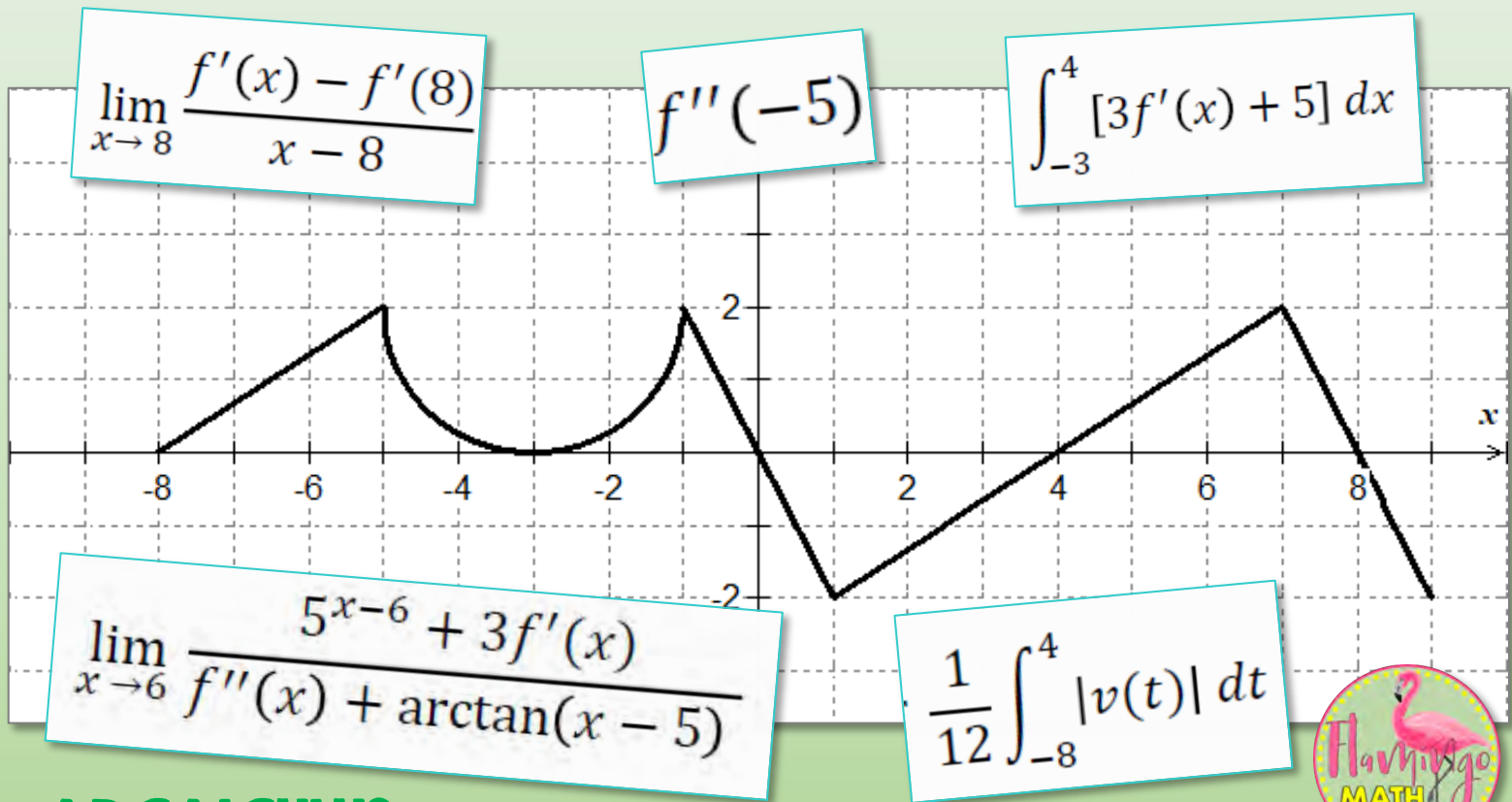


Graph Analysis & Interpretations

A Graphing Trifecta



AP CALCULUS



Sequence - Scaffold - Repeat

About this Activity:

As you progress through the year in AP Calculus, it is helpful to revisit concepts and big ideas to help students prepare for the rigor of the AP Exam.

This activity is designed to using the same graph that will be used in three different settings students could possibly see on the exam.

Activity #1: GRAPHICAL ANALYSIS & INTERPRETATION
Students analyze and interpret characteristics of a derivative function that include extrema, intervals of increase/decrease, derivatives, concavity, and mean value theorem.

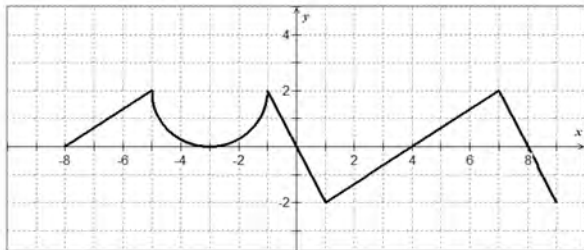
Activity #2: PARTICLE MOTION and VELOCITY
Students use the graph as a velocity function to answer questions related to the language used with particle motion, including topics for displacement, total distance, average speed, distance traveled, position of a particle given initial conditions, location of a particle and more.

Activity #3: INTEGRATION and ACCUMULATION
This activity will have students use the graph to evaluate concepts and topics related to integration, properties of limits, properties of integrals, and the fundamental theorem of calculus.



Graphical Analysis & Interpretation #1

The graph below consists of four line segments and a semi-circle. The graph shows $f'(x)$, the derivative of the function $f(x)$.

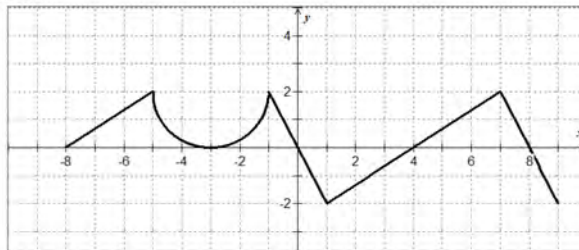


- For what values of x in the interval $-8 < x < 9$ does $f(x)$ have a relative maximum. Justify your answer.
- On what interval(s) of x is $f(x)$ increasing and concave down? Justify.
- On what interval(s) of x is $f(x)$ decreasing and concave up? Justify.
- Find the x -coordinate of each point of inflection on the graph of $f(x)$. Justify your answer.
- Determine the values of $f''(-5)$, $f''(-3)$, and $f''(3)$ or state that they do not exist.
- Determine the value of $\lim_{x \rightarrow 8} \frac{f'(x) - f'(8)}{x - 8}$, or state that it does not exist.
- Over what interval is there guaranteed to be a value $x = c$ such that $f''(c) = -1$? Justify your answer.

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Graphical Interpretation Particle Motion #2

The graph below consists of four line segments and a semi-circle. The graph represents the velocity $v(t)$ of a particle moving on the x -axis with position $x(t)$ measured in feet and time measured in minutes.

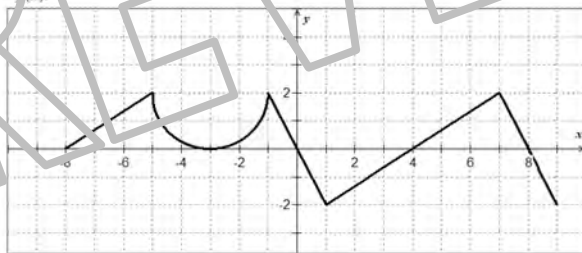


- Find the value of $v'(-6)$ and explain the meaning in the context of the problem. Use correct units.
- Find the displacement of the particle from $t = 1$ to $t = 8$.
- What is the total distance traveled by the particle from $t = 1$ to $t = 8$?
- Explain the meaning in context to the problem of $\frac{1}{12} \int_{-8}^4 |v(t)| dt$. Use Correct units.
- Find the position of the particle at $t = 4$ minutes, given that $x(0) = 2$.
- If $x(0) = 2$, when is the particle farthest to the left? Justify your answer.
- At $t = -6$, is the speed of the particle increasing or decreasing? Explain how you know.
- Name two intervals of time when the displacement of the particle would equal zero.

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Graphical Interpretation - Integration #3

The graph below consists of four line segments and a semi-circle. The graph represents the function $y = f(x)$.

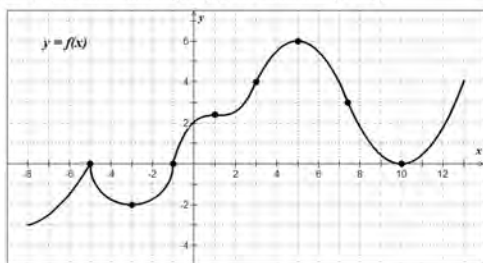


- Given $\int_{-8}^{15} f(x) dx = 18$, find $\int_8^{15} f(x) dx$
- Given $g(x) = \int_0^x f(t) dt$, find $g(7)$, $g'(7)$ and $g''(7)$ or explain why the value does not exist.
- Find $\lim_{x \rightarrow 6} \frac{5^{x-6} + 3f'(x)}{f''(x) + \arctan(x-5)}$
- Evaluate $\int_{-3}^4 [3f'(x) + 5] dx$
- The function g is given by $g(x) = x^3 \cdot \int_0^{2x} f(t) dt$. Find $g'(2)$.

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Graphical Interpretation - Extrema #1

This is the graph of some function $y = f(x)$ on the interval $[-8, 13]$:

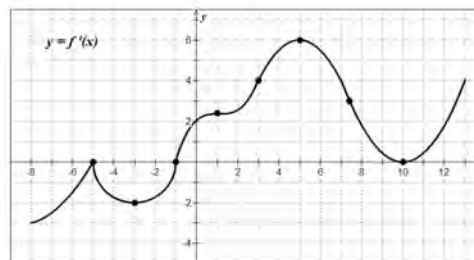


1. Sketch where tangent lines are zero or undefined. State the critical points of $f(x)$.
2. On what open interval(s) is $f(x)$ both increasing and concave up?
3. Determine the x -values of the relative extrema of f .
4. What are the absolute extrema of $f(x)$?
5. Where are the inflection points of f ?

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Graphical Interpretation - Reading the Derivative #2

Now suppose that this is the graph of $f'(x)$ the derivative of some function $f(x)$ on the interval $[-8, 13]$:

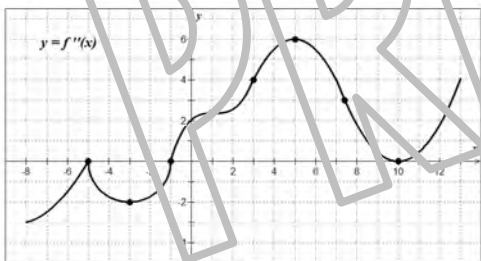


1. State the critical points of $f(x)$.
2. At what x -value(s) does $f(x)$ have a local maximum/minimum? Justify.
3. Determine the absolute extrema of $f(x)$, justify your answer.
4. On what open interval(s) is $f(x)$ concave down? Justify.
5. At what x -value(s) does $f(x)$ have an inflection point? Justify your answer.

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Graphical Interpretation - Reading the Second Derivative #3

Now suppose this is the graph of $f''(x)$ the second derivative of some function $f(x)$ on the interval $[-8, 13]$:

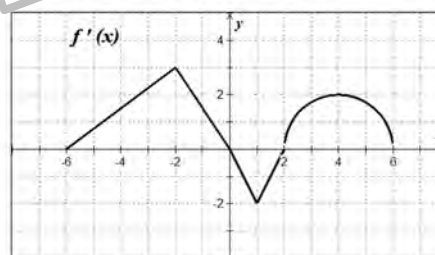


1. On what open interval(s) is $f(x)$ concave up? Justify.
2. On what open interval(s) is $f(x)$ concave down? Justify.
3. At what x -value(s) does $f(x)$ have an inflection point? Justify.

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Graphical Interpretation - More Integration #4

The function $y = f(x)$ is defined and differentiable on the closed interval $[-6, 6]$. The graph of $y = f'(x)$, the derivative of f , consists of line segments and a semi-circle, shown in the figure below. For each question below, find the value for $y = f(x)$ on the interval $-6 < x < 6$ and justify your reason.



1. Find the x -coordinate of each critical point for $y = f(x)$.
2. Find the x -coordinate of each relative extrema of $y = f(x)$ and label as a maximum or minimum.
3. Find the open interval(s) over which the function is increasing or decreasing.
4. Find the x -coordinate of each point of inflection for $y = f(x)$.
5. Find the interval(s) over which the function $y = f(x)$ is concave up or concave down.

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