

Precalculus Honors

Graphs of Functions



We read a graph in mathematics from left to right,
the same way we read a book.

EX #3: Consider the function $y = f(x)$ shown at right,
trace each interval where the function behavior is

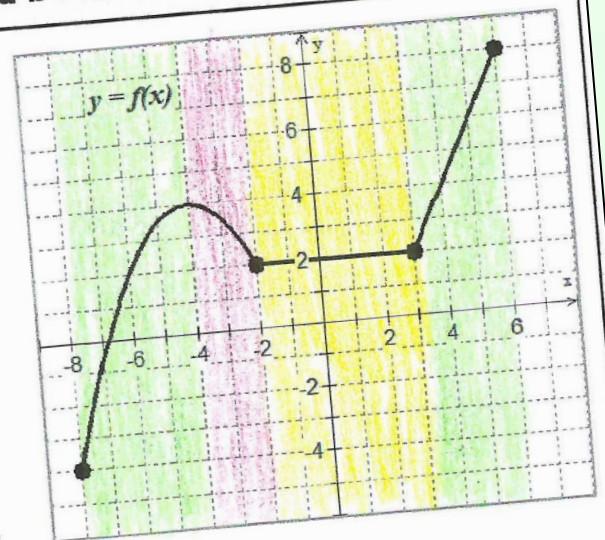
A. **Increasing**, using a GREEN pencil
Identify the interval(s) $(-8, -4)$ and $(3, 6)$

B. **Decreasing**, using a RED pencil
Identify the interval(s) $(-4, -2)$

C. **Constant**, using a YELLOW pencil
Identify the interval(s) $(-2, 3)$

D. State the **Domain** of the function $[-8, 6]$

E. State the **Range** of the function $[-4, 8]$



Completely Editable

1.2

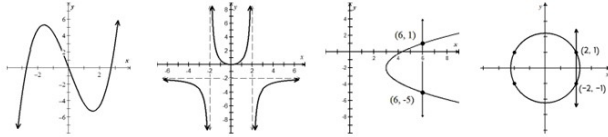
Lesson 2: Graphs of Functions

In the next two lessons we will focus on the characteristics and key features that will be used throughout our course to analyze and describe function behavior. You will develop skills to quickly graph a function by analyzing the characteristics of function behavior. We begin with some familiar terms.

Vertical Line Test

The vertical line test is a visual way to determine if a curve is a graph of a function or not. A set of points in a coordinate plane is the graph of y as a function of x if and only if no vertical line intersects the graph at more than one point.

If every vertical line intersects the graph of a relation in no more than one point, then the relation is a function.



Intercepts

x - intercepts

y - intercepts

EX #4: Use a graphing utility to determine the open intervals on which the function is increasing, decreasing, or constant.

A. $f(x) = x^3 - 3x^2 + 2$

Increase _____

Decrease _____

Constant _____

B. $g(x) = |x + 2| + |x - 1| - 2$

Increase _____

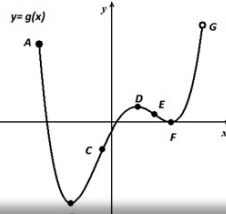
Decrease _____

Constant _____

Local and Absolute Extrema

When the graph of a function is increasing to the left of $x = c$ and decreasing to the right of $x = c$, then at $x = c$ the value of f is largest. This y -value is called a **local maximum** of f . If $f(c)$ is greater than (or equal to) all range values of f , then $f(c)$ is an **absolute maximum value** of f .

When the graph of a function is decreasing to the left of $x = c$ and increasing to the right of $x = c$, then at $x = c$ the value of f is smallest. This y -value is called a **local minimum** of f . If $f(c)$ is less than (or equal to) all range values of f , then $f(c)$ is an **absolute minimum value** of f .



Symmetry

The terms even and odd describe the symmetry that exists for the graph of a function.

Even

A function f is even if and only if, when the point (x, y) is on the graph of f , then the point $(-x, y)$ is also on the graph.

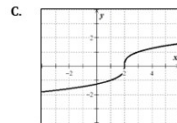
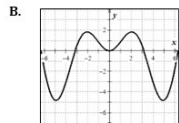
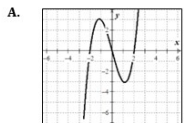
$$f(-x) = f(x)$$

Odd

A function f is odd if and only if, when the point (x, y) is on the graph of f , then the point $(-x, -y)$ is also on the graph.

$$f(-x) = -f(x)$$

EX #7: Determine whether each graph is even, odd or neither.



EX #8: Algebraically determine whether each of the following functions is even, odd or neither.

A. $y = |x| - 2$

B. $y = x^3 - 3$

C. $y = 2x^3 - x$

In general terms, we can say that a function **increases** over an open interval of its domain if its graph **rises from left to right on the interval**. Likewise, a function **decreases** over an open interval of its domain if its graph **falls from left to right on the interval**. Finally, a function is **constant** over an open interval of its domain if its graph is **horizontal on the interval**.

It is important to note that the **intervals** refer to the x -values where the y -values either increase, decrease, or remain constant.

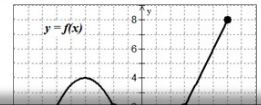
Definition of Increase, Decrease and Constant

Suppose that a function f is defined over an **open** interval, I and x_1 and x_2 are in I .

- KEY CONCEPT**
- ☐ f is increasing on interval I if, for any two points in the interval, a positive change in x results in a positive change in $f(x)$. $x_1 < x_2, f(x_1) < f(x_2)$.
 - ☐ f is decreasing on an interval I if, for any two points in the interval, a positive change in x results in a negative change in $f(x)$. $x_1 < x_2, f(x_1) > f(x_2)$.
 - ☐ f is constant on an interval I if, for any two points in the interval, a positive change in x results in a zero change in $f(x)$. For every x_1 and x_2 , $f(x_1) = f(x_2)$.

We read a graph in mathematics from left to right, the same way we read a book.

EX #3: Consider the function $y = f(x)$ shown at right, trace each interval where the function behavior is



A. **Increasing**, using a GREEN pencil

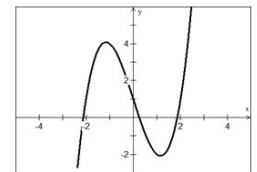
Identify the interval(s)

Approximating Extrema

We will explore techniques for finding extrema (maximum and minimum values) for polynomial functions in a later chapter. For now, we can use a graphing calculator to approximate extrema.

EX #6: Use a graphing utility, sketch $f(x) = x^3 - 4x + 1$.

Then, determine the local extrema. Give values rounded to two decimal places.



Local maximum value: _____

Occurs at: _____

Local minimum value: _____

Occurs at: _____

Boundedness

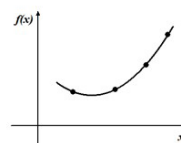
The idea of **boundedness** is conceptually similar to extrema. Let's take a look at the concept from a graphical point of view.



Concavity and Inflection Points

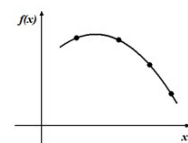
An **inflection point** is where a curve changes from **concave upward** to **concave downward** (or vice versa). We need Calculus to algebraically find where a curve goes from concave up to concave down.

Concave Up



Slope is increasing

Concave Down



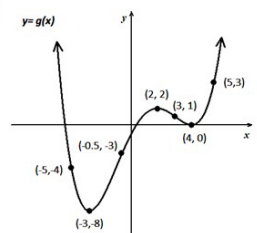
Slope is decreasing

EX #10: Use the graph of $g(x)$ shown at right to determine intervals of concavity and state the points of inflection.

Concave Up: _____

Concave Down: _____

Points of Inflection: _____



NOTES

(1, -1)

Problems 1 – 3, Find explicit form of the equation and use a graphing utility to determine whether each relation is a function by applying the vertical line test.

1. $x^2 - y^2 = 25$

2. $x^2 = 3xy - 6$

3. $2y - 4x^2 + 8 = 0$

Problems 4 – 7, Use a graphing utility to graph the function and determine the open intervals on which the function is increasing, decreasing, or constant.

4. $f(x) = x^{2/3}$

5. $g(x) = 5$

6. $h(x) = x\sqrt{x+2}$

7. $y = |x-2| + |x+1|$

Problems 8 – 11, Use a graphing utility to approximate any relative maximum or relative minimum values of the function. Give your estimate to the nearest hundredth.

8. $y = x^3 - 3x + 2$

9. $f(x) = (x-3)\sqrt{x}$

10. $g(x) = x^2 - 5x$

11. $h(x) = 2x^3 + 3x^2 - 4x$

Problems 12 – 15, Algebraically determine whether the function is even, odd, or neither:

12. $y = 2x^3 - 4x$

13. $y = \frac{2}{1+x^2}$

14. $g(x) = x^5 + 2x^3 - x$

15. $f(x) = \sqrt{4-x}$

Problems 16 – 19, Determine whether the function is bounded above, bounded below, or bounded on its entire domain.

16. $y = e^{-x}$

17. $y = 3 - x^2$

18. $y = 2x - x^3$

19. $y = \sqrt{4-x^2}$

Problems 20 – 23, Analytically determine what type(s) of symmetry, if any, the graph of the equation would possess. Show your work.

20. $x^2 + 2y = 7$

21. $y^2 - xy = 6$

22. $x^2 + y^2 = 3|x|$

23. $2x^2 - 3 = 4|y|$

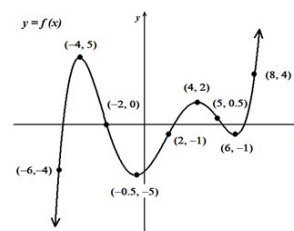
Problems 24 – 25, Given the continuous function, determine how many inflection points would be found on each graph. Show the approximate location of each estimate.

Problems 26 – 28, Use the graph of $y = f(x)$ shown below to complete the questions.

26. Name the points of inflection on the graph of $y = f(x)$.

27. On what interval(s) is the graph of f concave up?

28. On what interval(s) is the graph of f concave down?



FREE RESPONSE QUESTION

29. **Consider the scenario:**

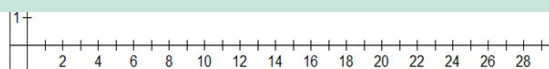
Hayden leaves home to walk to school, walks 2 blocks in 6 minutes at a constant speed. He forgot to bring his lunch, so he runs home in 3 minutes. It takes him two minutes to find his lunch and leave. Hayden walks the 5 blocks to school in 14 minutes and stops at the front of the school to talk to his friend Matt for 2 minutes.

A) Draw a graph of Hayden's distance from home (in blocks) as a function of time. Label the axes properly.

B) State the domain and range of the scenario.

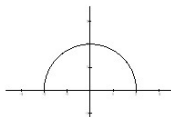
C) Determine the open intervals where the function is increasing, decreasing, or constant.

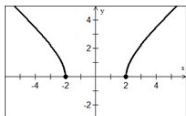
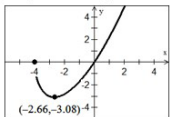
Practice Sets

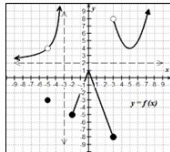
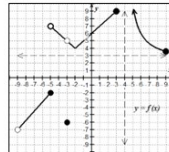


Name _____ Date _____ Period _____	
Lesson 1.2: Graphs of Functions Warm-Up # 1	
1. Find the domain and range of f .	2. Determine the intervals over which the function from #1 is increasing, decreasing, or constant.

Lesson Warm-Ups

$g(x) = x^3 + x^2 - 3x$	
Name _____ Date _____ Period _____	
Lesson 1.2: Graphs of Functions Warm-Up # 2	
1. Find the domain and range of f .	2. Determine the intervals over which the function from #1 is increasing, decreasing, or constant.
	
3. Using a graphing utility, approximate the maximum/minimum of the function. $g(h) = 2x - 3$	4. Determine if the function is even, odd, or neither. $y = \frac{3}{x^2 + 5}$

1.2 Graphs of Functions-Day 1 Form A Name _____ Date _____ Period _____ Use the graph of f to answer problems 1 & 2 	1.2 Graphs of Functions-Day 2 Form B Name _____ Date _____ Period _____ Use the graph of f to answer problems 1 & 2 
1. Find the domain and range of f . Domain: _____ Range: _____	1. Find the domain and range of f . Domain: _____ Range: _____
2. Determine the intervals over which the function is increasing, decreasing, or constant.	2. Determine the intervals over which the function is increasing, decreasing, or constant.
3. Use a graphing utility to approximate the relative minimum/relative maximum of the function. $f(x) = x^3 + x^2 - 6x$	3. Use a graphing utility to approximate the relative minimum/relative maximum of the function. $f(x) = x^3 + x^2 - 4x$

1.2 Graphs of Functions-Day 2 Form A Name _____ Date _____ Period _____ Use the graph of f to answer problems 1 - 4 	1.2 Graphs of Functions-Day 2 Form B Name _____ Date _____ Period _____ Use the graph of f to answer problems 1 - 4 
1. Find the domain and range of f . Domain: _____ Range: _____	1. Find the domain and range of f . Domain: _____ Range: _____
2. Determine the intervals over which the function is increasing or decreasing.	2. Determine the intervals over which the function is increasing, decreasing, or constant.
3. Name the x-value for each of the following: Point discontinuity: _____	3. Name the x-value for each of the following: Point discontinuity: _____

Daily Lesson Quiz

Grade: _____	Grade: _____	Grade: _____	Grade: _____
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