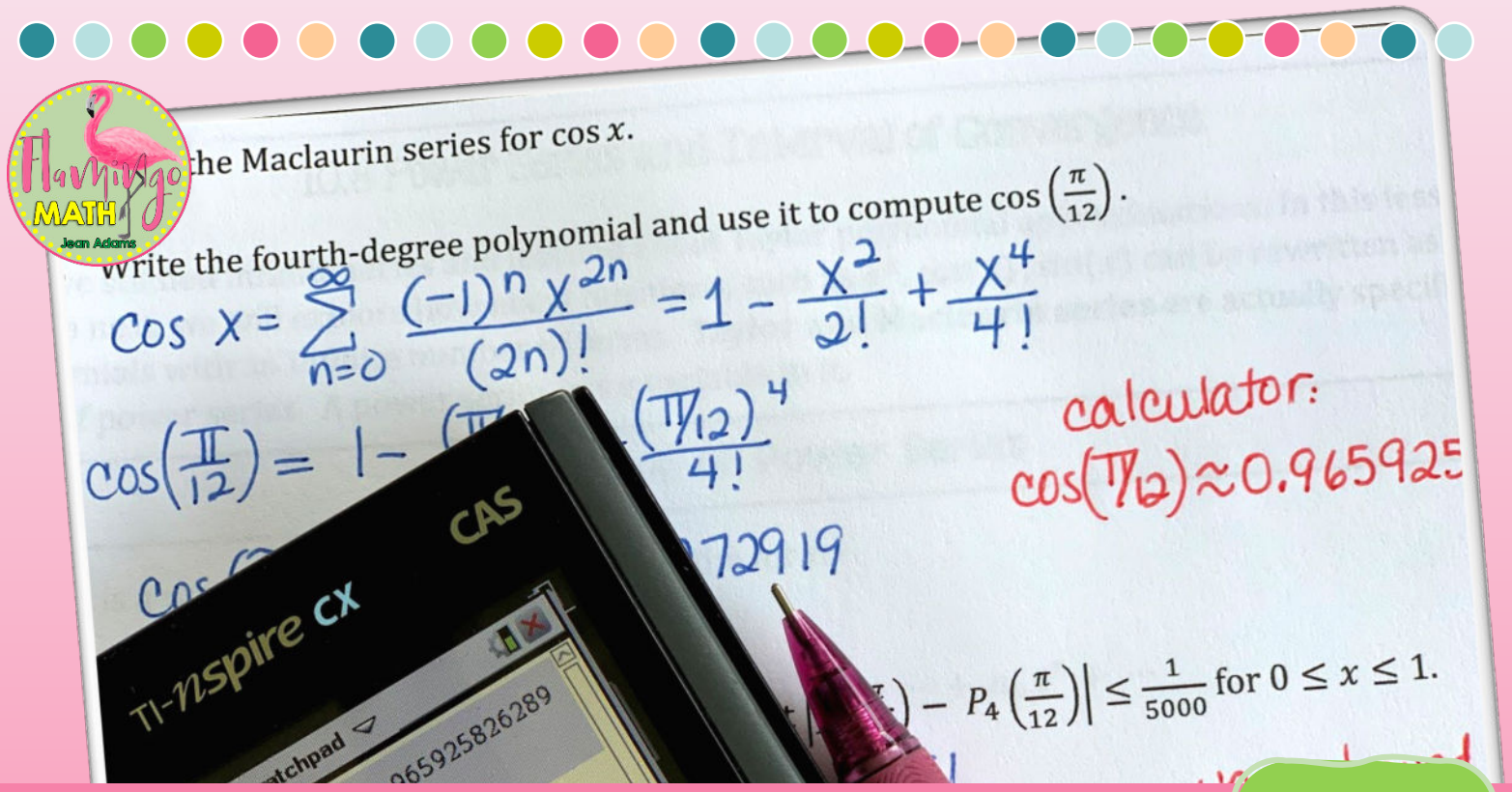
AP® Calculus-BC

Infinite Sequences and Series



Homework 10

Completely Editable

AP CALCULUS BC CURRICULUM

Scope and Sequence

Unit 10: Series and Convergence

Lesson	Skills & Learning Objectives
[10.1] Series and Convergence	❑ Use properties of infinite geometric series and Nth-Term Test for Divergence
[10.2] Integral Test p-Series	❑ Use the Integral Test to determine convergence and divergence of infinite series ❑ Use properties of convergence for p-series
[10.3] Comparison of Series	❑ Use the Direct Comparison Test and Limit Comparison Test for convergence
[10.4] Alternating Series	❑ Use the Alternating Series Test for convergence
[10.5] Ratio and Root Tests	❑ Use the Ratio Test and Root Test to determine whether a series converges or diverges
[10.6] Taylor Polynomials	❑ Find Taylor and Maclaurin polynomial approximations of elementary functions
[10.7] Lagrange Error Bound	❑ Use Taylor polynomials to estimate values of functions
[10.8] Power Series & Intervals of Convergence	❑ Find the radius and interval of convergence of a power series, find endpoint convergence
[10.9] Taylor and MacLaurin Series	❑ Differentiate and integrate a Taylor or Maclaurin series
[10.10] Representing Functions by Power Series	❑ Find a geometric power series that represents a function

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10.2 The Integral Test and p-Series Homework

Name _____

Date _____ Period _____

Problems 1 – 4, Expand the series or write an explicit rule to determine the sum of the convergent series. Find the sum, if geometric, or approximate the sum by using 6 terms.

1. $\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n$

2. $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n$

3. $\sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right)^n$

4. $\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\cdots$

Problems 5 – 12, Verify that the Integral Test can be applied. Then use the Integral Test to determine the convergence or divergence of each series.

5. $\sum_{n=1}^{\infty} \frac{3}{2n+5}$

6. $\sum_{n=1}^{\infty} 4^{-n}$

10.6 Taylor Polynomials Homework

Name _____

Date _____ Period _____

Problems 1-4, Find the *n*th term Taylor Polynomial for f centered at $x = c$.

1. $f(x) = \sqrt{x}\,,\ n = 3\,, c = 4$

2. $f(x) = \sin x\,, n = 5, c = \dfrac{\pi}{2}$

3. $f(x) = \cos x\,, n = 3, c = \dfrac{\pi}{4}$

4. $f(x) = \ln x, \qquad n = 5, c = 1$

5. Suppose $g(x)$ is a function which has continuous derivatives and is approximated near $x = 0$ by a fifth degree Taylor polynomial $T_5(x) = \frac{1}{5}x^5 - 3x^4 + 2x^3 - 7x + 1$. Give the value of each of the following.

A. $g(0)$ **B.** $g'(0)$ **C.** $g''(0)$

D. $g'''(0)$ **E.** $g^{(4)}(0)$ **F.** $g^{(5)}(0)$

10.7 Lagrange Error Bound and Accuracy Homework

Name _____

Date _____ Period _____

Problems 1 - 8, Use Taylor's Inequality to determine the error bounds of the approximations.

1. $e \approx 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$

2. $\cos(0.4) \approx 1 - \frac{(0.4)^2}{2!} + \frac{(0.4)^4}{4!}$

3. Estimate the value of $e^{1.5}$ by using a Taylor polynomial centered at $x = 2$, find the least degree of the polynomial needed to have an error less that 0.001.

4. Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 1$ is given by $P(x) = 6 - 9(x-1)^2 + 4(x-1)^3$. Suppose $\left|f^{(4)}(x)\right| \leq 6$ for all x on the interval [0,1]. Use the Lagrange error bound to justify why $f(0)$ is negative.

6. Let f be a function which has derivatives of all orders for all real numbers. Assume $f(4) = 2, f'(4) = -3, f''(4) = -1, f'''(4) = 5$.

A. Write the Taylor polynomial of degree 3 for f centered at $x = 4$.

B. Use the polynomial from part (A) to approximate $f(4.1)$

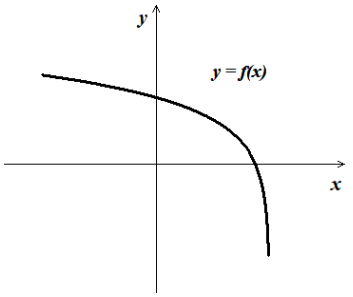
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7. Suppose that the function f is approximated near $x = 1$ by a third-degree Taylor polynomial $T_3(x) = 7 + 2(x-1)^2 - 5(x-1)^3$. Determine whether the function $f(x)$ has a local maximum, a local minimum, or neither at $x = 1$. Justify your answer.

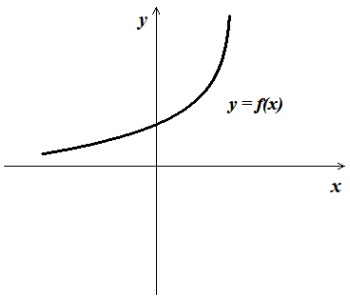
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8. Given $P_2(x) = a + bx + cx^2$ is the second-degree Taylor polynomial for f about $x = 0$. What can you say about the signs of a, b, and c if f has the graphs pictured below? Explain your reasoning.

A.



B.



10.9 Taylor and Maclaurin Series Homework

Name _____

Date _____ Period _____

Problems 1 – 8, Write the basic Maclaurin series representation, in general form, for each of the following:

1. $T(x)$ centered at $x = c$

2. $f(x) = \sin x$

3. $f(x) = \cos x$

4. $f(x) = e^x$

5. $f(x) = \dfrac{1}{1-x}$

6. $f(x) = \dfrac{1}{1+x}$

7. $f(x) = \tan^{-1} x$

8. $f(x) = \ln(1+x)$

9. Find the Maclaurin series for $f(x) = 3e^{-2x}$. Write the first three terms and the general term.

10. Find a Taylor polynomial for $f(x) = \sqrt{1+x}$ of degree 3, centered at $x = 0$.

18. Let $T_4(x) = 8 - 3(x-2) + 7(x-2)^2 - 4(x-2)^3 + 5(x-2)^4$

A. Find $f(2)$ and $f'''(2)$

B. Write a second-degree Taylor polynomial for $f'(x)$ about $x = 2$ and use it to approximate $f'(2.2)$.

C. Write the fourth-degree Taylor polynomial for $g(x) = \int_2^x f(t)dt$ about $x = 2$.

D. Is it possible to find $f(1)$ using the given information? Explain why or why not.

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19. Let $g(x)$ be a function with continuous derivatives and that $g(6) = 2, g'(6) = -3, g''(6) = 1$ and $g'''(6) = -2$.

A. Find a second-degree Taylor polynomial for g about $x = 6$.

B. Use your answer in part (A) to approximate $x = 5.9$