6. Apply the Intermediate Value Theorem, if possible, on $[1, 2]$ so that $f(c) = 9$ for the function $f(x) = x^3 + x$.

7. A delivery van travels along a straight road. During the time interval $0 \le t \le 30$ seconds, the van's velocity in feet per second is a continuous function. Use the table below to find the minimum number of times that the van must have been stopped. Justify your answer.

t (sec)	0	5	7	12	18	22	30
$V(t)$ (ft/sec)	−28	−60	−15	8	24	−4	10

8. Explain why the Intermediate Value Theorem does not apply for guaranteeing that a zero exists for the function $f(x) = x^2 + 2x + 5$ over $[0, 6]$.

9. The functions f and g are continuous. The function h is given by $h(x) = f(g(x)) - x$. The table below gives values of the functions. Explain why there must be a value c for $1 < c < 5$ such that $h(c) = -2$.

x	1	2	3	4	5
$f(x)$	0	9	7	−3	8
$g(x)$	4	6	−4	1	3

10. Given $f(x) = \frac{x}{x-3}$ on the interval $[-2, 2]$. Determine if the IVT applies. State why or why not. Then, find the value of c such that $f(c) = \frac{1}{3}$.

12. Does the IVT apply to the function $h(x) = \frac{x^2+x}{x-2}$ on the interval $[2.5, 5]$? If so, find the value of c guaranteed to exist, such that $h(c) = 12$.

13. Does the IVT apply to the function $f(x) = -\left(\frac{1}{2}\right)^{3-x} - 3$ on the interval $[2,5]$ for $f(c) = -4$?

between -2 and 7.

(C) $f(c) = 1$ for at least one c between -2 and 7

(D) $f(c) = 0$ for at least one c between -3 and 4

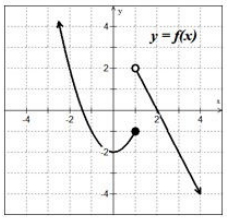
Practice Sets

1.9 Infinite Limits — Form A

Name _____

Date _____ Period _____

Problems 1 – 2: Use the graph of the function below.



1. Is there a value of c on $[-2,\ 2]$ such that $f(c) = -1$?

2. Does the I.V.T. guarantee a value of c such that $f(c) = -1$ on the interval $[-2, 2]$? Why or why not?

3. Can you show for $g(x) = \frac{x}{x-3}$ that $g(c) = -\frac{1}{2}$ on the closed interval $[-3, 2]$? If so, find the value of c.

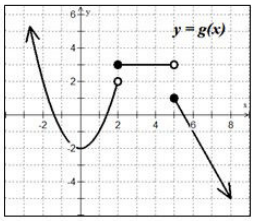
Grade:

1.9 Infinite Limits — Form B

Name _____

Date _____ Period _____

Problems 1 – 2: Use the graph of the function below.



1. Is there a value of c on $[-2,\ 4]$ such that $f(c) = -2$?

2. Does the I.V.T. guarantee a value of c such that $f(c) = -2$ on the interval $[-2, 4]$? Why or why not?

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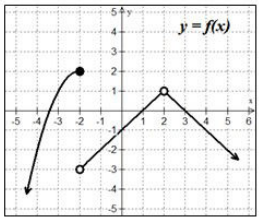
1.9 Infinite Limits — Form C

Name _____

Date _____ Period _____

Problems 1 – 2: Use the graph of the function below.



1. Is there a value of c on $[-4,\ 4]$ such that $f(c) = 1$?

2. Does the I.V.T. guarantee a value of c such that $f(c) = 1$ on the interval $[-4, 4]$? Why or why not?

3. Can you show for $f(x) = \frac{x}{x+4}$ that $f(c) = \frac{1}{2}$ on the closed interval $[-3, 5]$? If so, find the value of c.

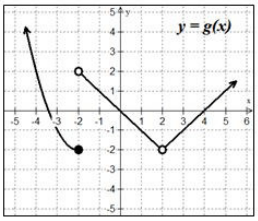
Grade:

1.9 Infinite Limits — Form D

Name _____

Date _____ Period _____

Problems 1 – 2: Use the graph of the function below.



1. Is there a value of c on $[-3,\ 4]$ such that $f(c) = 1$?

2. Does the I.V.T. guarantee a value of c such that $f(c) = 1$ on the interval $[-3, 4]$? Why or why not?

3. Can you show for $g(x) = -\left(\frac{1}{2}\right)^{3-x} - 2$ that $g(c) = -4$ on the closed interval $[3, 5]$? If so, find the value of c.

Grade:

Daily Lesson Quiz