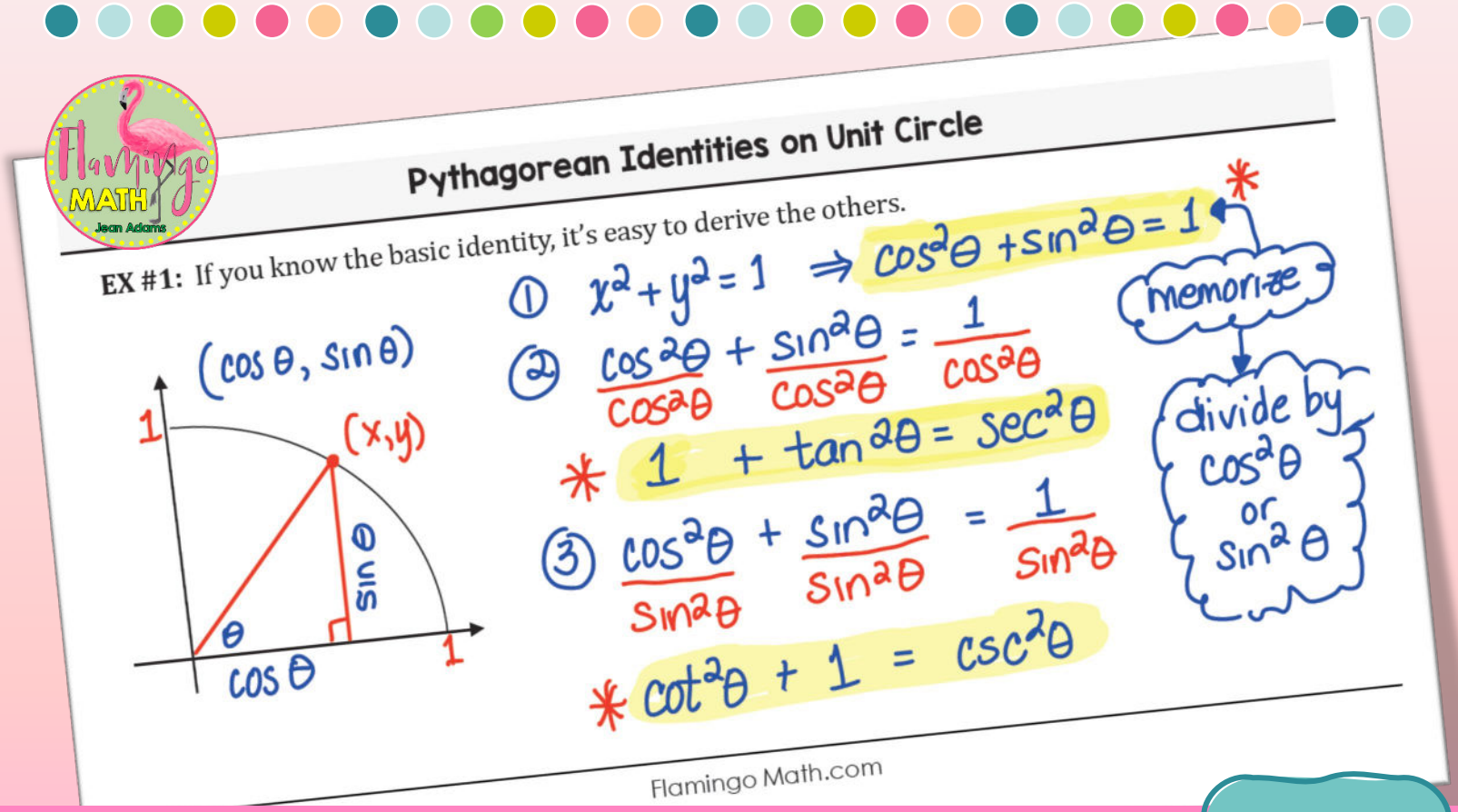
AP® Precalculus

Equivalent Representations of Trigonometric Functions



Completely Editable 3.12

Lesson I2-A: Equivalent Representations of Trigonometric Functions

Mathematical identities are two equated expressions that are equivalent in meaning and may be substituted for the other without losing the truth of a statement. For instance, 2+5=7 and 3+4 =7 so, we can say that 2+5=3+4. In trigonometric identities, we will work with expressions that can make a difficult problem easier to work with and remain equivalent.

The Fundamental Identities

Reciprocal Identities . . .

$\sin x = \frac{1}{\csc x}$ $\quad \cos x = \frac{1}{\sec x}$ $\quad \tan x = \frac{1}{\cot x}$

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cot x = \frac{1}{\tan x}$

Quotient Identities . . .

$\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities . . .

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

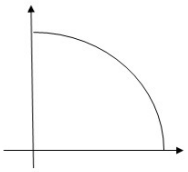
Even-Odd Identities . . .

$\sin(-x) = -\sin x$ $\quad \cos(-x) = \cos x$ $\quad \tan(-x) = -\tan x$

$\csc(-x) = -\csc x$ $\quad \sec(-x) = \sec x$ $\quad \cot(-x) = -\cot x$

Pythagorean Identities on Unit Circle

EX #1: If you know the basic identity, it's easy to derive the others.



Simplify and Rewrite Expressions to Verify Identities

EX #2: Use a quotient identity to simplify and rewrite.

A. $\cos\beta \tan\beta = \sin\beta$ **B.** $\sin x \sec x = \tan x$

EX #3: Use the idea of "decomposing" fractions to simplify and rewrite.

A. $\frac{\sin x - \csc x}{\csc x} = \sin^2 x - 1$ **B.** $\frac{\sec^2 x}{\tan x} = \sec x \csc x$

Changing to Sines and Cosines to Verify

EX #4: Verify the identities by changing to sine and cosine identities.

A. $\sin^2\theta \sec\theta \csc\theta = \tan\theta$ **B.** $\sin x \tan x + \cos x = \sec x$

Combining Fractional Expressions to Verify Identities

EX #5: Verify the identity using Pythagorean identities.

$\frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x} = \sec x$

EX #6: Verify the identity using algebraic techniques to simplify.

$\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

I2-B: Equivalent Representations of Trigonometric Functions

In this section we will study two different categories of trigonometric identities. We will use formulas that involve sum and differences to verify identities and double angle formulas.

Sum and Difference Formulas

Sums	Differences
$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$	$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Double-Angle Formulas

$\sin(2u) = 2\sin u\cos u$

$\cos(2u) = \cos^2 u - \sin^2 u$
$= 2\cos^2 u - 1$
$= 1 - 2\sin^2 u$

Sine and Cosine Formulas

EX #7: Find the exact value of $\sin\frac{5\pi}{12}$ using the fact that $\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$

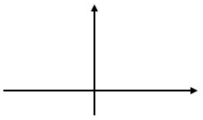
Guided Notes

Verify an Identity

EX #9: $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

Make A Sketch

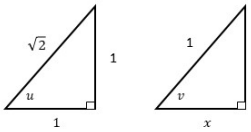
EX #10: Suppose that $\sin\alpha = \frac{4}{5}$ for a Quadrant II angle α and $\sin\beta = \frac{1}{2}$ for a Quadrant I angle β. Find the exact value of each of the following.



A. $\cos\alpha$	**B.** $\cos\beta$
C. $\cos(\alpha + \beta)$	**D.** $\sin(\alpha + \beta)$

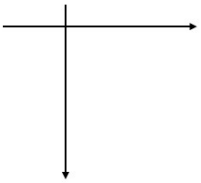
Application of a Sum Formula

EX #11: Write the trigonometric expression as an algebraic expression.
$\sin(\tan^{-1}(1) + \cos^{-1}x)$ Let $u = \tan^{-1}1$ and $v = \cos^{-1}x$



Evaluating Functions Involving Double Angles

EX #12: Find the exact values of $\sin 2u$ and $\cos 2u$ using the double-angle formulas.
Given $\cos\theta = \frac{3}{5}$ on $\frac{3\pi}{2} < \theta < 2\pi$



I2-C: Equivalent Representations of Trigonometric Functions

In this last section, we will solve trigonometric equations using the identities from the lesson.

EX #13: Find the solution(s) of the equation in the interval $[0, 2\pi)$. Verify with a graphing utility.
$\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

EX #14: Find the solution(s) of the equation in the interval $[0, 2\pi)$. Verify with a graphing utility.
$\sin 2x + \cos x = 0$

EX #16: Solve the equation $\cos 2x + 3\sin x - 2 = 0$ on $[0, 2\pi)$.

EX #17: Solve the equation: $\sin x - \cos x = 1$ on $0 \le x < 2\pi$

EX #18: Solve the equation: $\sin x - \sqrt{3}\cos x = 1$ on $0 \le x < 2\pi$

Guided Notes

3.12 **Equivalent Representations of Trigonometric Functions Practice Set 1**

Name _____

Date _____ Period _____

Problems 1 – 6, Use basic identities to simplify each expression.

1. $\frac{\cos\theta}{1-\sin^2\theta}$

2. $\frac{1-\cos^2\theta}{1-\sin^2\theta}$

3. $\cot x \sin x$

4. $\cos y \sec y$

5. $\sin x \csc(-x)$

6. $\tan x \cos x$

Problems 7 – 10, Factor each expression and use the fundamental identities to simplify.

7. $\frac{\csc(-\theta)}{\sec(-\theta)} = -\cot(\theta)$

8. $\cot^2 x - \cot^2 x \cos^2 x = \cos^2 x$

9. $\frac{\sin^2\theta - 1}{\sin^2\theta - \sin\theta} = 1 + \csc\theta$

10. $\frac{2\cos^2\theta - 2}{\cos^2\theta - 2\cos\theta + 1} = \frac{2(\cos\theta + 1)}{\cos\theta - 1}$

Problems 12- 15, Prove each identity.

12. $\sin\theta \csc\theta - \sin^2\theta = \cos^2\theta$

13. $\cos x(\tan x + \cot x) = \csc x$

14. $\csc x - \cot x = \frac{\sin x}{1 + \cos x}$

15. $\frac{\sin^2\theta}{1-\cos\theta} - 1 = \cos\theta$

3.12 **Equivalent Representations of Trigonometric Functions Practice Set 2**

Name _____

Date _____ Period _____

Problems 1 – 8, Find the exact value of each expression.

1. $\sin\frac{\pi}{18}\cos\frac{5\pi}{18} + \cos\frac{\pi}{18}\sin\frac{5\pi}{18}$

2. $\sin\frac{5\pi}{12}\cos\frac{\pi}{4} + \cos\frac{5\pi}{12}\sin\frac{\pi}{4}$

3. $\cos\frac{5\pi}{12}\cos\frac{7\pi}{12} - \sin\frac{5\pi}{12}\sin\frac{7\pi}{12}$

4. $\cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

5. $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

6. $\cos\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)$

Problems 9 – 12, Verify each identity.

9. $\sin(\pi - \theta) = \sin\theta$

10. $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$

11. $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

12. $\sin(\alpha - \beta)\sin(\alpha + \beta) = \sin^2\alpha - \sin^2\beta$

Practice Sets

14. If $\cos\theta = \frac{1}{4}$, θ in Quadrant IV, find $\cos\left(\theta + \frac{\pi}{3}\right)$

15. Given $\sin u = \frac{5}{13}$ for Quadrant I and $\cos v = -\frac{3}{5}$ for Quadrant II. Find $\cos(v-u)$.

Problems 16-17, Use the trigonometric substitution to create a right triangle and resolve each statement.

16. Use the substitution $x = 2\cos\theta$, to write $\sqrt{4-x^2}$ as a trigonometric function of θ, where $0 < \theta < \frac{\pi}{2}$.

17. Write the trigonometric expression $x = \sin\theta$, as the algebraic expression. $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$

Problems 18 – 20, Use the information given about the angle θ, $0 \le \theta \le 2\pi$, to find the exact values for $\sin(2\theta)$ and $\cos(2\theta)$.

18. $\tan\theta = \frac{3}{5}$; $\pi < \theta < \frac{3\pi}{2}$

19. $\cos\theta = -\frac{\sqrt{6}}{3}$; $\frac{\pi}{2} < \theta < \pi$

20. $\sec\theta = 3$; $\sin\theta > 0$

3.12 **Equivalent Representations of Trigonometric Functions Practice Set 3**

Name _____

Date _____ Period _____

Problems 1 – 4, Find all solutions to the equation in the interval $[0, 2\pi)$

1. $\sin 2x = 2\sin x$

2. $\cos 2x + \cos x = 0$

3. $\sin 2x - \sin x = 0$

4. $\cos 2x - \cos x = 0$

7. $\sin(2\theta) = \cos\theta$

8. $\cos(2\theta) = 3 - \sin\theta$

9. $\cos 2x + 6\sin^2 x = 4$

10. $\cos 2x + 5\cos x + 3 = 0$

Practice Sets

3.12 Equivalent Representations of Trigonometric Functions Set 1 **Form A**

Name _____

Date _____ Period _____

Use basic identities to simplify each expression.

1. $\frac{1-\cos^2 x}{\sin x}$

2. $\cot x \sec x$

Prove each identity

3. $\sin^2 x \cot^2 x + \sin^2 x = 1$

4. $\frac{\cos^2 x - 1}{\cos^2 x + \cos x} = 1 - \sec x$

Grade:

3.12 Equivalent Representations of Trigonometric Functions Set 1 **Form B**

Name _____

Date _____ Period _____

Use basic identities to simplify each expression.

1. $\frac{1-\sin^2 x}{\cos x}$

2. $\tan x \csc x$

Prove each identity

3. $\cos^2 x \tan^2 x + \cos^2 x = 1$

4. $\frac{\sin^2 x - 1}{\sin^2 x + \sin x} = 1 - \csc x$

Grade:

3.12 Equivalent Representations of Trigonometric Functions Set 2 **Form A**

Name _____

Date _____ Period _____

Find the exact value of each expression

1. $\cos\left(\frac{7\pi}{12}\right)$

2. $\sin\left(\frac{13\pi}{12}\right)$

3. Use the information to find the exact values for $\sin 2x$ and $\cos 2x$.

$$\tan x = \frac{3}{4};\ \pi < x < \frac{3\pi}{2}$$

Grade:

3.12 Equivalent Representations of Trigonometric Functions Set 2 **Form B**

Name _____

Date _____ Period _____

Find the exact value of each expression

1. $\sin\left(\frac{7\pi}{12}\right)$

2. $\cos\left(\frac{13\pi}{12}\right)$

3. Use the information to find the exact values for $\sin 2x$ and $\cos 2x$.

$$\tan x = \frac{4}{3};\ \pi < x < \frac{3\pi}{2}$$

Grade:

3.12 Equivalent Representations of Trigonometric Functions Set 3 **Form A**

Name _____

Date _____ Period _____

Find all solutions on the interval $[0, 2\pi)$

1. $\sin 2x = 2\cos x$

2. $\cos 2x + 6\sin^2 x = 3$

3. $\cos 2x + 9\cos x + 5 = 0$

Grade:

3.12 Equivalent Representations of Trigonometric Functions Set 3 **Form B**

Name _____

Date _____ Period _____

Find all solutions on the interval $[0, 2\pi)$

1. $\sin 2x = 2\sin x$

2. $\cos 2x + 6\cos^2 x = 3$

3. $\cos 2x - 9\sin x - 5 = 0$

Grade:

Daily Lesson Quiz