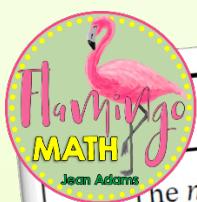


Precalculus Honors

Geometric Sequences & Series



General Form and Common Ratios

The n th term (general term) of a geometric sequence with first term a_1 and common ratio r is

$$a_n = a_1 r^{n-1}$$

Geometric sequences have common ratios between consecutive terms.

$$\text{Common ratio: } r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_n}{a_{n-1}}$$

Each term in a geometric sequence can be obtained recursively from its previous term by multiplying by r :

Formula:	
General Form:	$a_n = a_1 r^{n-1}$
Recursion Formula:	$a_1 = a$ $a_n = a_{n-1} \cdot r$ for $n > 1$

Completely Editable

9.3

9.3 Geometric Sequences and Series

Geometric Sequences

A **geometric sequence** is any sequence with a constant ratio between consecutive terms. With a starting value " a " and common ratio " r " is in the form $a, ar, ar^2, ar^3, \dots, ar^n$.

EX. 4, 8, 16, 32, 64, $\dots 4(2)^n$

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Identifying Geometric Sequences

EX #1: Determine whether each sequence is geometric. If so, find the common ratio, r .

A. 3, 6, 12, 24, $\dots$

B. 3, -9, 27, -81, $\dots$

C. 1, 5, 9, 13, 17, $\dots$

D. 48, -12, 3, $\dots$

Finding Terms of a Geometric Sequence

EX #2: Find the indicated terms of the geometric sequences.

A. Find a_6 and a_n for the geometric sequence
3, -15, 75, $\dots$

B. Determine r and a_1 for the geometric sequence
with $a_4 = -8$ and $a_{11} = \frac{1}{16}$

EX #3: When a radioactive substance decays, the amount remaining will form a geometric sequence when measured over constant intervals of time. The table shows the amount of an unknown substance, initially and after 2 hours. Find the amounts left after 1 hour, 3 hours, and 4 hours.

Hours Lapsed	0	1	2	3	4
Grams of radioactive substance	1320		384		

Sums of Geometric Series

A **geometric series** is the sum of the terms of a geometric sequence. When a_1 is the first term, r is the common ratio, and n is the number of terms, the sum, S_n , of a geometric sequence is:

Sum of a Finite Geometric Sequence
with ratio $r \neq 1$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Alternate Form of the Finite Formula

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r}$$

Sum of an Infinite Geometric Series

if $|r| < 1$

$$S_n = \frac{a_1}{1 - r}$$

EX #4: Find the sum of the first 10 terms of the geometric sequence 2, -6, 18, -54, 162, $\dots$

EX #5: Find the infinite sum.

A. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

B. $1 - 0.6 + 0.36 - 0.216 + \dots$

Using S_n to Evaluate a Summation

EX #6: Find the following sum:

A. $\sum_{i=1}^{10} (3 \cdot 2^i)$

B. $\sum_{i=1}^{\infty} \left(\frac{2}{3}\right)^n$

Using the Geometric Mean

The geometric mean of two positive numbers x and y is $\sqrt{xy}$

In a sequence the formula is:

$$\text{Mean} = a_n = \sqrt{a_{n-1} \cdot a_{n+1}}$$

EX #8: Insert the geometric mean between the following.

A. 45, $\square$, 80, $\dots$

B. 972, $\square$, $\square$, $\square$, 12, $\dots$

Applications

EX #9: You make a contract with your parents to work eight hours every day during the month of July. The first day you will be paid one penny, the second day two cents, on the third day you get 4 cents, the fourth day 8 cents, and so on. Is this a good plan for you or do your parents get the most out of the contract?

NOTES

9.3

Geometric Sequences
& Series Homework

Name _____

Date _____ Period _____

Problems 1–4, write the first five terms of the geometric sequence. Determine the common ratio and write the n th term of the sequence as a function of n .

1. $a_1 = 8$; $a_{k+1} = 2a_k$

2. $a_1 = 5$; $a_{k+1} = -2a_k$

3. $a_1 = 60$; $a_{k+1} = -\frac{1}{2}a_k$

4. $a_1 = 12$; $a_{k+1} = -\frac{3}{2}a_k$

Problems 5–8, write an expression for the n th term of the geometric sequence. Then find the indicated term.

5. $a_1 = 64$; $r = -\frac{1}{4}$; $n = 10$

6. $a_1 = 1$; $r = \sqrt{2}$; $n = 12$

7. $a_1 = 5$; $r = \frac{5}{2}$; $n = 8$

8. $a_1 = 1000$; $r = -\frac{1}{2}$; $n = 4$

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Problems 9–12, determine the common ratio, the fifth term, and the n th term of the geometric sequence.

9. 144, 24, 4, $\frac{2}{3}$, ...

10. $5, \frac{10}{3}, \frac{20}{9}, \frac{40}{27}, \dots$

11. $-8, -2, -\frac{1}{2}, -\frac{1}{8}, \dots$

12. $1, \sqrt{2}, 2, 2\sqrt{2}, \dots$

Problems 13–16, find the sum of the first n terms of the geometric sequence.

13. Find the sum of the first 9 terms for the sequence. 4, 12, 36, 108, ...

14. Find the sum of the first 8 terms for the sequence. $\frac{3}{2}, -3, 6, -12, \dots$

15. Find the sum of the first 10 terms for the sequence. 5, 10, 20, 40, ...

16. Find the sum of the first 6 terms for the sequence. $18, 9, \frac{9}{2}, \frac{9}{4}, \dots$

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Problems 17–20, find the sum of the infinite geometric series if possible. If not possible, explain why.

17. $3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \dots$

18. $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \dots$

19. $6 + 8 + \frac{32}{3} + \frac{128}{9} + \dots$

20. $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \dots$

Problems 21–24, express each repeating decimal as a fraction in lowest terms.

21. $0.\overline{7} = \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \frac{7}{10000} + \dots$

22. $0.\overline{43} = \frac{43}{100} + \frac{43}{10000} + \frac{43}{1000000} + \dots$

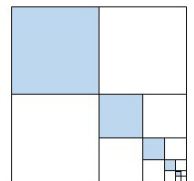
Problems 25–29, solve.



25. You are offered your dream job with an annual salary of \$35,000 to start, with an expected annual increase of 5%. Find your salary at the start of your fifth year.

26. Caliber One Construction purchased a new service vehicle for \$25,000. Each year they will depreciate the value of the vehicle for tax purposes by 12%. What will be the value of the vehicle after 5 years?

27. In the figure shown at right, what fraction of the square is eventually shaded if the indicated shading process continues indefinitely?



28. A family-owned screen-printing company sells t-shirts and earns a \$24,000 profit during its first year. The owners set a goal of increasing profits by \$5000 each year for the next four years. Assuming that this goal is met, what is the total profit earned during their first 5 years of business?

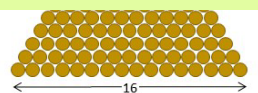
Practice Sets

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Lesson Warm-Ups

	$\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$
3. Find the sum of the first 7 terms of the geometric sequence. $3 + 6 + 12 + 24 \dots$	4. Determine whether the series converges. If it converges, give the sum. $\sum_{k=1}^{\infty} 2(0.3)^{k-1}$

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Name _____ Date _____ Period _____	
Lesson 9.3: Geometric Sequences and Series Warm-Up # 2	
1. Write an expression for the n th term of the geometric sequence. Then find the 4 th term. $a_1 = 26, r = \frac{1}{2}, n = 4$	2. Find the number of terms, the first term, the last term, and the sum. $\sum_{n=2}^6 \left(\frac{n}{3}\right)$
3. Find the sum of the first 7 terms of the geometric sequence. $6 + 12 + 24 + 48 \dots$	4. Determine whether the series converges. If it converges, give the sum. $\sum_{n=0}^{\infty} (-0.25)^{n-1}$

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Form A

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9.3 Geometric Sequences & Series Form B
Name _____
Date _____ Period _____
1. Write an expression for the n th term of the geometric sequence then find the indicated term. $a_1 = 9, r = 2, n = 8$
2. Write a formula for the general term of the geometric sequence. $12, 6, 3, \frac{3}{2}, \dots$
3. Find the sum. $\sum_{n=2}^7 \left(\frac{n}{2}\right)$
4. Write the first five terms of the geometric sequence. $a_1 = 64, a_{k+1} = \frac{1}{2}a_k$
5. Determine whether the series converges.

2. Write a formula for the general term of the geometric sequence. $18, 6, 2, \frac{2}{3}, \dots$
3. Find the sum. $\sum_{n=1}^6 \left(\frac{2n}{3}\right)$
4. Write the first five terms of the geometric sequence. $a_1 = 81, a_{k+1} = \frac{1}{3}a_k$
5. Determine whether the series converges.

Daily Lesson Quiz

Grade: _____

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Grade: _____