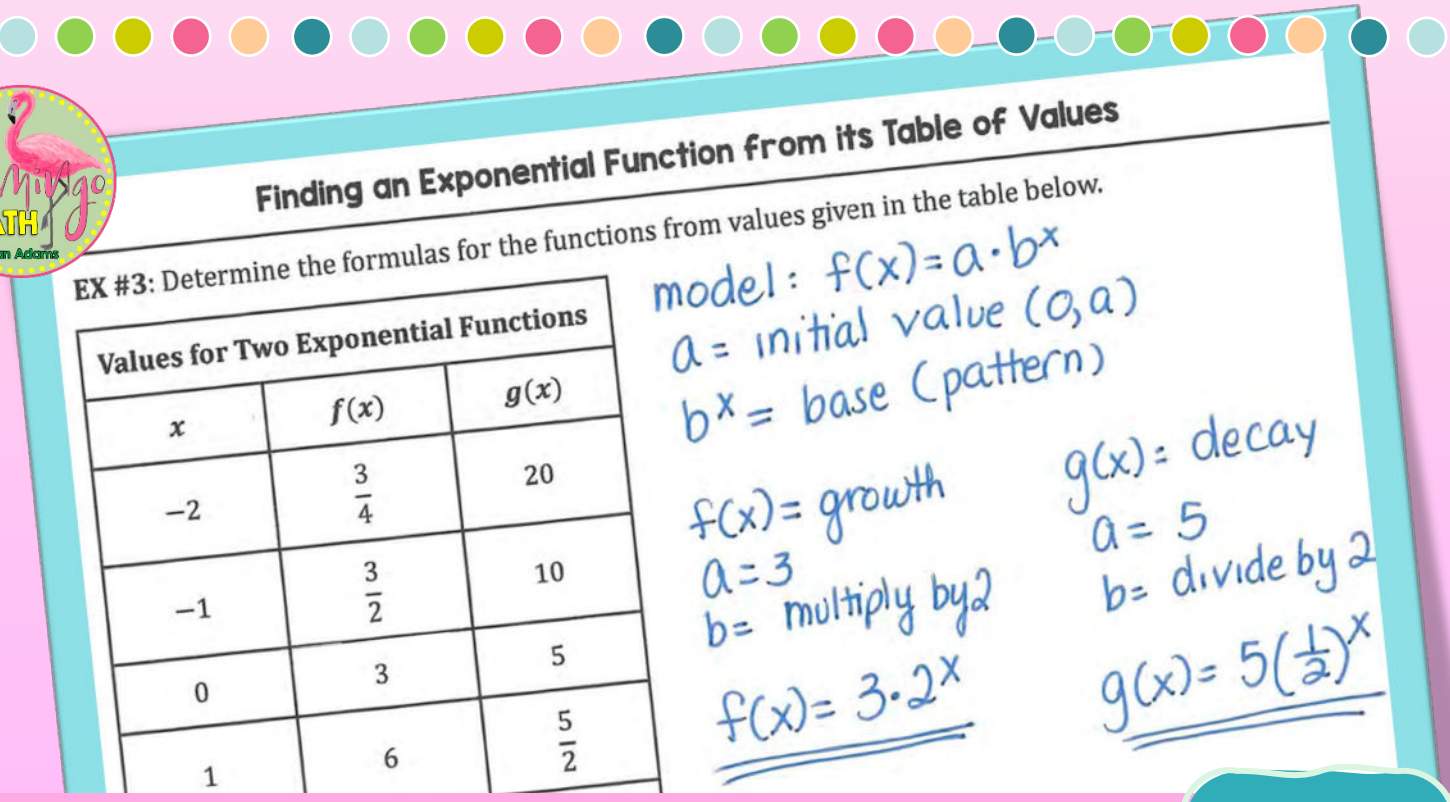
Precalculus Honors

Exponential Logistic & Logarithmic Functions



Guided Notes 3

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PRECALCULUS CURRICULUM

Scope and Sequence

Unit 3: Exponential, Logistic & Logarithmic Functions

Lesson	Skills & Learning Objectives
[3.1] Exponential & Logistic Functions	▪ Investigate the basic behaviors of exponential functions, logistic functions, and their graphs ▪ Analyze the function behavior related to population models ▪ Analyze exponential and logistic graphs for domain, range, end-behavior, continuity, symmetry, asymptotes, increase and decrease.
[3.2] Logarithmic Functions & Their Graphs	▪ Convert equations between logarithmic form and exponential form ▪ Evaluate common and natural logarithms ▪ Graph common and natural logarithmic functions ▪ Solve problems involving exponential and logarithmic expressions
[3.3] Properties of Logarithmic Functions	▪ Apply the properties of logarithms to evaluate expressions and graph functions ▪ Use properties of exponents and logarithms to simplify and evaluate expressions
[3.4] Equation Solving – Logarithmic Functions	▪ Apply the properties of logarithms to solve exponential and logarithmic equations algebraically ▪ Solve exponential and logarithmic equations related to real-world scenarios
[3.5] Exponential, Logistic & Logarithmic Modeling	▪ Use exponential growth, decay, and regression to model real-life problems.
[3.6] Analyzing Exponential Graphs	▪ Use properties of exponents to analyze the behavior of an exponential function without the aid of a calculator.

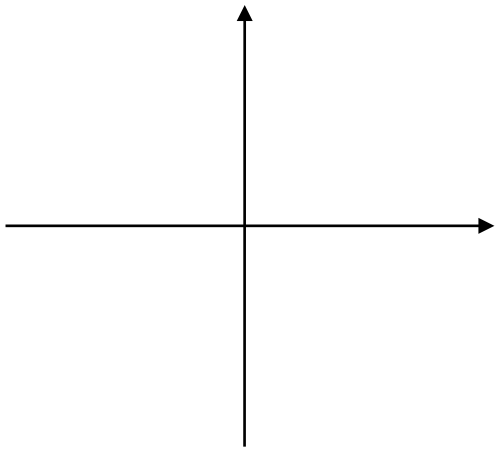
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Lesson 2: Logarithmic Functions & Their Graphs

Logarithmic Functions are Inverses of Exponential Functions

If $a > 0$ *and* $b > 0, b \neq 1$, then $y = \log_b x$ if and only if $b^y = x$.

Graph: $y = b^x$
$y = \log_b x$
$y = x$



Basic Properties of Logarithms

(Note: These are also valid for base 10 and base *e*)
For $b > 0, b \neq 1, x > 0,$ and any real number y

$\log_b 1 = 0$	$\log_b b = 1$	$\log_b b^y = y$	$b^{\log_b x} = x$
$\ln 1 = 0$	$\ln e = 1$	$\ln e^y = y$	$e^{\ln x} = x$

Evaluating Logarithmic and Exponential Expressions

EX #1: Give a reason for the following logarithms

A.	$\log_2 8 = 3$	because …
B.	$\log_3 \sqrt{3} = \frac{1}{2}$	because …
C.	$\log_5 \frac{1}{25} = -2$	because …
D.	$\log_7 7 = 1$	because …
E.	$6^{\log_6 11} = 11$	because …

Solving Simple Logarithmic Equations

To solve an exponential equation, change it to a logarithmic equation.
To solve a logarithmic equation, change it to an exponential equation.

EX #4: Solve each equation by changing it to its exponential form.

A. $\log x = 3$

B. $\log_2 x = 5$

Natural Logarithms, Base e

Notation: The logarithmic function $\log_e x = \ln x$

EX #5: Evaluate the following logarithmic and exponential expressions.

A. $\ln \sqrt{e}$

B. $\ln e^5$

C. $e^{\ln 4}$

Evaluating Natural Logarithms with a Calculator

EX #6: Use a calculator to evaluate the logarithmic expressions.

A. $\ln 23.5$

B. $\ln 0.48$

C. $\ln(-5)$

Using Basic Properties for Natural Logarithms

EX #7: Evaluate the natural logarithms below:

A. $\ln e^7$

B. $e^{\ln 81}$

C. $e^{\ln \sqrt{9x}}$

D. $\ln \frac{1}{e^5}$

Lesson 3: Properties of Logarithmic Functions

Properties of Logarithms

For any positive numbers M, N, and b where $b \neq 1$, and p any real number the following properties apply:

Product Property	
Quotient Property	
Power Property	

Note: These are also valid for base 10 and base e.

Expanding Logarithmic Expressions

EX #1: Expand the following logarithms using the properties.

A. $\log(8xy^4)$

B. $\ln\frac{\sqrt{x^2+5}}{x}$

Condensing Logarithmic Expressions

EX #2: Condense the following logarithms using the properties.

A. $\ln x^5 - 2\ln(xy)$

B. $\log 5 - \log x$

C. $3\ln x^3y + 2\ln(yz^2)$

Lesson 4: Equation Solving

Vocabulary

Richter Scale Model measures the magnitude *R* of an earthquake, *a* is amplitude in micrometers of the vertical ground motion at the receiving station, *T* is period of the seismic wave in seconds, and *B* is the dampening of the seismic wave as the distance from the epicenter increases.	$R = \log\frac{a}{T} + B$
pH Scale Model is a scale that measures how acidic or basic a substance ranks. The scale ranges from 0 to 14, where 7 represents neutral and each whole pH value below 7 is ten times more acidic than the next higher value.	$pH = -\log[H^+]$
Newton's Law of Cooling – The temperature T of an object at time *t*, where *Tm* = temperature of the surrounding medium, and T_0 = initial temperature of the object.	$T(t) = T_m + (T_0 - T_m)e^{-kt}$

One-to One Properties

Some logarithmic equations can be solved by changing the exponential form and vice versa.

For any Exponential Function	For any Logarithmic Function
$f(x) = b^y$	$f(x) = \log_b x$
If $b^u = b^v$, then	If $\log_b u = \log_b v$, then

Solving Exponential Equations Algebraically

EX #1: Solve. $24\left(\frac{1}{2}\right)^{x/3} = 6$

EX #2: Solve: $e^x = \pi^{1+x}$

Lesson 6: Analyzing Exponential Graphs

Growth and decay are not synonymous with increasing and decreasing.

In this unit, we have seen a wide variety of equations and graphs. Moving forward you need to distinguish between the difference between exponential growth and decay models. It is important to consider the behavior of the function in relation to the horizontal asymptote. Notice whether the function is moving away from or toward the horizontal asymptote. We have explored the following formulas that normally represent growth and decay in real-world applications:

Exponential Growth $k > 0$	Exponential Decay $k < 0$
$A(t) = A_0 e^{kt}$	$A(t) = A_0 e^{kt}$
$P(t) = P_0(1 + r)^t$	$P(t) = P_0(1 - r)^t$
$y = Ce^{kt}$	$T(t) = T_m + (T_0 - T_m)e^{kt}$

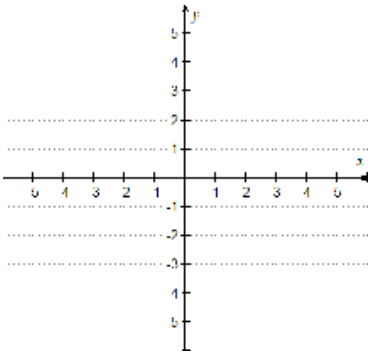
We will need to find the behavior of a graph without the aid of a calculator in this course. By using the rules for transformations of exponential functions, you can find patterns that will be helpful to graphing.

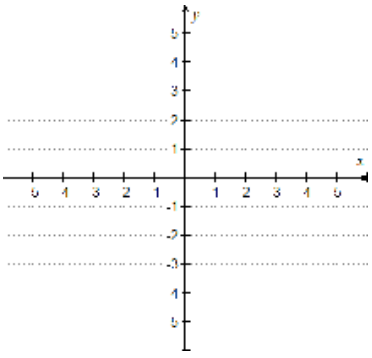
Analyzing Exponential Functions

EX #1: Recall that exponential equations are written in the form $y = ab^{(x-h)} + k$.
In this unit we have seen the following rules without considering the behavior for negative a-values.
Let's explore the effect a negative coefficient on x does to the behavior of a function.

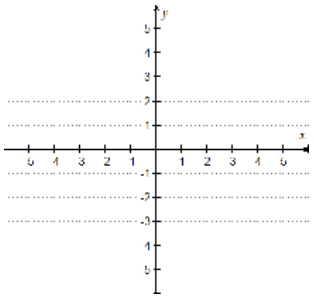
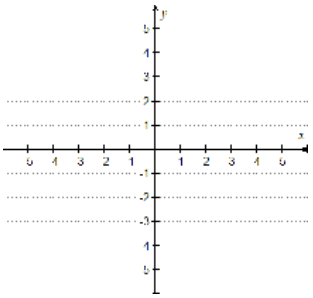
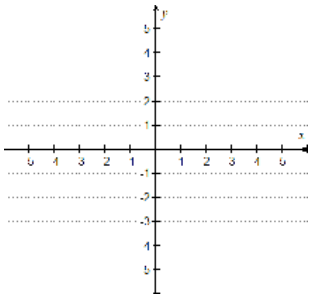
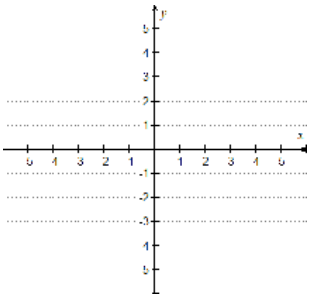
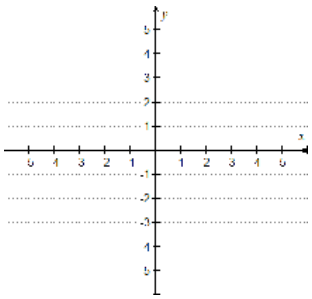
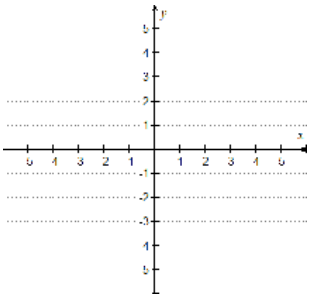
A. Graph and analyze: $y = -\left(\frac{1}{2}\right)^x$

B. Graph and analyze: $y = -2^x$





EX #3: Graph, describe the behavior as increasing or decreasing and state the end-behavior.

Exponential Growth	Exponential Decay
A. $f(x) = \left(\frac{3}{2}\right)^{x-1} - 2$ 	**D.** $f(x) = (2)^{-x-2} - 3$ 
B. $f(x) = -e^{x-1} + 3$ 	**E.** $f(x) = -\left(\frac{1}{4}\right)^{x+1} + 2$ 
C. $f(x) = \left(\frac{1}{2}\right)^{-x}$ 	**F.** $f(x) = 3^{-x} - 2$ 

EX #4: The graph at right can be modeled by the equation $y = a \cdot b^{-x} + c$. Complete the following.

a: _____

b: _____

c: _____

Left end behavior: _____

Right end behavior: _____

Growth or Decay Justification:

