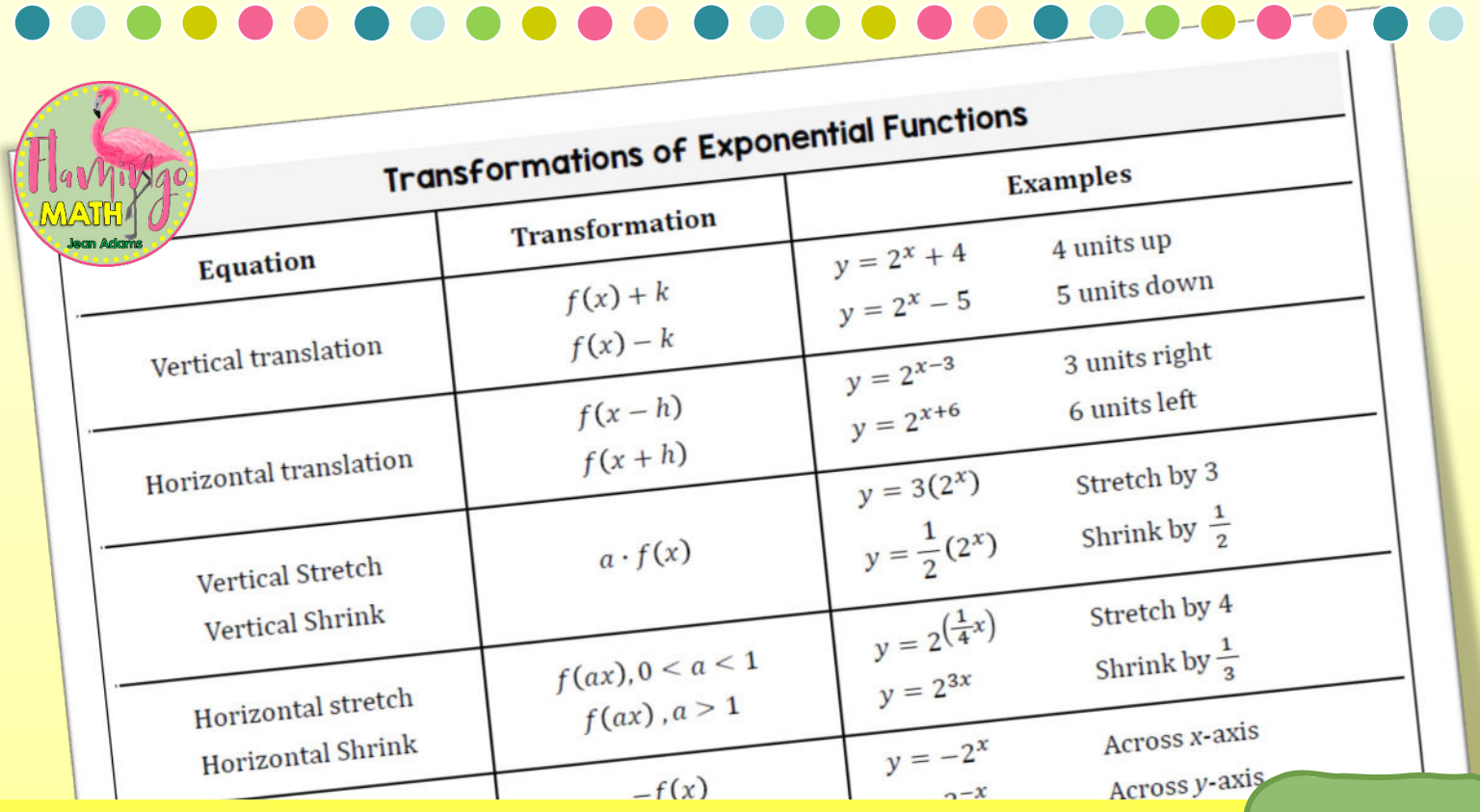
AP® Precalculus

Exponential Function Manipulation



Transformations of Exponential Functions

Equation	Transformation	Examples	
Vertical translation	$f(x)+k$ $f(x)-k$	$y=2^x+4$ $y=2^x-5$	4 units up 5 units down
Horizontal translation	$f(x-h)$ $f(x+h)$	$y=2^{x-3}$ $y=2^{x+6}$	3 units right 6 units left
Vertical Stretch Vertical Shrink	$a\cdot f(x)$	$y=3(2^x)$ $y=\frac{1}{2}(2^x)$	Stretch by 3 Shrink by $\frac{1}{2}$
Horizontal stretch Horizontal Shrink	$f(ax), 0<a<1$ $f(ax), a>1$	$y=2^{\left(\frac{1}{4}x\right)}$ $y=2^{3x}$	Stretch by 4 Shrink by $\frac{1}{3}$
	$-f(x)$	$y=-2^x$	Across x-axis Across y-axis

Completely Editable 2.4

Lesson 4-A: Exponential Function Manipulation

Properties of exponents are used to simplify exponential expressions. Let's recall those properties here and explore how they are related to graphs of exponential functions.

Properties of Exponents

If a, b, m and n are real numbers with $a > 0$ and $b > 0$, then

$(ab)^m = a^m \cdot b^m$ $\quad$ $(b^m)^n = b^{mn}$ $\quad$ $b^m \cdot b^n = b^{m+n}$

$b^{\frac{1}{k}} = \sqrt[k]{b}$ $\quad$ $b^{-n} = \frac{1}{b^n}$ $\quad$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

EX #1: Evaluate each number to three decimal places.

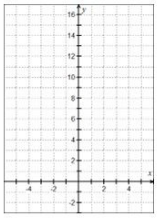
A. $5^{2.3}$ $\quad$ **B.** $3^{\sqrt{7}}$ $\quad$ **C.** $6^{-1.5}$

Characteristics of Exponential Functions of the Form $f(x) = b^x$

1. The domain of $f(x) = b^x$ consists of all real numbers. The range of $f(x) = b^x$ consists of all positive real numbers.
2. The graphs of all exponential functions of the form $f(x) = b^x$ pass through the point $(0, 1)$ because $f(0) = b^0 = 1$ $(b \neq 0)$. The y-intercept is 1.
3. If $b > 1$, $f(x) = b^x$ is an increasing function, continuous and concave up.
4. If $0 < b < 1$, $f(x) = b^x$ is a decreasing function, continuous and concave up.
5. The graph of $f(x) = b^x$ approaches, but does not cross, the x-axis. The x-axis is a horizontal asymptote.
6. $f(x) = b^x$ is one-to-one and has an inverse that is a function.

EX #2: Graph $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$ on the grid at right.

A. What can you conclude about the relationships between the graphs of $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$

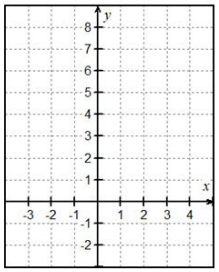


Transformations of Exponential Functions

Equation	Transformation	Description
$f(x) = b^x + k$ $f(x) = b^x - k$	Vertical translation	▪ Shifts graph k units upward ▪ Shifts graph k units downward
$f(x) = b^{x+k}$ $f(x) = b^{x-k}$	Horizontal translation	▪ Shifts graph k units left ▪ Shifts graph k units right
$f(x) = b^{x+k} = b^x b^k = ab^x$ $f(x) = b^{cx} = (b^c)^x$		▪ Vertical dilation where $a = b^k$ ▪ Horizontal dilation b^c is constant
$f(x) = -b^x$ $f(x) = b^{-x}$	Reflection	▪ Reflects graph about the x-axis ▪ Reflects graph about the y-axis
$f(x) = ab^x$	Vertical Stretch Vertical Shrink	▪ For $a > 1$, vertical stretch ▪ For $0 < a < 1$, vertical shrink

EX #3: Graph $f(x) = 3^x$ on the grid at right. Then, use the graph to obtain the graph of $g(x) = 3^{x+2}$

x	$f(x) = 3^x$	$g(x) = 3^{x+2}$
-2		
-1		
0		
1		
2		

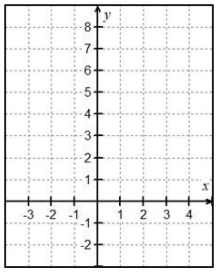


EX #4: Show that the function $g(x) = 3^{x+2}$ can be written in the form of $h(x) = ab^x$ using the transformation properties in the table above. Describe the transformation of the y-intercept from the parent function $f(x) = 3^x$ for $g(x)$ and $h(x)$.

Lesson 4-B: Exponential Function Manipulation

EX #5: Graph $f(x) = 3^x$ on the grid at right. Then, use the graph to obtain the graph of $g(x) = 3^{x+2}$

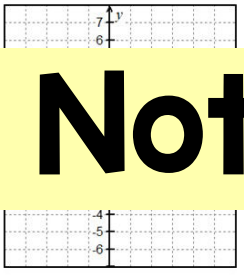
x	$f(x) = 2^{-x}$	$g(x) = 2^{-2x}$
-2		
-1		
0		
1		
2		



EX #6: Show that the function $g(x) = 2^{-2x}$ can be written in the form of $h(x) = ab^x$ using the transformation properties in the table shown on the previous page. Describe the transformation of the y-intercept from the parent function $f(x) = 2^{-x}$ for $g(x)$ and $h(x)$.

EX #7: Use the grid at right to graph $f(x) = -2^x$ and $g(x) = -2^x + 3$.

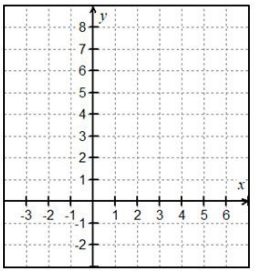
A. Label the y-intercept for each function.



EX #8: Graph $f(x) = \left(\frac{1}{3}\right)^x$ on the grid at right. Then, use the graph to obtain the graph of $g(x) = \left(\frac{1}{3}\right)^{x-4} + 2$

A. Find the y-intercept for $g(x)$.

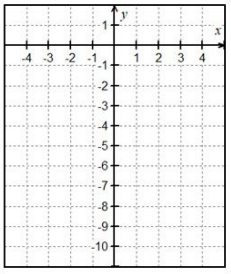
B. Describe the transformation for $g(x)$ compared to the parent function $f(x) = \left(\frac{1}{3}\right)^x$



EX #9: Graph the function and label the asymptote and y-intercept.

$y = -\frac{1}{2}\left(\frac{1}{2}\right)^x$

Describe the transformations from $y = 2^x$



EX #10: Write an equation for the transformation of the graph of $f(x) = 4^x$ then explain the

Guided Notes

2.4 **Exponential Function Manipulation Practice Set 1**

Name _____

Date _____ Period _____

Problems 1-4, Use a calculator to evaluate the function at the given value for x. Round your answer to three decimal places.

1. $f(x) = 4^x$; $x = -\pi$

2. $g(x) = 5.3^x$; $x = 7.8$

3. $h(x) = 2.6^{-3x}$; $x = -\sqrt{2}$

4. $k(x) = 1.8^{-x}$; $x = \frac{1}{4}$

Problems 5-8, Use the graph of f to describe the transformation that yields the graph of g.

5. $f(x) = 3^x, g(x) = 3^x + 4$

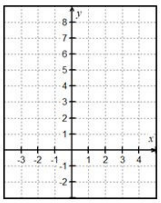
6. $f(x) = -4^x, g(x) = -4^{x-3}$

7. $f(x) = \left(\frac{1}{2}\right)^x, g(x) = \left(\frac{1}{2}\right)^{-(x+1)}$

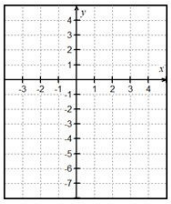
8. $f(x) = 0.2^x, g(x) - 0.2^x - 5$

Problems 9-12, Graph the exponential function by hand, identify asymptotes and intercepts. Determine whether the function is increasing or decreasing, concave up or concave down.

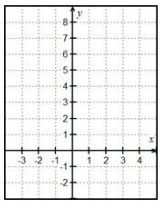
9. $g(x) = \left(\frac{3}{2}\right)^x$



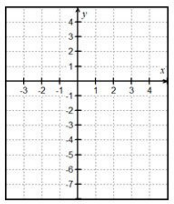
10. $h(x) = -2^x + 3$



11. $g(x) = \left(\frac{3}{2}\right)^{-x}$



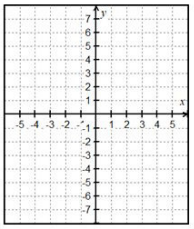
12. $h(x) = -2^{-x} + 4$



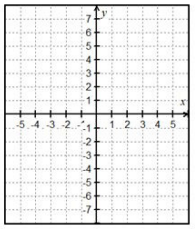
Problems 13-14, Given the function $f(x)$,

A. Show that the function $f(x) = b^{x+k}$ can be algebraically rewritten in the form of $h(x) = ab^x$, where $a = b^k$.
B. State the domain and range of the function.
C. Graph each function on the grid provided.
D. State the y-intercept and asymptotes.
E. Describe the transformation from each parent function.

13. $f(x) = 2^{x+3}$



14. $f(x) = -\left(\frac{1}{3}\right)^{-(x+2)}$



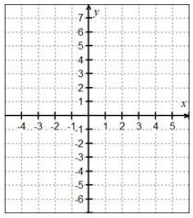
2.4 **Exponential Function Manipulation Practice Set 2**

Name _____

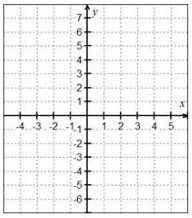
Date _____ Period _____

Problems 1-4, Graph the function $f(x)$, then use the graph to obtain the graph of $g(x)$. Label the asymptote and y-intercept of each graph. Describe the domain and range of $g(x)$.

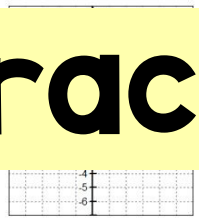
1. $f(x) = \left(\frac{1}{2}\right)^x$; $g(x) = \left(\frac{1}{2}\right)^{x-3} - 2$



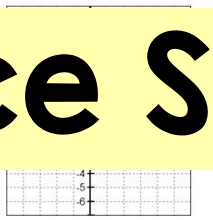
2. $f(x) = -3^{-x}, g(x) = -3^{2-x} + 5$



3. $f(x) = 2^{-x}, g(x) = 2^{-2x} - 3$



4. $f(x) = 2^x, g(x) = -2^x + 3$



Problems 5-8, Write an equation for the transformation of the graph of $f(x) = 3^x$ then explain the transformation for each graph.

5. $g(x) = f(x + 5)$

6. $h(x) = f(x) - 6$

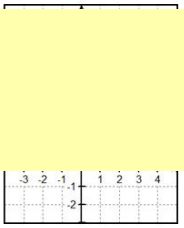
7. $k(x) = f(-2x)$

8. $p(x) = -3 \cdot f(x - 2) + 1$

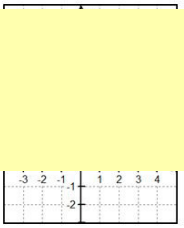
Problems 9-10, Given the function $f(x)$,

A. Show that the function $f(x) = b^{(kx)}$ can be algebraically rewritten in the form of $h(x) = ab^x$, where $a = b^k$ is a constant.
B. State the domain and range of the function.
C. Graph each function on the grid provided.
D. State the y-intercept and asymptotes.
E. Describe the transformation from each parent function.

9. $f(x) = 2^{-3x}$



10. $f(x) = \left(\frac{1}{4}\right)^{-2x}$



Practice Sets

2.4 Exponential Function Manipulation Set 1 **Form A**

Name _____

Date _____ Period _____

For 1 – 3, describe the transformations of f that yield the graph of g.

1. $f(x) = 2^x, g(x) = -2^x + 1$

2. $f(x) = \left(\frac{1}{5}\right)^x, g(x) = \left(\frac{1}{5}\right)^{-(x-1)}$

3. $f(x) = e^x, g(x) = e^{x+2} - 3$

For 4 and 5, identify the asymptote and whether the graph is concave up or concave down.

4. $f(x) = -(0.5)^x + 2$

5. $f(x) = 2^{-x} - 3$

Grade:

2.4 Exponential Function Manipulation Set 1 **Form B**

Name _____

Date _____ Period _____

For 1-3, describe the transformations of f that yield the graph of g.

1. $f(x) = e^x, g(x) = -e^x - 2$

2. $f(x) = \left(\frac{2}{3}\right)^x, g(x) = \left(\frac{2}{3}\right)^{x-3} + 1$

3. $f(x) = 4^x, g(x) = 4^{-(x+2)}$

For 4 and 5, identify the asymptote and whether the graph is concave up or concave down.

4. $f(x) = -3^x - 5$

5. $f(x)$

Grade

2.4 Exponential Function Manipulation Set 2 **Form A**

Name _____

Date _____ Period _____

Use the following function for 1 – 4.
$f(x) = 3^{-2x}$

1. Show that the function can be algebraically rewritten in the form $h(x) = ab^x$.

2. State the domain and range of the function.

3. State the y-intercept and asymptote.

4. Describe the transformation from the parent function $g(x) = 3^x$.

5. Write an equation for the transformation of the graph of $f(x) = 2^x$ then explain the transformation for the graph.
$p(x) = -3f(x+2) - 1$

Grade:

2.4 Exponential Function Manipulation Set 2 **Form B**

Name _____

Date _____ Period _____

Use the following function for 1 – 4.
$f(x) = 2^{-4x}$

1. Show that the function can be algebraically rewritten in the form $h(x) = ab^x$.

2. State the domain and range of the function

3. State the y-intercept and asymptote

4. Describe the transformation from the parent function $g(x) = 2^x$.

5. Write an equation for the transformation of the graph of $f(x) = 5^x$ then explain the transformation for the graph.
$p(x) = -2f(x-3) + 4$

Grade:

Daily Lesson Quiz