

Precalculus Honors

Graphs of Rational Functions



Lesson 6: Graphs of Rational Functions

Rational Functions

Let $N(x)$ (numerator) and $D(x)$ (denominator) be polynomial functions, where $D(x) \neq 0$.

Then, a Rational Function is:

$$R(x) = \frac{N(x)}{D(x)}$$

Finding Domain

The domain of x includes all reals except the values that cause division by zero.

(Domain restrictions come from the values that make the denominator zero.)

EX #1: Find the domain for $f(x) = \frac{x+7}{x^2-3x+2}$.

Domain:
 $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

Completely Editable

2.6

Lesson 6: Graphs of Rational Functions

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Then, a Rational Function is:

Finding Domain

The domain of x includes _____ except the values that cause _____ by _____.

(Domain restrictions come from the values that make the _____.)

EX #1: Find the domain for $f(x) = \frac{x+7}{x^2-3x+2}$.

Horizontal and Vertical Asymptotes

Vertical Asymptotes: _____

The graph behavior will approach the vertical asymptote as x increases or decreases without bound. That is,

$$\lim_{x \rightarrow a^-} f(x) = \text{or} \quad \lim_{x \rightarrow a^+} f(x) =$$

Horizontal Asymptotes: _____

The graph behavior will approach the horizontal asymptote as x increases or decreases without bound. That is,

$$\lim_{x \rightarrow -\infty} f(x) = \text{or} \quad \lim_{x \rightarrow \infty} f(x) =$$

Finding Holes, Vertical Asymptotes and x -intercepts

Both holes and vertical asymptotes are found when setting the denominator equal to zero.

Steps:

- (1.) Simplify the rational function by _____ both the numerator and denominator.
- (2.) Find the _____ of both (numerator and denominator)
- (3.) If a zero is common to the denominator and numerator, you have a _____.
- (4.) If a zero is unique to the denominator, you have a _____.
- (5.) If a zero is unique to the numerator, you have an _____.

EX #2: Find the holes, vertical asymptote(s) and x -intercepts of the following rational function.

$$f(x) = \frac{x^2 - 3x}{3x^2 + 6x}$$

Horizontal Asymptotes

Example:

denominator
horizontal asymptote.

denominator
coefficients is the

denominator
horizontal asymptote.

EX #3: Find the horizontal asymptotes for the given functions and provide a reason.

(A) $f(x) = \frac{1}{3x^2 + 3x - 18}$

(B) $f(x) = \frac{2x-2}{x+2}$

(C) $f(x) = \frac{3x^2 + 3x - 18}{3x - 5}$

Slant (Oblique) Asymptotes

A slant (oblique) asymptote occurs when the numerator is one degree higher than the degree of the denominator.

The slant asymptote is the quotient polynomial found by using synthetic or long division.

EX #4: Determine the slant asymptote of the function $f(x) = \frac{x^2 - 6x + 7}{x + 5}$.

It will become important, as we move through our course, to be able to graph rational functions completely **without** access to a graphing utility. You should be thinking to help you along this pathway.

Graphing and Analyzing a Rational Function

Process:

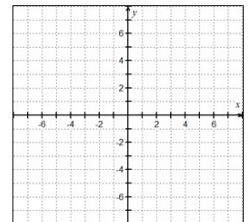
1. Determine the domain
2. Find the asymptote(s) and hole(s)

EX #5: Find the asymptotes and intercepts of

$$f(x) = \frac{x-3}{x^2 + x - 6}$$

EX #7: Graph the function using the process above:

$$f(x) = \frac{2x^2 - 5x - 3}{x^2 + x - 12}$$



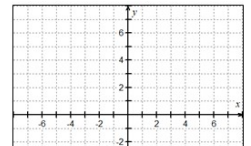
EX #8: Analyze and draw the graph of $f(x) = \frac{x^2 - 2x - 8}{x^2 - 4}$

x -intercept(s):

y -intercept:

Vertical Asymptote(s):

Hole:



NOTES

2.6 Graphs of Rational Functions Homework

Name _____

Date _____ Period _____

Problems 1 – 2, Find the domain of the function and use limits to describe the behavior of f at value(s) of x not in the domain.

1. $f(x) = \frac{-2}{x-3}$

2. $f(x) = \frac{1}{x^2-4}$

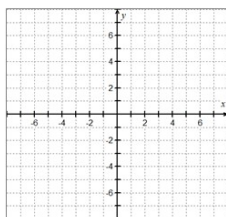
Problems 3 – 4, Analyze the function: determine the domain and find any asymptotes/holes

3. $f(x) = -\frac{2x^2}{x^2+3x-4}$

4. $f(x) = \frac{x^2}{x+5}$

Problems 5 – 8, For each of the given functions, find the domain, holes, asymptotes, intercepts, and end-behavior. Then sketch a graph.

5. $f(x) = \frac{x^2-x-2}{x^2-2x-3}$



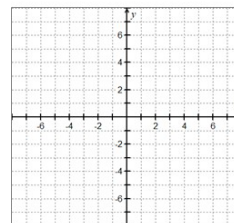
6. $f(x) = \frac{x-1}{x^2-x-12}$

Domain

Intercepts

Asymptotes/Holes

End behavior



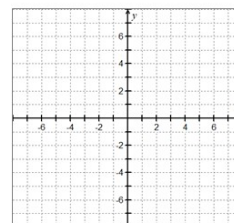
7. $f(x) = \frac{x^2-1}{x-1}$

Domain

Intercepts

Asymptotes/Holes

End behavior



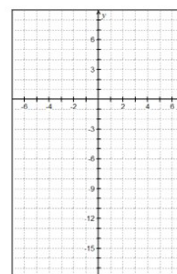
8. $f(x) = \frac{2x^2+x-3}{x+2}$

Domain

Intercepts

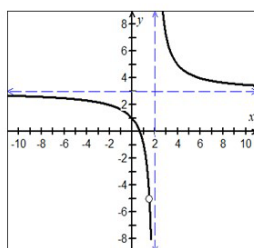
Asymptotes/Holes

End behavior



Problems 9 – 14, The graph of the rational function, $g(x)$ is shown below. There is a removable discontinuity at the point $(\frac{3}{2}, -5)$ and the x -intercept is $(\frac{2}{3}, 0)$.

9. State the domain and range of the function $g(x)$.



Graph of $g(x)$

10. Use proper notation to describe the behavior of the function as x approaches the infinite discontinuity from the left and from the right.

11. What factor(s) will be common to both the numerator and denominator? How do you know?

12. Describe the end-behavior of the function. How is this represented on the graph of $g(x)$?

13. Name the missing factor of the numerator. How is this represented on the graph?

Free Response Table Problem

x	-80	-6	-4	-3	-2.005	-2	-1.995	0	1	1.995	2	2.005	6	80
$f(x)$	1.948	1	0	-2	-798	undefined	802	4	3.333	3.001	undefined	2.998	2.5	2.048

The table above represents values on the graph of the function $f(x) = \frac{2x^2 + 4x - 16}{x^2 - 4}$

A. For what value(s) of x does the graph of $f(x)$ have a vertical asymptote? Give a reason for your answer.

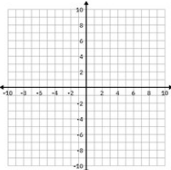
B. Does the function contain a point discontinuity? If so, name the coordinates of the hole. Explain.

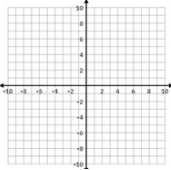
C. Use limit notation to describe the end behavior of the function.

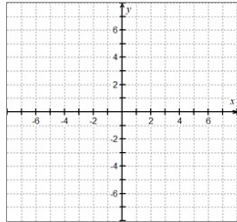
D. Consider the values in the table, what factor will be guaranteed in the numerator? Give a reason for your answer.

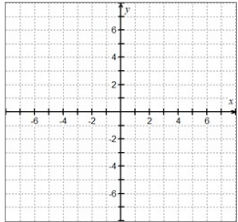
Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 2.6: Graphs of Rational Functions Warm-Up # 1	
1. Find the vertical asymptotes of: $f(x) = \frac{3x^2}{x^2 - 16}$	2. Find the horizontal asymptotes of the function in #1. State the end behavior of $f(x)$.
3. State the intercepts and domain and range.	4. Sketch the graph. 

Name _____ Date _____ Period _____	
Lesson 2.6: Graphs of Rational Functions Warm-Up # 2	
1. Find the vertical asymptotes of: $f(x) = \frac{-3x^2}{x^2 - 1}$	2. Find the horizontal asymptotes of the function in #1. State the end behavior of $f(x)$.
3. State the intercepts and domain and range.	4. Sketch the graph. 

2.6 Graphs of Rational Functions Form A	
Name _____	
Date _____ Period _____	
Problems 1-4: $f(x) = \frac{2x^2}{x^2 - 4}$	
1. Find the vertical asymptotes.	
2. Find the horizontal asymptotes. State the end-behavior of $f(x)$	
3. State the intercepts, domain and range.	
4. Sketch the graph. 	
Grade: _____	

2.6 Graphs of Rational Functions Form B	
Name _____	
Date _____ Period _____	
Problems 1-4: $f(x) = \frac{-2x^2}{x^2 - 9}$	
1. Find the vertical asymptotes.	
2. Find the horizontal asymptotes. State the end-behavior of $f(x)$	
3. State the intercepts, domain and range.	
4. Sketch the graph. 	
Grade: _____	

Daily Lesson Quiz