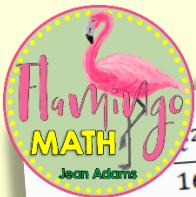


Precalculus Honors

Hyperbolas



$$\frac{x^2}{16} - \frac{(y+2)^2}{9} = 1$$

$$a = \frac{4}{\leftrightarrow} \quad b = \frac{3}{\updownarrow} \quad c = \frac{5}{\leftrightarrow}$$

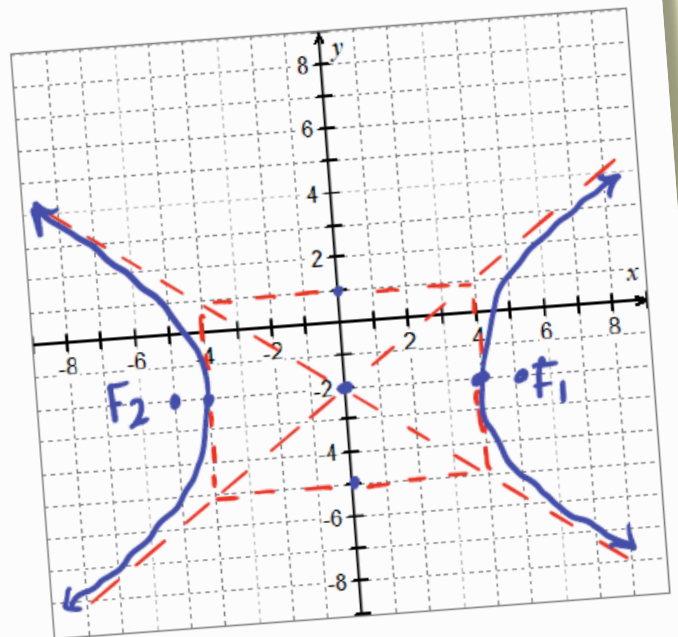
$$(h, k) \quad (0, -2)$$

$$\text{Foci} \quad (5, -2) \quad (-5, -2)$$

$$\text{Asymptotes} \quad y + 2 = \pm \frac{3}{4}x$$

$$\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \pm \frac{3}{4}$$

$$\text{int } (0, -2)$$



Completely Editable 8.4

Lesson 8.4: Hyperbolas

DEFINITION OF A HYPERBOLA

A **hyperbola** is the set of all points in a plane whose distances from two fixed points in the plane have a constant difference. The fixed points are the **foci** of the hyperbola. The line through the foci is the **focal axis**. The point on the focal axis midway between the foci is the **center**. The points where the hyperbola intersects its focal axis are the **vertices** of the hyperbola.

Verbal Vocabulary

Exploration: Label the hyperbolas in the visual vocabulary at the right, using the highlighted words below.

The point on the focal axis midway between the foci is the **center** of the hyperbola.

The foci, plural of focus, are the fixed points **equidistant** from the center and located on the **focal axis** (major axis).

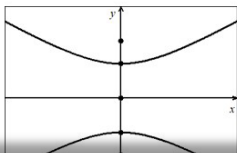
The **vertices** are the points where the hyperbola intersects its axis.

A **chord** is any line segment with endpoints on the hyperbola. The chord lying on the focal axis connecting the vertices is the **transverse axis** ($2a$).

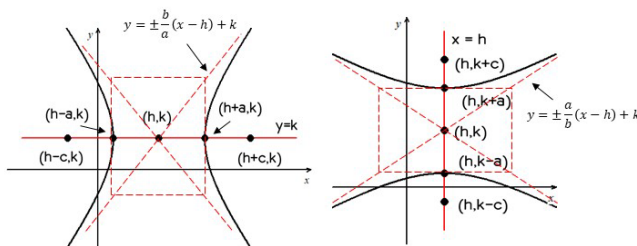
The **conjugate axis** is the line segment of length $2b$ that is perpendicular to the focal axis, with the center of the hyperbola as its midpoint.

The number a is the **semi-transverse axis** and the number b is the **semi-conjugate axis**.

Visual Vocabulary



Hyperbolas with Center (h, k)



Major Axis

Horizontal

Vertical

Equation

Focal Axis

Foci

Vertices

Semi-transverse

Semi-conjugate

Pythagorean Relation

Asymptotes

Graphing Basic Hyperbolas

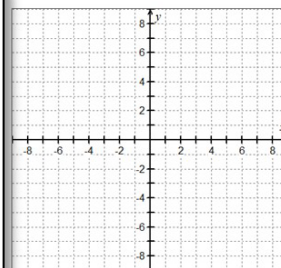
EX #1: Find the vertices and the foci of the hyperbola

$$9x^2 - 16y^2 = 144$$

Major vertices: _____

Minor vertices: _____

Foci: _____



Finding Equations from

EX #3: Write an equation for the hyperbola. Then sketch the location for each focus point.

$$a = \quad b = \quad c =$$

Equation: _____

(h, k) _____

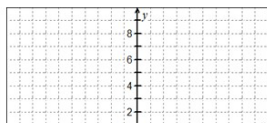
Foci _____

Asymptotes: _____

Finding an Equation of a Hyperbola

EX #4: Find the equation in standard form for a hyperbola whose transverse axis endpoints are $(-1, -3)$ and $(-1, 7)$ and the conjugate axis length is 8.

$$a = \quad b = \quad c =$$



Eccentricity

The **eccentricity** of a hyperbola is $e = \frac{c}{a}$ where a is the semi-transverse axis, b is the semi-conjugate axis, and c is the distance from the center of the hyperbola to either focus.

EX #5: Prove that the graph of the equation is a hyperbola, find its vertices, foci, asymptotes, and eccentricity.

$$4x^2 - 9y^2 - 16x - 72y - 164 = 0$$

Major vertices _____ Foci _____

NOTES

Asymptotes: _____

8.4

Hyperbolas Homework

Name _____

Date _____ Period _____

Problems 1 – 6, the graph of an ellipse is given. Match each graph to its equation.

A. $\frac{x^2}{9} - y^2 = 1$

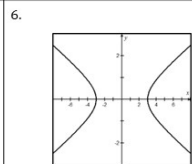
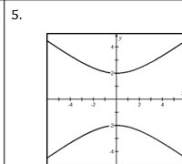
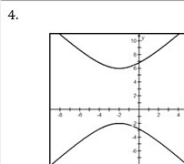
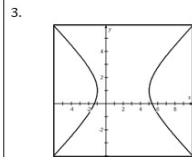
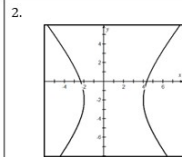
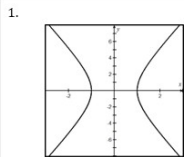
B. $x^2 - \frac{y^2}{9} = 1$

C. $\frac{y^2}{4} - \frac{x^2}{9} = 1$

D. $\frac{(x-1)^2}{9} - \frac{(y+2)^2}{16} = 1$

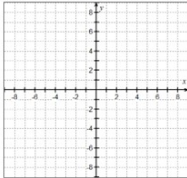
E. $\frac{(x-2)^2}{9} - \frac{(y-1)^2}{4} = 1$

F. $\frac{(y-2)^2}{16} - \frac{(x+2)^2}{9} = 1$

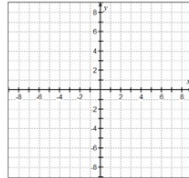


Problems 7 – 12, find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation.

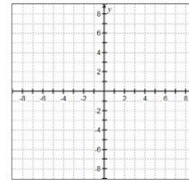
7. $\frac{x^2}{25} - \frac{y^2}{16} = 1$



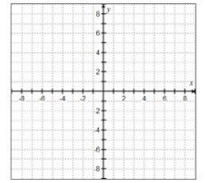
8. $\frac{y^2}{4} - \frac{x^2}{25} = 1$



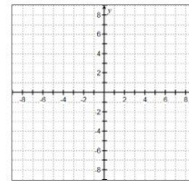
9. $4x^2 - 9y^2 = 36$



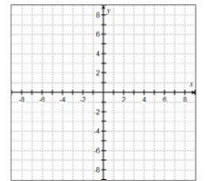
10. Foci at $(\pm 5, 0)$ vertex at $(3, 0)$



11. Focus at $(0, 6)$ vertices at $(0, \pm 2)$

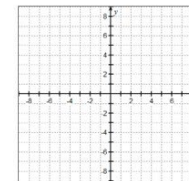


12. Vertices at $(\pm 3, 0)$ asymptotes $y = \pm \frac{2}{3}x$

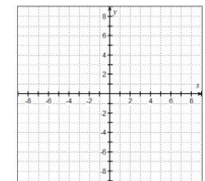


Problems 13 – 16, find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation.

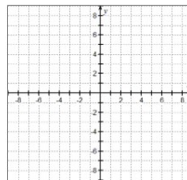
13. $\frac{(x+2)^2}{9} - \frac{(y-3)^2}{16} = 1$



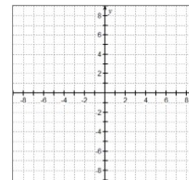
14. $\frac{(y-3)^2}{25} - \frac{(x+2)^2}{16} = 1$



15. $x^2 - y^2 - 4x + 4y - 9 = 0$



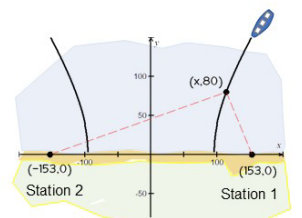
16. $4y^2 - x^2 - 24y - 4x + 16 = 0$



Problems 17-19, solve.

17. Voyager 2, a space probe, was launched by NASA in August, 1977 to study the outer planets. As it approached Saturn the gravitational pull of the planet changed the probe's path to a hyperbola that diverges from its asymptote. Assuming that the center of the hyperbola is at the origin and the transverse axis is horizontal, find an equation that models the path of the Voyager given that $a = 332,956$ km and $c = 492,788.2$ km.

19. LORAN or Long Distance Radio Navigation uses synchronized pulses between widely separated stations traveling at the speed of light (186,000 miles per second). The difference between the times of arrival of the pulses at an airplane or ship is constant on a hyperbola where the stations are located at the foci. If two stations are 306 miles apart and the ship is located at $(x, 80)$. [Picture not drawn to scale.]



A. Find the x-coordinate of the location of the ship if the time difference between the pulses from the stations is 1022 microseconds (0.001022 second).

B. When the ship reaches the shore, what is the distance to station 1?

Problems 20 – 27, classify the graph of the equation as a line, circle, parabola, ellipse, or hyperbola.

20. $x^2 + 9y^2 - 4x - 18y - 23 = 0$

21. $x^2 + y^2 + 6x - 10y + 6 = 0$

26. $25x^2 - 7y^2 - 175 = 0$

27. $y^2 + 12x + 4y + 28 = 0$

Practice Sets

Lesson Warm-Ups

$$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{16} = 1$$



3. Write the equation for the hyperbola with center at the origin, foci $F(0, \pm 5)$ and vertices $V(0, \pm 4)$.

Name _____ Date _____ Period _____

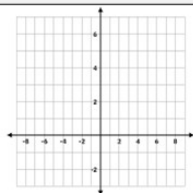
Lesson 8.4: Hyperbolas

Warm-Up # 2

1. Find the vertices, foci, and equation of the asymptotes for the hyperbolas.

$$\frac{(x+2)^2}{16} - \frac{(y-3)^2}{9} = 1$$

2. Sketch the graph.



3. Write the equation for the hyperbola with center at the origin, foci $F(\pm 6, 0)$ and vertices $V(\pm 2, 0)$.

Form A

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8.4 Hyperbolas

Form B

Name _____

Date _____ Period _____

1-2: Find the vertices, foci, and equations of the asymptotes for the hyperbolas, and sketch the graph in the grid below.

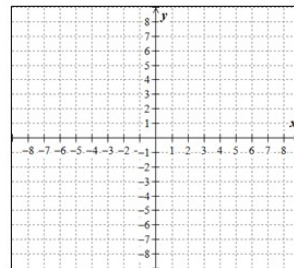
1. $\frac{(x+3)^2}{9} - \frac{(y-2)^2}{16} = 1$

Vertices: _____

Foci: _____

Asymptotes: _____

2.

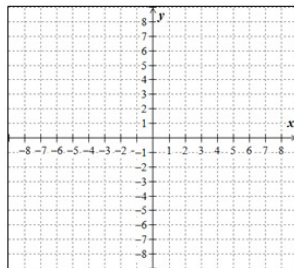


3. Write the equation for the hyperbola with center at origin, foci $F(\pm 5, 0)$ and vertices $V(\pm 3, 0)$.

Foci: _____

Asymptotes: _____

2.



3. Write the equation for the hyperbola with center at origin, foci $F(0, \pm 4)$ and vertices $V(0, \pm 2)$.

Daily Lesson Quiz

Grade: _____

Grade: _____