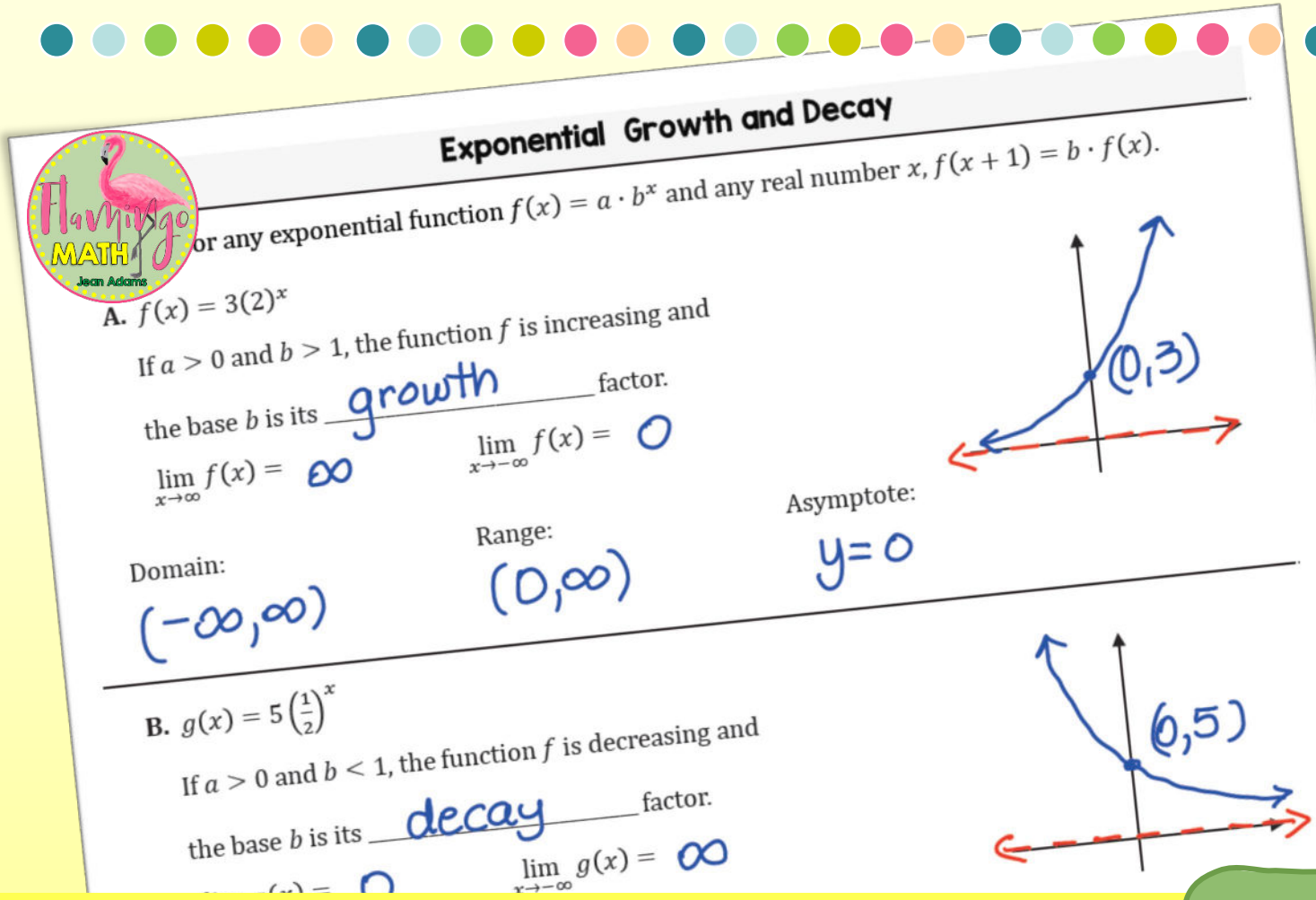
AP® Precalculus

Exponential Functions



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2.3

Lesson 3: Exponential Functions

While exploring linear growth, we observed a constant rate of change. For example, in the equation $f(x) = 2x + 3$, the slope tells us the output will increase by 2 each time the input increases by 1. In this lesson, we will explore what it means to grow exponentially. We will look at real world scenarios that have a *percent change per unit of time* rather than a constant (linear) change.

Definition of an Exponential Function

An ***exponential function in x*** is a function that can be written in the form
$f(x) = a \cdot b^x$
where b is a positive real number ($b > 0$), $b \neq 1$, and $a \neq 0$ is a real number.
The **domain** of f is the set of all real numbers. The base b is the **growth factor**, and the constant a is the ***initial value of f*** ($f(0) = ab^0 = a$).

Characteristics of Exponential Functions

- ❑ If $a > 0$ and $b > 1$ the exponential function will demonstrate *exponential growth.*
- ❑ If $a > 0$ and $0 < b < 1$, the exponential function will demonstrate *exponential decay.*

Graphing an Exponential Function

EX #1: Graph $y = 2^x$

x	-2	-1	0	1	2	3
2^x						
y						

Domain:

Range:

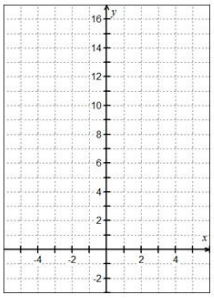
As $x \to \infty, f(x) \to$

As $x \to -\infty, f(x) \to$

Asymptote(s):

Intercept(s):

Concavity:



EX #2: Which of the following are exponential functions? Explain why or why not.

Identifying Exponential Functions

Function	Initial Value	Base	Why?
A. $f(x) = 2^x$			
B. $g(x) = 4x^{-3}$			
C. $h(x) = 5(e^{-x})$			
D. $f(x) = 3 \cdot 4^e$			

EX #3: Compute the exact value without a calculator.

A. $f(x) = 6(3)^{x+1}$ *for* $x = 2$

B. $f(x) = -5(8)^x$ *for* $x = \frac{1}{3}$

EX #4: Find an equation $y = g(x)$ that describes the function characteristics.

1. The value of g at $x = 0$ is 3.
2. The output value of g doubles at every 1-unit increase in the input value.

x	$g(x)$
0	3
1	
2	
3	
4	

Finding an Exponential Function from its Table of Values

EX #5: Determine the formulas for the functions from values given in the table below. For each function, compute the average rate of change of y with respect to x and the ratio of consecutive outputs.

A.

x	-2	-1	0	1	2
$f(x)$	$\frac{3}{4}$	$\frac{3}{2}$	3	6	12

B.

x	-2	-1	0	1	2
$g(x)$	20	10	5	$\frac{5}{2}$	$\frac{5}{4}$

Exponential Growth and Decay

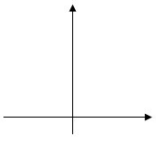
EX #6: For any exponential function $f(x) = a \cdot b^x$ and any real number x, $f(x+1) = b \cdot f(x)$.

A. $f(x) = 3(2)^x$

If $a > 0$ and $b > 1$, the function f is increasing and

the base b is its _____ factor.

$\lim_{x \to \infty} f(x) =$ $\lim_{x \to -\infty} f(x) =$



Domain: Range: Asymptote:

Compare and Contrast

EX #7: Big Tech Industries has 25 locations and is expanding by 10 new locations each year. The company growth can be represented by the function $B(x) = 25 + 10x$. Central Computing, Inc. has 25 locations, and they expand by increasing their total locations by 40% each year. The growth can be represented by the function $C(x) = 25(1 + 0.4)^x$.

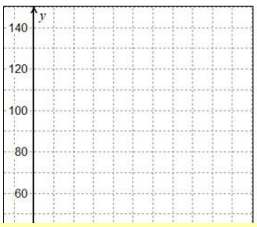
A. Complete the table to show the growth of each company for the first 5 years.

Year, x	Big Tech Industries	Central Computing, Inc.
0		
1		
2		
3		
4		
x	$B(x) = 25 + 10x$	$C(x) = 25(1 + 0.4)^x$

B. Graph each function on the grid provided to compare the growth for each company.

C. State the domain and range for $B(x)$ and $C(x)$.

D. When do both companies have the same number of locations?



Guided Notes

2.3 **Exponential Functions Practice**

Name _____

Date _____ Period _____

Problems 1-4, use a calculator to evaluate the function at the indicate value of x. Round your result to the nearest hundredth.

1. $f(x) = 2.7^x$ at $x = 3.4$

2. $g(x) = 4^x$ at $x = \sqrt{2}$

3. $h(x) = 1.3^x$ at $x = \frac{1}{3}$

4. $k(x) = 6.8^{-2x}$ at $x = -\pi$

Problems 5-10, identify whether the statement represents an exponential function. Explain how you know.

5. A population of bacteria decreases by a factor of $\frac{1}{4}$ every 12 hours.

6. The average yearly population increase of a flock of geese is 65.

7. Your personal trainer will discount each session by \$7 for every new referral you bring.

8. The value of your stamp collection has increased by 4.5% annually over the last 5 years.

9. The height of a book dropped from a building is modeled by $h(t) = -16t^2 + 30$

10. A certain radioactive isotope used in medicine has a half-life of 15 hours.

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Problems 11-14, determine whether the equation represents exponential growth, exponential decay, or neither. Explain.

11. $y = 250(1 - x)^3$

12. $f(x) = 18.4(1.025)^x$

13. $g(t) = 215(0.95)^t$

14. $h(x) = 7(1.025)^{\frac{1}{x}}$

Problems 15-18, determine whether the table could represent a function that is linear, exponential or neither. If it is exponential, find a function that passes through the points.

15.

x	-1	0	1	2	3
$f(x)$	$-\frac{1}{4}$	-1	-4	-16	-64

16.

x	-1	0	1	2	3
$g(x)$	-7	-4	-1	2	5

17.

x	-2	-1	0	1	2
$h(x)$	12	15	16	15	12

18.

x	-1	0	1	2	3
$k(x)$	3.5	7	14	28	56

Problems 19-20, use the table of values to determine whether $f(x)$ is an exponential function. Explain your reasoning and state the equation $f(x)$.

19. $g(x) = f(x) - 3$

x	-2	-1	0	1	2
$f(x)$					
$g(x)$	-2.75	-2.5	-2	-1	1

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20. $h(x) = f(x) + 2$

x	-3	-2	-1	0	1
$f(x)$					
$h(x)$	66	18	6	3	2.25

Problems 21 – 22, graph the function, label intercept(s), asymptotes, and analyze completely.

21. $f(x) = 4e^x$

Domain: Range:

Continuity: Increase / Decrease

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$

22. $g(x) = 3\left(\frac{1}{2}\right)^x$

Domain: Range:

Continuity: Increase / Decrease

$\lim_{x\to-\infty} g(x) =$ $\lim_{x\to\infty} g(x) =$

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Problems 23 – 24, use the table and given information to answer the questions.

23. An exponential function of the form $F(x) = a \cdot b^x + c$ is shown in the table below.

x	-8	-6	-3	-1	1	3	6
$F(x)$	4.9997	4.9973	4.9259	4.3333	-1	-49	-1453

A. Describe the end behavior of the function using limit notation.

B. What is the equation of the horizontal asymptote?

C. What conclusion can be made about the value of a? Why?

D. What is the value of c? Give a reason.

24. An exponential function of the form $G(x) = a \cdot b^{-x} + c$ is shown in the table below.

x	-8	-6	-3	-1	1	3	8
$G(x)$	487	92	17	4	3.25	2.477	2.003

A. Describe the end behavior of the function using limit notation.

B. What is the equation of the horizontal asymptote?

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Practice Sets

2.3 Exponential Functions **Form A**

Name _____

Date _____Period _____

Problems 1 – 3. Determine whether the equation represents exponential growth, exponential decay or neither and find $f(3)$. If necessary, round to the nearest hundredth.

1. $f(x) = 12(1.5)^x$

2. $f(x) = 12 + 1.5x$

3. $f(x) = 500(0.6)^x$

Use the following function for 4 and 5.

$$f(x) = 5e^x$$

4. Is the graph increasing or decreasing? Concave up or concave down?

5. Write the end behavior in limit notation.

2.3 Exponential Functions **Form B**

Name _____

Date _____Period _____

Problems 1-3. Determine whether the equation represents exponential growth, exponential decay or neither and find $f(4)$. If necessary, round to the nearest hundredth.

1. $f(x) = 0.75x + 5$

2. $f(x) = 800(0.75)^x$

3. $f(x) = 8(1.6)^x$

Use the following function for 4 and 5.

$$f(x) = 2\left(\frac{1}{3}\right)^x$$

4. Is the graph increasing or decreasing? Concave up or concave down?

5. Write the end behavior in limit notation.

Daily Lesson Quiz

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