

Precalculus Honors

Functions and Their Inverses



Functions and Their Inverses

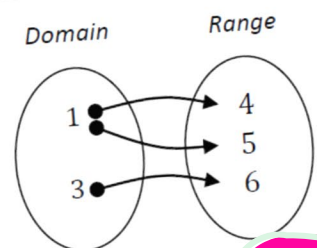
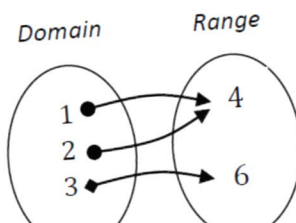
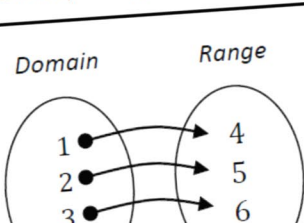
A **function** is a set of ordered pairs. A **function** is a relation in which, for each distinct x -value, there is exactly one corresponding y -value. If a function f is a set of ordered pairs (x, y) , then the inverse of f is the set of ordered pairs (y, x) . When the inverse of a function f is itself a function, then f is said to be a **one-to-one function**.

One-to-One Functions

A function f is a **one-to-one function** if, for elements a and b in the domain of f , $a \neq b$ implies $f(a) \neq f(b)$.

That is, different values of the domain correspond to different values of the range.

EX #1: Justify each mapping with an observation.



Completely Editable

1.8

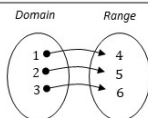
1.8 Functions and Their Inverses

A **relation** is a set of ordered pairs. A **function** is a relation in which, for each distinct x -value, there is *exactly one* corresponding y -value. If a function f is a set of ordered pairs (x, y) , then the inverse of f is the set of ordered pairs (y, x) . When the inverse of a function f is itself a function, then f is said to be a **one-to-one function**.

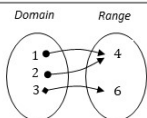
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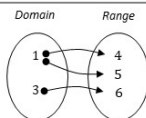
EX #1: Justify each mapping with an observation.



One-to-One Function:



Not a one-to-one function:

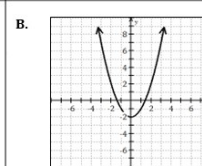
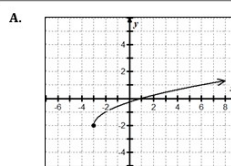


Not a function:

Graphing the Inverse Function

EX #2: For each of the graphs below, draw the line $y = x$.

- ☐ Sketch the inverse of the function on the same grid.
☐ Is the function one-to-one?



Definition of the Inverse of a Function

Let f and g be two functions such that

Finding the Equation of the Inverse of $y = f(x)$

Given a formula for a function f , find an inverse by:

Determine that an inverse exists, that is, check that f is one-to-one.
(Ask yourself, "Is it necessary to restrict the domain to get a one-to-one version of f ?").

Step 1: Interchange x and y in the formula.

Step 2: Solve for y to get the formula for the inverse $f^{-1}(x)$.
State any restrictions on the domain of $y = f^{-1}(x)$

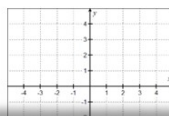
Step 3: Replace y by $f^{-1}(x)$ in the new equation.

Graphing and Algebraic Practice

EX #4: For each of the following functions:

- A. Sketch the function and $y = x$,
B. Find the inverse,
C. Sketch the inverse, and
D. Verify that they are inverses, algebraically.

A. $f(x) = \sqrt{x+3} - 2$; $x \geq -3$



Function Notation

EX #5: Let $C(m)$ be the cost in dollars of m ounces of mangoes. What do the following statements tell you? Be sure to use units!

A. $C(12) = 0.75$

B. $C(8) = 0.5$

C. $C^{-1}(3) = 48$

D. $C^{-1}(0.75) = 12$

Understanding Function Notation

EX #6: Suppose $w = p(x)$ is the average daily quantity of water (in gallons) required by a palm tree of height x feet.

- A. What does the expression $p(25)$ represent?

E. Using the definitions of z and k from part D, what do the following expressions represent?

i. $p(2z)$

ii. $2p(z)$

iii. $p(z + 10)$

iv. $p(z) + 10$

v. $p^{-1}(2k)$

vi. $p^{-1}(k + 10)$

vii. $p^{-1}(k) + 10$

viii. $p^{-1}(k) - 2$

Finding the Inverse of a Rational Function

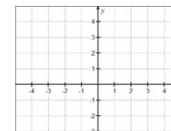
EX #8: The following rational function is one-to-one. $f(x) = \frac{2x+3}{x+2}$

- A. Find its inverse.

- B. Find the domain and range of $f(x)$.

If a function is not one-to-one, then its inverse is not a function. However, if an appropriate restriction on the domain is made, then we can find an inverse that is also a function.

EX #9: Find the inverse of $y = 2 - x^2$ if $x \geq 0$. Graph f and its inverse, f^{-1}



NOTES

3	-3	4	-1
5	-4	8	6

C. $f^{-1}(g(1))$

D. $g^{-1}(f(0))$

1.8

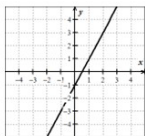
Functions and Their Inverses
Practice

Name _____

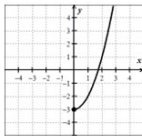
Date _____ Period _____

Problems #1-4: Sketch the graph of the inverse function for each function shown. State the domain for each.

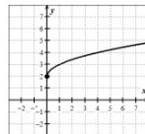
1.



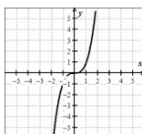
2.



3.



4.

**Problems #5-8:** Use a graphing utility to graph each function and use the Horizontal Line Test to determine whether the function is one-to-one and has an inverse.

5. $f(x) = \frac{1}{2}(x+3)^2 - 4$



6. $g(x) = \frac{x-2}{3x^2}$



7. $h(x) = -x\sqrt{9-x^2}$



8. $f(x) = \sqrt[3]{x-4}$

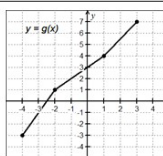
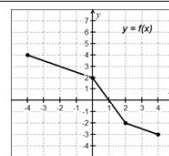
**Problems #9-12,** Restrict the domain of the function f so that the function is one-to-one and has an inverse function. Then find the inverse function f^{-1} . State the domain and ranges of f and f^{-1} .

9. $f(x) = (x-3)^2$

10. $f(x) = |x-3|$

11. $f(x) = 3-2x^2$

12. $f(x) = |x+3|-2$

Problems #13-18, Use the graphs of $y = f(x)$ and $y = g(x)$ to evaluate the expressions.

13. $f^{-1}(0)$

14. $g^{-1}(0)$

15. $(f \circ g)(1)$

16. $g(f(-2))$

17. $f^{-1}(g(-4))$

18. $(f^{-1} \circ g^{-1})(-1)$

Problems #19-22, Answer each question.

19. The circumference of a child's head
- C
- , is related to the height of the child
- H
- , by the equation
- $H(C) = 2.15C - 10.53$
- where both
- C
- and
- H
- are in inches.

A. Express the head circumference C , as a function of height H .

B. What is the predicted circumference of a child's head who measures 32 inches tall?

20. Find the inverse of the linear function
- $f(x) = mx + b$
- ;
- $m \neq 0$

21. In June 2015, you are planning a trip to Japan and find the exchange rate for converting US dollars (
- x
-) to yen (
- y
-) was
- $y = 123.65x$
- .

A. How many yen could you get for \$100 U.S.?

B. What is the inverse function, and what conversion does it represent?

C. While in Tokyo you have lunch in an outside café at a cost of 2000 yen. How much was that in U.S. dollars?

A.) what does the inverse represent?

B.) Is f^{-1} a function, explain?

Practice Sets

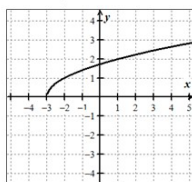
Lesson Warm-Ups

1.8 Functions and Their Inverses Form A

Name _____

Date _____ Period _____

1. Sketch the inverse of the function f , shown on the grid below.



2. Write an equation for the function f , shown above.

3. Find the inverse function of f , algebraically.

4. State the domain of f and f^{-1} .

Grade: _____

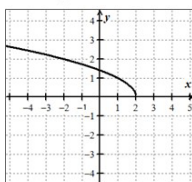
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1.8 Functions and Their Inverses Form B

Name _____

Date _____ Period _____

1. Sketch the inverse of the function f , shown on the grid below.



2. Write an equation for the function f , shown above.

3. Find the inverse function of f , algebraically.

4. State the domain of f and f^{-1} .

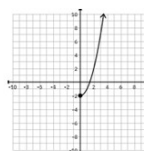
Grade: _____

Name _____ Date _____ Period _____

Lesson 1.8: Relations and their Inverses

Warm-Up # 1

1. Sketch the inverse of the function f , shown on the graph.



3. Find the inverse of f , algebraically.

4. State the domain of f and f^{-1} .

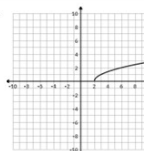
2. Write an equation for the function f .

Name _____ Date _____ Period _____

Lesson 1.8: Relations and their Inverses

Warm-Up # 2

1. Sketch the inverse of the function f , shown on the graph.



3. Find the inverse of f , algebraically.

4. State the domain of f and f^{-1} .

2. Write an equation for the function f .

Daily Lesson Quiz