

Algebra 2 Honors

Geometric Series



Series and Summation

Write the summation notation and find the sum of the first 6 terms of the geometric series $3 + 15 + 75 + 375 + \dots$

1) Rule: $a_n = 3(5)^{n-1}$

2) Summation:

$$\sum_{n=1}^6 [3(5)^{n-1}]$$

3) Sum $S_6 = 3 \left[\frac{1-5^6}{1-5} \right]$

$$\begin{aligned} a_1 &= 3 \\ r &= 5 \\ n &= 6 \end{aligned}$$

Completely Editable

9.4

Lesson 9.4: Geometric Series

Vocabulary	Example:
The sum of the terms in a geometric sequence is known as a _____.	The series $2.5 + 5 + 10 + 20 + 40$ corresponds to the sequence 2.5, 5, 10, 20, 40. The sum of the series is 77.5.
An infinite series $a_1 + a_2 + \dots + a_n + \dots$ is said to _____ if the sum $a_1 + a_2 + \dots + a_n$ gets closer and closer to a real number as n increases.	
An infinite series will _____ if it does not converge.	
Sums of Geometric Series Formula	
When a_1 is the first term, r is the common ratio, $r \neq 1$, and n is the number of terms.	
Sum of a Finite Geometric Sequence with ratio $r \neq 1$	$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$
Alternate Form of the Finite Formula	$S_n = \frac{a_1 - a_1 \cdot r^n}{1-r}$
Sum of an Infinite Geometric Series if $ r < 1$	$S = \frac{a_1}{1-r}$
NOTE: An infinite geometric series with $ r \geq 1$ does not have a finite sum. The series is said to diverge .	
EX #1: What do you need to find the sum of the first 8 terms in the series $3 + 6 + 12 + 24 + \dots$?	

Finding the Sum of a Finite Series

EX #2: Find the sum of the finite geometric series

A. $3 + 6 + 12 + 24 + \dots + 1536$

B. $-15 + 30 - 60 + 120 - 240 + 480$

EX #3: Find the sum of the finite geometric series.

A. $\sum_{i=1}^{10} 5(-2)^{i-1}$

B. $\sum_{k=0}^{24} 3\left(\frac{1}{2}\right)^k$

Using the Geometric Series Formula

EX #4: You make a contract with your parents to work eight hours every day during the month of July. The first day you will be paid one penny, the second day two cents, on the third day you get 4 cents, the fourth day 8 cents, and so on. Is this a good plan for you or do your parents get the most out of the contract?

Series and Summation

EX#5: Write the summation notation and find the sum of the first 6 terms of the geometric series $3 + 15 + 75 + 375 + \dots$

EX #6: Write the summation notation and find the sum of the first 10 terms of the geometric series $2 - 1 + \frac{1}{2} - \frac{1}{4} + \dots$

Analyzing Infinite Geometric Series

EX #7: Determine whether the series converges or diverges. If it converges, find the sum.

A. $\frac{1}{2} + \frac{3}{4} + \frac{9}{8} + \dots$

B. $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \dots$

NOTES

Geometric Series Practice

Problems 1 – 4, evaluate the series to the given term.

1. $2 + 4 + 5 + \cdots; S_7$

2. $-3 - 6 - 12 - \cdots; S_7$

3. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots; S_6$

4. $-\frac{1}{5} + 1 - 5 + 25 \cdots; S_5$

Problems 5 – 8, decide whether each infinite geometric series *diverges* or *converges*.
State whether each series has a sum.

5. $1 + \frac{1}{2} + \frac{1}{4} + \cdots$

6. $3 - 3 + 3 - \cdots$

7. $1 - \frac{1}{4} + \frac{1}{16} - \cdots$

8. $\frac{1}{4} + \frac{1}{2} + 1 + 2 + \cdots$

Problems 9 – 12, evaluate each infinite series that has a sum.

9. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1}$

10. $\sum_{n=1}^{\infty} 4(2^{n-1})$

11. $\sum_{n=1}^{\infty} 7\left(\frac{1}{4}\right)^{n-1}$

12. $\sum_{n=1}^{\infty} 5(1.5)^{n-1}$

Problems 13 – 16, determine whether each series is *arithmetic* or *geometric*. Then evaluate the series to the given term.

13. $3 + 6 + 9 + 12 + \cdots; S_{20}$

14. $3 + 6 + 12 + 24 + \cdots; S_{10}$

15. $-2 + 2 + 6 + 10 + \cdots; S_{12}$

16. $-4 + 16 - 64 + 256 - \cdots; S_9$

Problems 17 – 20, evaluate each infinite geometric series.

17. $1.1 + 0.11 + 0.011 + \cdots$

18. $4 + 1 + \frac{1}{4} + \frac{1}{16} + \cdots$

19. $1.1 - 0.11 + 0.011 - \cdots$

20. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

Problems 21 – 23, solve.

21. A ball is bounced and reaches heights of 15 inches, 9 inches, and 5.4 inches on the first three bounces.

A. If the ball started at a height of 25 inches, write a geometric series for the distances the ball travels from its starting height in a downward direction.

B. Write a geometric series for the upward distances the ball travels after its first bounce.

C. What is the total vertical distance the ball travels before it comes to a rest?

22. The length of the outside shell of each closed chamber of the chambered nautilus is 0.9 times the length of the larger chamber next to it. If the outside edge of the largest enclosed chamber is 36 mm long, estimate the total length of the outside shell for the enclosed chambers.



Practice Sets

Lesson Warm-Ups

3. Evaluate the series for the first 6 terms
 $100 + 200 + 400 + \dots$

4. Use a calculator to evaluate the series.

$$\sum_{n=1}^{25} \left(\frac{5}{4}\right)^{n-1}$$

Name _____ Date _____ Period _____

Lesson 9.4: Geometric Series

Warm-Up #2

1. Is the series *arithmetic* or *geometric*? Evaluate for the number of terms.
 $2 + 4 + 8 + 16 + \dots; n = 10$

2. What is the value of a_1 in the series?

$$\sum_{n=0}^{26} 7 \left(\frac{1}{2}\right)^n$$

3. Evaluate the series for the first 6 terms
 $10 + 30 + 90 + \dots$

4. Use a calculator to evaluate the series.

$$\sum_{n=1}^{25} \left(\frac{5}{2}\right)^{n-1}$$

Form A

9.4 Geometric Series

Form B

Name _____

Date _____ Period _____

metric series.

1. Find the sum of the geometric series.

$$\sum_{k=1}^5 (3^k)$$

2. Evaluate the series for the first 20 terms

$$15 + 12 + 9.6 + \dots$$

2. Evaluate the series for the first 20 terms

$$6.4 + 8 + 10 + \dots$$

3. Evaluate the infinite geometric series.

$$1.1 + 0.11 + 0.011 + \dots$$

3. Evaluate the infinite geometric series.

$$3 + 2 + \frac{4}{3} + \frac{8}{9} + \dots$$

Daily Lesson Quiz

Grade: _____

Grade: _____