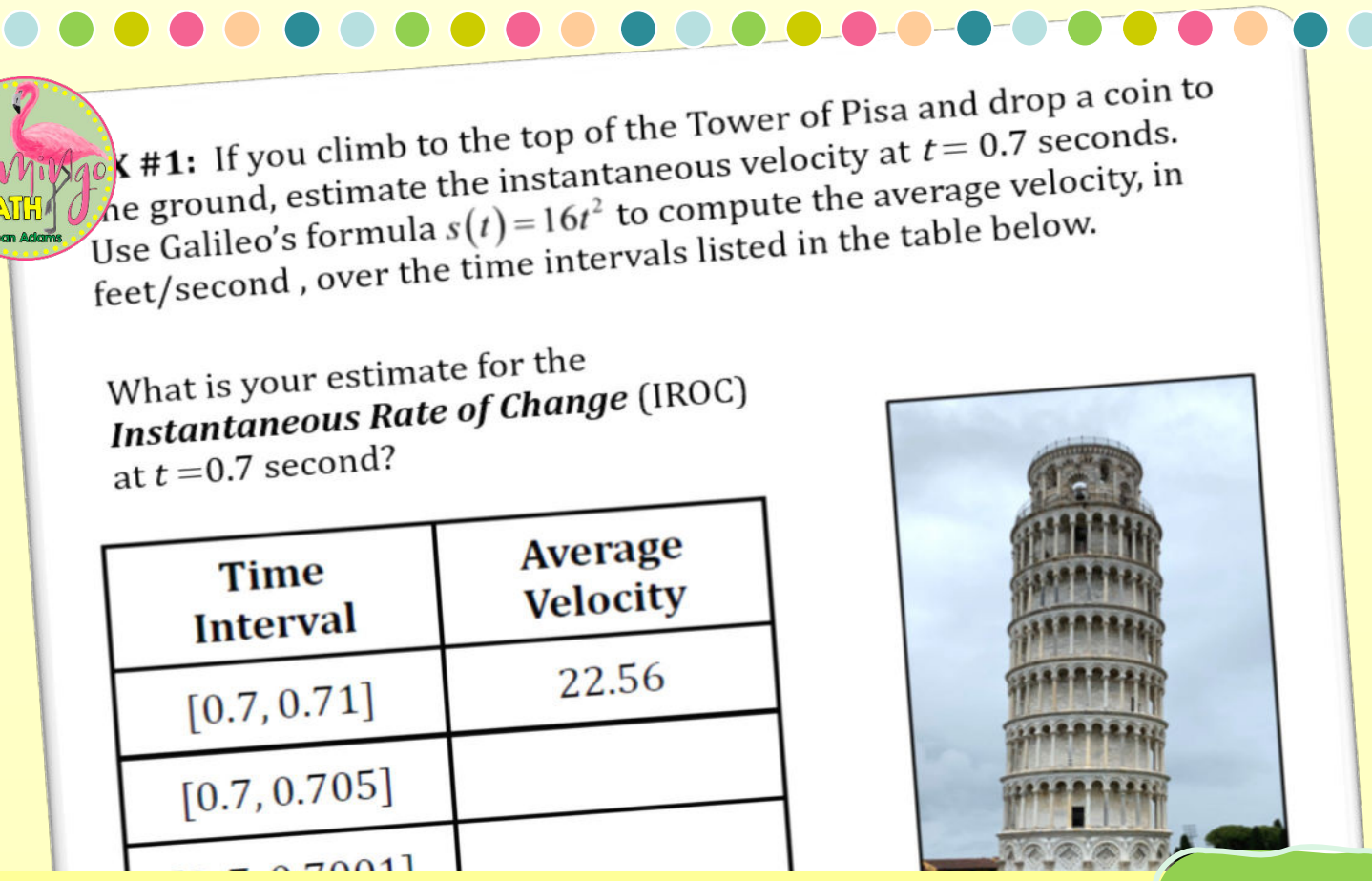
AP® Calculus

The Concept of Instantaneous Rate



X #1: If you climb to the top of the Tower of Pisa and drop a coin to the ground, estimate the instantaneous velocity at $t = 0.7$ seconds. Use Galileo's formula $s(t) = 16t^2$ to compute the average velocity, in feet/second , over the time intervals listed in the table below.

What is your estimate for the ***Instantaneous Rate of Change*** (IROC) at $t = 0.7$ second?

Time Interval	Average Velocity
[0.7, 0.71]	22.56
[0.7, 0.705]	
[0.7, 0.7001]	

Completely Editable 1.1

Lesson I: The Concept of Instantaneous Rate of Change

Topic 1.1: Introducing Calculus: Can Change Occur at an Instant?

As students of life, and algebra, you have studied many rates of change for different situations: how fast we drive, how our savings grow over time, the rate that the world population changes, our growth rate every year, unemployment rates, and so much more. In each of these studies, we can express one variable in terms of another, that is, as a function, $y = f(x)$.

Velocity

Consider an object that travels in a straight line. The **average velocity** over some given time interval can be defined as the ratio of the change in position (**displacement** or **net change**) to the elapsed time period:

Average Velocity =

However, when we think of velocity, we usually mean **instantaneous velocity**, which indicates the speed and direction of some object at a particular moment, which could be faster or slower than the average velocity.

Can you explain why we can't define instantaneous velocity as a ratio?

How can you estimate an instantaneous velocity?

Let's generalize that big idea with this principle:

Average velocity over a very small time interval is very close to instantaneous velocity.

EX #1: If you climb to the top of the Tower of Pisa and drop a coin to the ground, estimate the instantaneous velocity at $t = 0.7$ seconds. Use Galileo's formula $s(t) = 16t^2$ to compute the average velocity, in feet/second , over the time intervals listed in the table below.

Time Interval	Average Velocity
$[0.7, 0.71]$	22.56
$[0.7, 0.705]$	
$[0.7, 0.7001]$	
$[0.7, 0.70005]$	
$[0.7, 0.700001]$	

What is your estimate for the ***Instantaneous Rate of Change*** (IROC) at $t = 0.7$ second?



Photo: Jean Adams

THE STUDY OF RATE OF CHANGE IS FUNDAMENTAL TO THE STUDY OF CALCULUS.

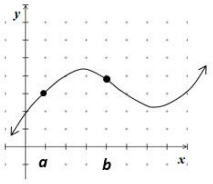
In our first example, we allowed the time interval to shrink to zero, so can say that *the average velocity **converges** to the instantaneous velocity* or that the *instantaneous velocity is the **limit** of the average velocity.*

Graphical Representation of Velocity

Average Speed (AROC) is defined by the total distance traveled, d, divided by the elapsed time period, t.

Graphically, the average speed is the slope of the secant line.

Analytically, average speed AROC $= \frac{\Delta d}{\Delta t}$

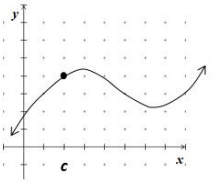


Sketch the secant line through a and b. *Estimate the average velocity.*

Instantaneous Speed (IROC) is the speed at a specific instant in time, where $t = t_0$ is the limit of the average rates of change.

Graphically, the instantaneous speed is the slope of the tangent line at a specific point.

Analytically, we need ***calculus*** to determine this instantaneous speed.



Sketch the tangent line through point c. *Can you estimate the instantaneous velocity at $x = c$?*

EX #2: Spin Class Heart Rate and Exercise

Have you ever taken a Spin Class at the gym? Then you know that after the class is over, your rapid heart rate decreases as time passes. Let $H(t)$ represent your heart rate. Write an expression for the rate of change of your heart rate over the period from $t = 0$ seconds to $t = 90$ seconds, after you have stopped exercising.

EX #3: Roller Coaster

A **hypercoaster** is a special circuit roller coaster with a height measuring greater than 200 feet. If a new hypercoaster is under design to have its largest drop on the circuit modeled by the equation below, where $d(t)$ is measured in feet, and time t, is measured in seconds.

$$d(t) = 1.87t^3 - 20.64t^2 + 112.79t + 132.56$$

A. Find the average speed (velocity) of the hypercoaster from 2 seconds to 8 seconds.

B. What is the average speed (velocity) of the hypercoaster from 4 seconds to 6 seconds?

C. Estimate the instantaneous speed (velocity) of the hypercoaster at exactly 5 seconds.

Time Interval	Average Velocity
$[5, 5.1]$	

D. Explain the meaning of the instantaneous speed (velocity)in the context of the question at $t = 5$ seconds.

EX #4: Airport Departures

The traffic pattern of departing flights at Orlando International Airport on a fall afternoon can be modeled by the function V defined by $V(t) = 92 - 15 \sin\left(\frac{t}{3}\right)$ where $V(t)$ is measured in vehicles and t is measured in minutes $0 \le t \le 30$.

A. Find $V(t)$ for $t = 18$.

B. Make a sketch of the graph of time *vs.* number of vehicles. Be sure your graph has a scale.



C. What is the average rate of change of the vehicles at the departing flights over the time interval $15 \le t \le 20$? Indicate units of measure.

D. Estimate the instantaneous rate of change of vehicles at $t = 18$ by finding the average rates from $t = 18$ to $t = 18.1$, $t = 18$ to $t = 18.01$, and $t = 18$ to $t = 18.001$.

Time Interval	Average Velocity

E. Why can't the instantaneous rate of change of traffic in the departure lane with respect to time be calculated using the method in part D?

NOTES

1.1 **The Concept of Instantaneous Rate of Change**

Name _____

Date _____ Period _____

1. An object dropped from a state of rest at time $t = 0$ travels a distance $s(t) = 4.9t^2$ meters in t seconds.

A. How far does the object travel during the time interval $[2.5, 3]$.

B. Find the average velocity of the object over $[2.5, 3]$.

C. Estimate the object's instantaneous velocity at $t = 2.5$ seconds.

Interval	$[2.5, 2.51]$	$[2.5, 2.505]$	$[2.5, 2.5001]$
Average Velocity			

2. Suppose Neil Armstrong decided to throw a golf ball into the air while he was standing on the moon and that the height of the golf ball was modeled by the equation below, where s is measured in feet and t is measured in seconds $s(t) = -2.72t^2 + 26.9t + 6$. Find the best approximation for the instantaneous rate of change (velocity) of the golf ball at 7 seconds.

3. A pendulum swings from the ceiling. Its distance, d, in feet, from one wall of the room depends on the number of seconds, t, since it was set in motion. Assume that the equation for d as a function of t is $d(t) = 20 + 16\cos\left(\frac{\pi t}{3}\right)$. You want to find out how fast the pendulum is moving at a given instant, t, and whether it is approaching or going away from the wall.

A. Find d when $t = 4$. What mode should your calculator be set?

B. Estimate the instantaneous rate of change of d with respect to t when t
the pendulum approaching the wall or moving away from it? Explain ho

4. Let $C(t)$ be the concentration of a drug in the bloodstream. As the body eliminates the drug, $C(t)$ decreases at a rate that is proportional to the amount of the drug that is present at the time. Write an algebraic expression to represent the average rate of change of the drug's concentration for the period $t = 0$ hour to $t = 4$ hours after the drug has been administered.

5. Compute $\Delta y/\Delta x$ for the interval $[2,6]$, where $y = 4x - 7$. What is the instantaneous rate of change of y with respect to x at $x = 2$?

6. An initial deposit of \$500 in your bank will have a balance after t years given by the equation $P(t) = 500(1.06)^t$ dollars.

A. What are the units of the rate of change of $P(t)$?

B. Find the average rate of change over $[0, 1]$.

C. Estimate the instantaneous rate of change at $t = 1$ by computing the average rate of change over intervals to the left and right of $t = 1$.

7. Each position graph shown below represents particle motion as a function of time. Match each graph with the proper description:

A) Speeding up **B)** Speeding up and then slowing down

C) Slowing down **D)** Slowing down and then speeding up

Practice Sets

1.1 Concept of Instantaneous Rate **Form A**

Name _____

Date _____Period _____

The height of a small weight is oscillating at the end of a spring measured in centimeters per second is modeled by $h(t) = 8\cos\left(\frac{3\pi}{2}t\right)$

1. Calculate the weight's average velocity over the time interval [3, 3.5]. **Show your work.**

2. Estimate the instantaneous velocity at $t = 3$. **Show your work.**

1.1 Concept of Instantaneous Rate **Form B**

Name _____

Date _____Period _____

The height of a small weight is oscillating at the end of a spring measured in centimeters per second is modeled by $h(t) = 6\cos\left(\frac{\pi}{2}t\right)$

1. Calculate the weight's average velocity over the time interval [3, 3.5]. **Show your work.**

2. Estimate the instantaneous velocity at $t = 3$. **Show your work.**

Daily Lesson Quiz