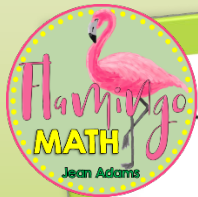


Precalculus Honors

Trigonometric Identities



Pythagorean Identities

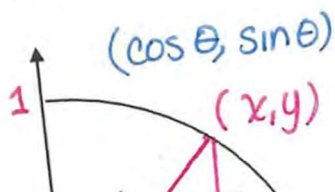
$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

Pythagorean Identities on Unit Circle

EX #1: If you know the basic identity, it's easy to derive the others.



$$1) x^2 + y^2 = 1$$

$$* \cos^2 \theta + \sin^2 \theta = 1$$

$$2) \frac{\cos^2 \theta}{\sin^2 \theta} + \frac{\sin^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$
$$= \csc^2 \theta$$

memorize this one

divide by $\sin^2 \theta$

Completely Editable

5.1

Lesson I: Fundamental Identities

Mathematical identities are two equated expressions that are equivalent in meaning and may be substituted for the other without losing the truth of a statement. For instance, $2+5=7$ and $3+4=7$ so we can say that $2+5=3+4$. In trigonometric identities, we will work with expressions that can make a difficult problem easier to work with and remain equivalent.

Reciprocal Identities

$$\csc \theta = \quad \quad \quad \sec \theta = \quad \quad \quad \cot \theta =$$

$$\sin \theta = \quad \quad \quad \cos \theta = \quad \quad \quad \tan \theta =$$

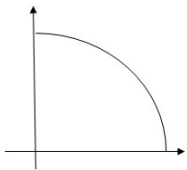
Quotient Identities

$$\tan \theta = \frac{\quad}{\quad} \quad \quad \quad \cot \theta = \frac{\quad}{\quad}$$

Pythagorean Identities

Pythagorean Identities

EX #1: If you know the basic identity, it's easy to



EX. #5: Simplify. $\frac{(\sec x + 1)(\sec x - 1)}{\sin^2 x}$

Simplifying by Combining Fractions and Using Identities

EX #6: Simplify the expression: $\frac{\cos x}{1 - \sin x} - \frac{\sin x}{\cos x}$

Even, Odd and Co-Functions

Co-functions are complements of some trig function. Namely, angles that are complements add to 90 degrees and have the same trig ratios.

Co-Function Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \quad \quad \quad \cos\left(\frac{\pi}{2} - \theta\right) =$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \quad \quad \quad \csc\left(\frac{\pi}{2} - \theta\right) =$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \quad \quad \quad \cot\left(\frac{\pi}{2} - \theta\right) =$$

EX #9: Use Co-function identities to find the following:

A. If $\sin \theta = -\frac{1}{2}$, find $\cos\left(\frac{\pi}{2} - \theta\right)$

B. If $\cos \theta = 0.38$, find $\sin\left(\theta - \frac{\pi}{2}\right)$

Even-Odd Identities

EVEN

$$\cos(-x) = \quad \quad \quad \sin(-x) =$$

$$\sec(-x) = \quad \quad \quad \csc(-x) =$$

EX #7: Even and Odd. If $\sin \theta = \frac{1}{2}$, find $\sin(-\theta)$

EX #8: Find all values of x in the interval $[0, 2\pi]$

Technique:

1. Consider any domain restrictions.
2. Change everything to sine & cosine.
3. Multiply both sides by $\sin x$ to clear fractions.
4. Factor.
5. Simplify using Pythagorean Identity.
6. Solve using the unit circle:

NOTES

Lesson I: Fundamental Identities

Mathematical identities are two equated expressions that are equivalent in meaning and may be substituted for the other without losing the truth of a statement. For instance, $2+5=7$ and $3+4=7$ so we can say that $2+5=3+4$. In trigonometric identities, we will work with expressions that can make a difficult problem easier to work with and remain equivalent.

Reciprocal Identities

$\csc \theta =$	$\sec \theta =$	$\cot \theta =$
$\sin \theta =$	$\cos \theta =$	$\tan \theta =$

Quotient Identities

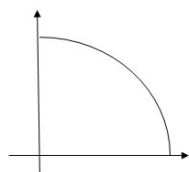
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

Pythagorean Identities

EX #1: If you know the basic identity, it's easy to derive the others.



Simplify and Factor Expressions

EX #2: Use a quotient identity to simplify and rewrite.

A. $\cos \beta \tan \beta$

B. $\sec x \cot x \sin x$

EX #3: Use the idea of "decomposing" fractions to simplify and rewrite.

A. $\frac{\csc \theta}{\sin \theta}$

B. $\frac{\sin x - \csc x}{\csc x}$

Simplifying by Factoring and Expanding

EX #4: Factor completely.

B. $(16 - \sec^2 x)$

D. $\csc^2 \beta - \cot \beta - 3$

EX #5: Simplify. $\frac{(\sec x + 1)(\sec x - 1)}{\sin^2 x}$

Simplifying by Combining Fractions and Using Identities

EX #6: Simplify the expression: $\frac{\cos x}{1 - \sin x} - \frac{\sin x}{\cos x}$

Even-Odd Identities

EVEN

$\cos(-x) =$

$\sin(-x) =$

$\sec(-x) =$

$\csc(-x) =$

EX #7: Even and Odd. If $\sin \theta = \frac{1}{2}$, find $\sin(-\theta)$

Even, Odd and Co-Functions

Co-functions are complements of some trig function. Namely, angles that are complements add to 90 degrees and have the same trig ratios.

Co-Function Identities

$\sin\left(\frac{\pi}{2} - \theta\right) =$

$\cos\left(\frac{\pi}{2} - \theta\right) =$

$\sec\left(\frac{\pi}{2} - \theta\right) =$

$\csc\left(\frac{\pi}{2} - \theta\right) =$

$\tan\left(\frac{\pi}{2} - \theta\right) =$

$\cot\left(\frac{\pi}{2} - \theta\right) =$

EX #9: Use Co-function identities to find the following:

A. If $\sin \theta = -\frac{1}{2}$, find $\cos\left(\frac{\pi}{2} - \theta\right)$

B. If $\cos \theta = 0.38$, find $\sin\left(\theta - \frac{\pi}{2}\right)$

EX #8: Find all values of x in the interval $[0, 2\pi)$ that satisfy the equation.

Technique:

1. Consider any domain restrictions.
2. Change everything to sine & cosine.
3. Multiply both sides by $\sin x$ to clear fraction.
4. Factor.
5. Simplify using Pythagorean Identity.
6. Solve using the unit circle:

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 5.1: Fundamental Identities Warm-Up # 1	
1. Use the given values to evaluate the remaining trig functions. $\csc \theta = \frac{5}{3}; \tan \theta = -\frac{3}{4}$	2. Factor: $\sin^2(\theta) + 2 \sin(\theta) + 1$
3. Simplify: $(\sin(x))(\sec(x))$	4. Write the expression in terms of sine. $\cot(\theta) \csc(\theta)$

Name _____ Date _____	
Lesson 5.1: Fundamental Identities	
1. Use the given values to evaluate the remaining trig functions. $\csc \theta = -\frac{5}{4}; \tan \theta = \frac{3}{4}$	2. Factor: $\tan^2(\theta)$
3. Simplify: $\frac{1}{\sin(x) \cot(x)}$	4. Write the expression in terms of sine. $\cot(\theta) \cos(\theta)$

5.1 Fundamental Identities	Form A
Name _____	
Date _____ Period _____	
1. Use the given values to evaluate the remaining trig functions. $\csc \theta = \frac{5}{3}, \tan \theta = \frac{3}{4}$	
2. Factor: $\tan^2 \theta + 2 \tan \theta + 1$	
3. Simplify: $\sin^2 x \csc^2 x - \sin^2 x$	
4. Write the expression in terms of sine.	

5.1 Fundamental Identities	Form B
Name _____	
Date _____ Period _____	
1. Use the given values to evaluate the remaining trig functions. $\sec \theta = \frac{13}{12}, \tan \theta = \frac{5}{12}$	
2. Factor: $\cos^2 \theta - 2 \cos \theta + 1$	
3. Simplify: $\cos^2 x \sec^2 x - \cos^2 x$	
4. Write the expression in terms of sine.	

Daily Lesson Quiz

Grade: _____

Grade: _____
