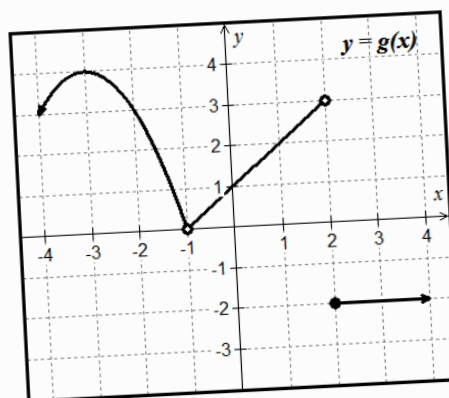
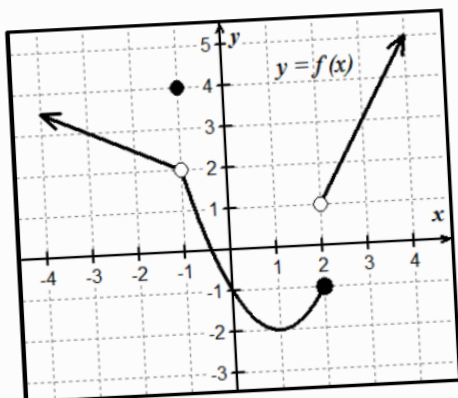


Precalculus Honors

Finding Limits Analytically

More Limit Properties

#2: Find each of the following limits if possible. If a limit does not exist, state why.



A. $\lim_{x \rightarrow 2^-} [f(x) + g(x)]$

$$\lim_{x \rightarrow 2^-} f(x) + \lim_{x \rightarrow 2^-} g(x)$$

B. $\lim_{x \rightarrow 1} [2f(x) - 3g(x)]$

$$2 \cdot \lim_{x \rightarrow 1} f(x) - 3 \cdot \lim_{x \rightarrow 1} g(x)$$

$$2(-2) - 3(2)$$

Completely Editable 10.2

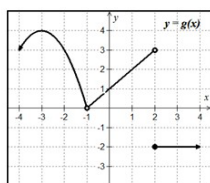
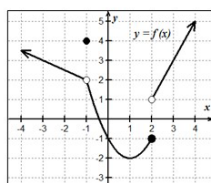


Lesson 10.2: Finding Limits Analytically

Properties of Limits	Verbal Description
1. Constant Rule: $\lim_{x \rightarrow c} k = k$	
2. Identity Rule: $\lim_{x \rightarrow c} x = c$	
3. Sum or Difference Rule: $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \lim_{x \rightarrow c} [f(x)] \pm \lim_{x \rightarrow c} [g(x)]$	
4. Product Rule: $\lim_{x \rightarrow c} [f(x) \cdot g(x)] = \lim_{x \rightarrow c} [f(x)] \cdot \lim_{x \rightarrow c} [g(x)]$	
5. Constant Multiple Rule: $\lim_{x \rightarrow c} [k \cdot f(x)] = k \cdot \lim_{x \rightarrow c} [f(x)]$	

More Limit Properties

EX #2: Find each of the following limits if possible. If a limit does not exist, state why.



A. $\lim_{x \rightarrow 2} [f(x) + g(x)]$

B. $\lim_{x \rightarrow 1} [2f(x) - 3g(x)]$

C. $\lim_{x \rightarrow -3} [f(x) - g(x)]$

D. $\lim_{x \rightarrow 2} [f(x)]^2$

Rationalization Technique

EX #5: Find the limits, if possible.

A. $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$

B. $\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x}$

C. $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2}$

Using Limit Properties

EX #1: Suppose $\lim_{x \rightarrow c} f(x) = 4$, $\lim_{x \rightarrow c} g(x) = -3$, and $\lim_{x \rightarrow c} h(x) = 9$. Find each of the following limits.

A. $\lim_{x \rightarrow c} [f(x) + g(x)]$

B. $\lim_{x \rightarrow c} [3f(x) - 5g(x)]$

C. $\lim_{x \rightarrow c} (7[f(x) \cdot g(x)])$

D. $\lim_{x \rightarrow c} -5f(x)$

EX #3: Direct Substitution

A. $\lim_{x \rightarrow -1} x^3 + 2x^2 - 5x + 4$

B. $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 5}{x + 3}$

C. $\lim_{x \rightarrow 2} \sqrt{3x^3 - 2x + 5}$

Cancellation Technique

EX #4: Sometimes it is necessary to factor and simplify before you can apply the Substitution Theorem.

A. $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$

Special Trigonometric Limits

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \quad \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} = 0$$

EX #6: Find the limit if it exists.

A. $\lim_{x \rightarrow \frac{\pi}{4}} 5 \cos x$

B. $\lim_{x \rightarrow \frac{\pi}{2}} 7 \sin x$

C. $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \cos \theta}{\theta}$

D. $\lim_{\theta \rightarrow 0} \frac{\sin 4\theta}{\theta}$

NOTES

10.2

Finding Limits Analytically Homework

Name _____
Date _____ Period _____

Problems 1 – 10, find each of the following limits analytically.

1. $\lim_{x \rightarrow -4} \frac{5x}{4-x}$

2. $\lim_{x \rightarrow \frac{1}{2}} (6 - 5x)$

3. $\lim_{x \rightarrow 2} \frac{x^2-8}{x-2}$

4. $\lim_{x \rightarrow -3} \frac{2x^2+7x+3}{x^2+8x+15}$

5. $\lim_{x \rightarrow 6} \frac{x^2-6x}{x^2-36}$

6. $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$

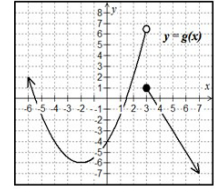
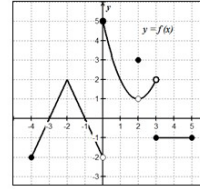
7. $\lim_{x \rightarrow -2} \frac{x^2-x-6}{2x+4}$

8. $\lim_{x \rightarrow 0} \frac{(x+2)^2-9}{x}$

9. $\lim_{x \rightarrow 0} \frac{\frac{1}{x+2} - \frac{1}{2}}{x}$

10. $\lim_{h \rightarrow 0} \frac{(x+h)^2-3(x+h)-x^2+3x}{h}$

Problems 11 – 19, use the graph to find the limit (if it exists). If the limit does not exist, explain why.



11. $\lim_{x \rightarrow 2} [f(x) + g(x)]$

12. $\lim_{x \rightarrow -2} [3f(x) - g(x)]$

13. $\lim_{x \rightarrow 5} [g(x)]^2$

14. $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)}$

15. $\lim_{x \rightarrow 1} [f(x) - g(x)]$

16. $\lim_{x \rightarrow 0^+} [f(x) \cdot g(x)]$

17. $\lim_{x \rightarrow -2} \sqrt{f(x) \cdot g(x)}$

18. $\lim_{x \rightarrow 4} \frac{5g(x)}{f(x)}$

19. $\lim_{x \rightarrow -4^+} \frac{f(x)}{g(x)}$

Problems 20– 23, find the limits if $\lim_{x \rightarrow c} f(x) = -3$ and $\lim_{x \rightarrow c} g(x) = 5$

20. $\lim_{x \rightarrow c} [f(x) + 2]^2$

21. $\lim_{x \rightarrow c} [f(x) \cdot 2g(x)]$

22. $\lim_{x \rightarrow c} \frac{[f(x)]^2}{1 - g(x)}$

23. $\lim_{x \rightarrow c} \frac{4f(x) + 3g(x)}{g(x) - f(x)}$

Problems 24 – 31, find the value of each limit. For a limit that does not exist, state why.

24. $\lim_{x \rightarrow -2} (x - 6)^{4/3}$

25. $\lim_{\theta \rightarrow \frac{\pi}{6}} \theta^2 \tan \theta$

26. $\lim_{x \rightarrow 3} e^x \cos\left(\frac{\pi}{x}\right)$

27. $\lim_{x \rightarrow 4^+} \frac{x+4}{x-4}$

28. $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$

29. $\lim_{x \rightarrow 0} \frac{\sin 2x}{2x^2 + x}$

Practice Sets

Lesson Warm-Ups

3. $\lim_{\theta \rightarrow \frac{\pi}{2}} \theta^2 \cos(\theta)$

Name _____ Date _____ Period _____

Lesson 10.2: Finding Limits Analytically

Warm-Up # 2

1. $\lim_{x \rightarrow 3} (x + 1)^{\frac{4}{3}}$

2. $\lim_{x \rightarrow 2} \frac{2x^2 - 11x + 5}{x^2 + x - 30}$

3. $\lim_{\theta \rightarrow \frac{\pi}{4}} \theta^2 \sin(\theta)$

Form A

10.2 Finding Limits Analytically Form B

Name _____

Date _____ Period _____

1. $\lim_{x \rightarrow -3} (x - 5)^{4/3}$

2. $\lim_{x \rightarrow 5} \frac{2x^2 - 11x + 5}{x^2 - 11x + 30}$

3. $\lim_{\theta \rightarrow \frac{\pi}{2}} \theta^2 \sin \theta$

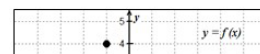
4. Evaluate, if possible.

A. $\lim_{x \rightarrow 4} f(x)$

B. $\lim_{x \rightarrow 2} f(x)$

C. $\lim_{x \rightarrow 2^-} f(x)$

D. $\lim_{x \rightarrow 1} 4 \cdot f(x)$



2. $\lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x^2 + 6x - 16}$

3. $\lim_{\theta \rightarrow \frac{\pi}{4}} \theta^2 \cos \theta$

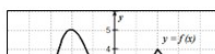
4. Evaluate, if possible.

A. $\lim_{x \rightarrow 2} f(x)$

B. $\lim_{x \rightarrow 1} 3 \cdot f(x)$

C. $\lim_{x \rightarrow -1^+} f(x)$

D. $\lim_{x \rightarrow -2} f(x)$



Daily Lesson Quiz

Grade: _____

Grade: _____