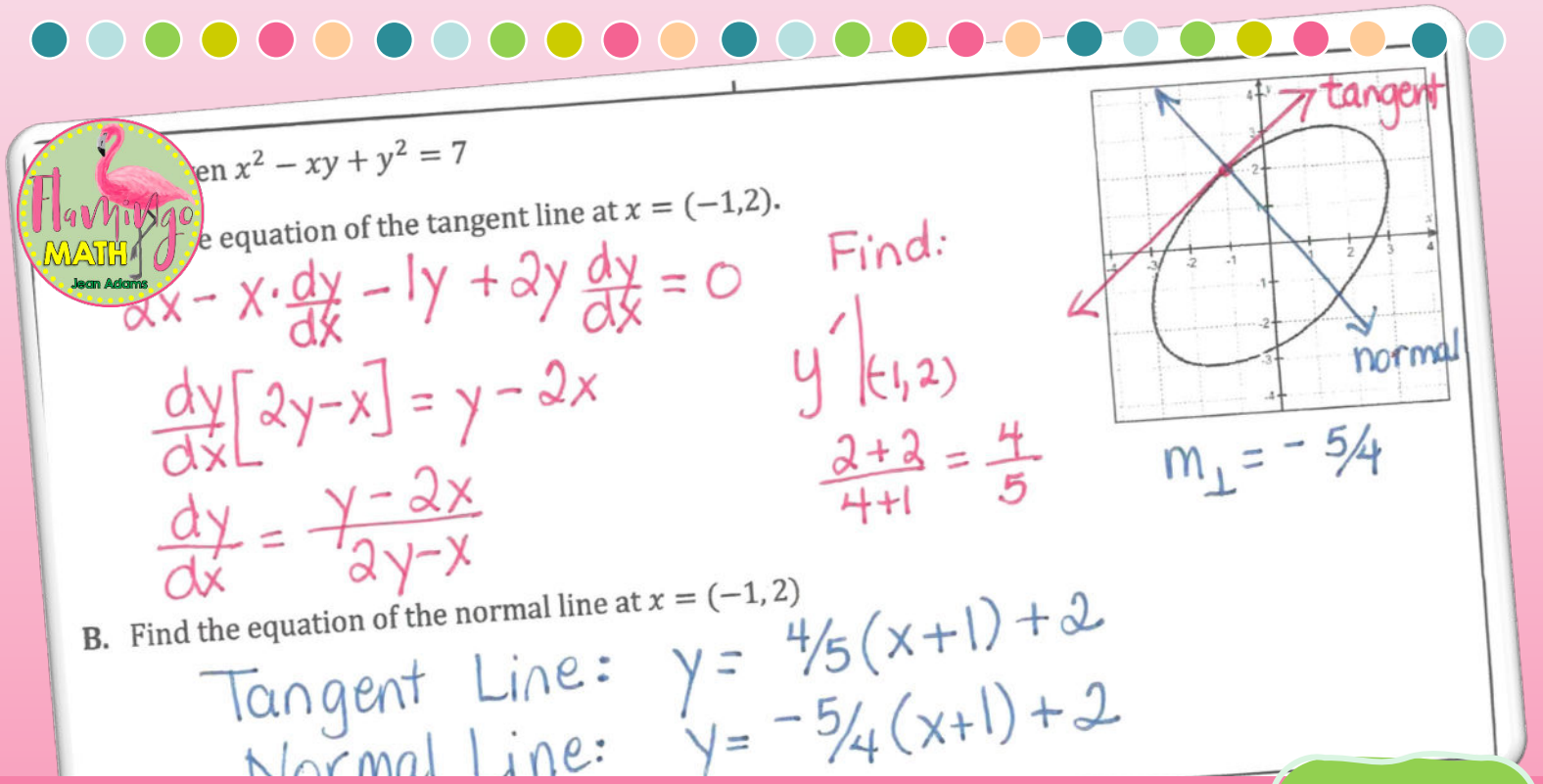
AP® Calculus

Implicit Differentiation



Completely Editable

3.3

Lesson 3: Implicit Differentiation

Topic 3.2: Implicit Differentiation

We have been able to differentiate functions that are solved for y explicitly, up to this point. Now we want to consider functions of the type $x^2 - 2y^3 + 4y = 2$. You can see that it would be quite challenging to solve for y as a function of x, explicitly.

Some basic facts to consider related to implicit differentiation:

- ❑ Realize differentiation is taking place with respect to x.
- ❑ When you differentiate terms involving x alone, you can differentiate as usual.
- ❑ When you differentiate terms involving y, you must apply the Chain Rule (because you are assuming that y is defined implicitly as a differentiable function of x.
- ❑ $\frac{d}{dx}[y] = \frac{dy}{dx}$
- ❑ $\frac{d}{dx}[x] = 1$

Guidelines for Implicit Differentiation:

1. Differentiate **both sides** with respect to x. *OBEY THE CHAIN RULE* by multiplying by dy/dx every time you differentiate an expression containing y.
2. Isolate dy/dx by getting all of the dy/dx terms onto one side of the equation, and all other terms onto the other side.
3. Factor out dy/dx if necessary.
4. Solve for dy/dx.

EX #1: Find $\frac{dy}{dx}$ for $xy + y = 8$ at $(3,2)$

EX #2: Differentiate $x + 3xy - 2y^2 = 2$ at $(1,1)$

EX #3: Find $\frac{dy}{dx}$: $y^3 + 5y^2 - 5y - x^2 = -4$

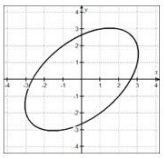
EX #4: Find y' for $ye^x = 3\pi$

EX #5: Differentiate: $\cos xy = x + 3y$

EX #6: Differentiate: $\ln(xy) = \sin x$

EX #7: Given $x^2 - xy + y^2 = 7$

A. Find the equation of the tangent line at $x = (-1,2)$.



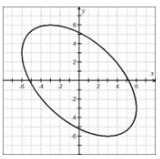
B. Find the equation of the normal line at $x = (-1, 2)$

Topic 3.6: Calculating Higher-Order Derivatives

When finding higher-order derivatives using implicit differentiation, be sure to solve for $\frac{dy}{dx}$ before finding the second derivative, or higher. The subsequent derivatives will have a $\frac{dy}{dx}$ term. For each of these terms, you can make a substitution to be able to put your final derivative in terms of x and y.

EX. #8: Find $\frac{d^2y}{dx^2}$ given $\cos y = x$

EX #9: Find the points at which the graph of $x^2 + xy + y^2 = 27$ has a either a horizontal or vertical tangent line.



NOTES

3.3 **Implicit Differentiation Homework**

Name _____

Date _____ Period _____

Problems 1 – 8, find dy/dx by implicit differentiation.

1. $8x^2 + y^2 = 10$

2. $\sqrt{x} + \sqrt{y} = 100$

3. $\frac{x}{y} + \frac{y}{x} = 3y$

5. $xe^y = 2xy + y^3$

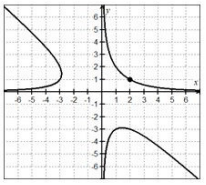
6. $\sin x + 2\cos 2y = 1$

7. $\cos(xy) = xy$

Problems 9 – 11, find the equation of the tangent line and the equation of the normal line to the curve at the indicate point.

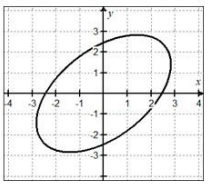
9. $x^3 + 2xy - y^2 = 11$ at $(2,3)$

10. $\sqrt{xy} = 12y - x$ at $(9,1)$

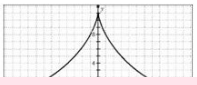


Problems 12 – 15, complete each of the following.

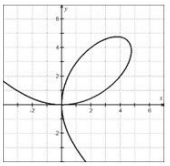
12. Given $x^2 - xy + y^2 = 6$, find the x-intercepts. Then, show that the tangent lines to the curve at these intercepts are parallel.



13. Write the equations of the tangent line for $x^{2/3} + y^{2/3} = 5$ at $(8,1)$ and $(-8,1)$. Are the tangent lines to the two points perpendicular? Explain.



, 2) and (2, 4).



ertical tangent line?

15. Find $\frac{d^2y}{dx^2}$ in terms of x and y, given $5x^2 - 2y^2 = 4$

Practice Sets

3.3 Implicit Differentiation Form A

Name _____

Date _____ Period _____

1. Find the derivative with respect to x.

$y^3 - 4y = 3x^2 + 2x$

2. If $3x^2 - xy^2 + y^{-1} = 1$, find y' at $(1, 1)$

3. Find $\frac{dy}{dx}$ for $y = \cos(xy)$

3.3 Implicit Differentiation Form B

Name _____

Date _____ Period _____

1. Find the derivative with respect to x.

$x^3 + 5x = 3y^2 - 2y$

2. If $5x^2 + 2x^2y + y^2 = 8$, find y' at $(1, 1)$

3.

3.3 Implicit Differentiation Form C

Name _____

Date _____ Period _____

1. Find the derivative with respect to x.

$x^5 + 7x = 5y - 3y^3$

2. If $x^{1/2} + 1 = y\left(1 + y^{\frac{1}{2}}\right)$, find y' at $(1, 1)$

3. Find $\frac{dy}{dx}$ for $y^2 = x\cos(y)$

3.3 Implicit Differentiation Form D

Name _____

Date _____ Period _____

1. Find the derivative with respect to x.

$y^4 + 6y = 7x - 4x^3$

2. If $y^2 - (xy)^{1/2} + 3x = 3$, find y' at $(1, 1)$

3. Find $\frac{dy}{dx}$ for $x = \sin(xy)$

Daily Lesson Quiz