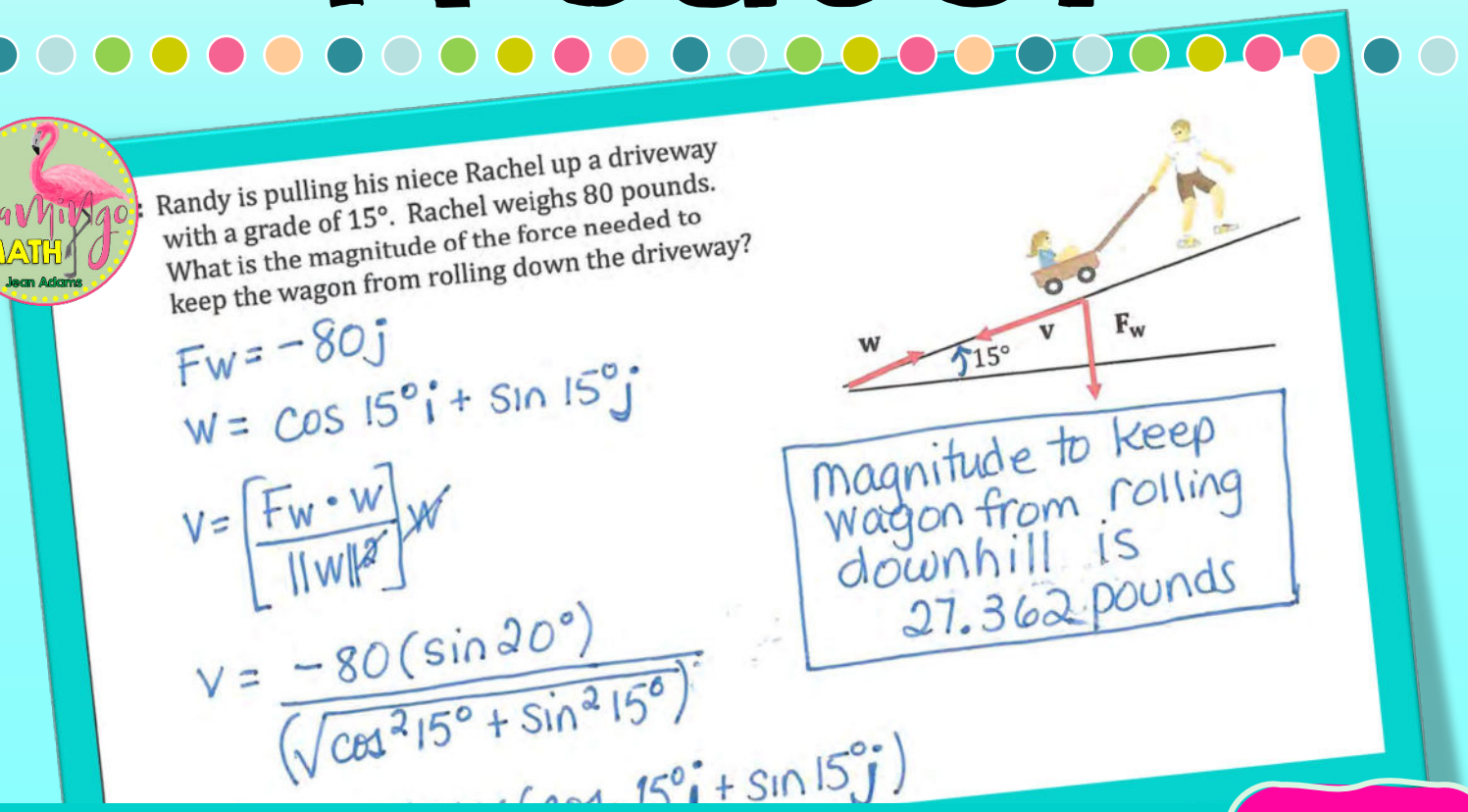
Precalculus Honors

The Dot Product



Completely Editable

6.6

Lesson 6.6: The Dot Product

In this section, we explore one of the ways vectors can be multiplied. The **dot product of two vectors** results in a scalar. So, the dot product of two vectors is not a vector but a real number.
Another method of multiplying vectors, known as the cross product, is a vector that is perpendicular to the plane of the two vectors being multiplied. This takes us into three dimensions, so it is outside of this chapter's scope.

The Language of Vectors

A **dot product** of $\mathbf{u} = \langle a_1, b_1 \rangle$ and $\mathbf{v} = \langle a_2, b_2 \rangle$ is $\mathbf{u} \bullet \mathbf{v} = a_1a_2 + b_1b_2$

Properties of Dot Products

Let **u, v** and **w** be vectors and let c be a scalar

1. $\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u}$
2. $0 \bullet \mathbf{v} = \mathbf{0}$
3. $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) = \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$
4. $\mathbf{v} \bullet \mathbf{v} = \mathbf{v}^2$
5. $c(\mathbf{u} \bullet \mathbf{v}) = c\mathbf{u} \bullet \mathbf{v} = \mathbf{u} \bullet c\mathbf{v}$

Angle Between Two Vectors:

$\cos\theta = \frac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \qquad 0 \le \theta \le \pi$

Alternate Form Of Dot Product:

$\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos\theta$

Orthogonal Vectors

Orthogonal (perpendicular or normal) vectors have a dot product of zero
Two vectors **u** and **v** are orthogonal if and only if $\mathbf{u} \bullet \mathbf{v} = 0$

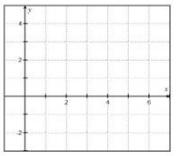
EX #1: Find the dot product $\mathbf{u} \bullet \mathbf{v}$, given: $\mathbf{u} = \langle 5, 12 \rangle$ and $\mathbf{v} = \langle -3, 2 \rangle$

EX #2: Use the dot product to find magnitude of $\mathbf{u} = \langle 2, -4 \rangle$

EX #3: Show that the vectors are orthogonal. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} + 6\mathbf{j}$

Finding the Angle Between Two Vectors

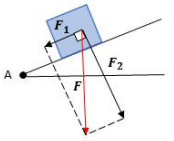
EX #4: Find the angle between the vectors $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j}$



Projecting One Vector onto Another

What if you want to decompose a resultant vector into the sum of two component vectors where $\mathbf{F_1} + \mathbf{F_2} = \mathbf{F}$? And, if we did, how could we find these "component parts?"

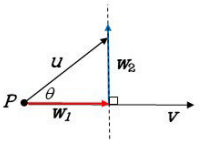
Often, we need to find "how much" a vector force is applied in a given direction. In the figure at right, the force **F** due to gravity is pulling straight down (toward the center of Earth) on the crate. To study the effect of gravity on the crate, we want to find how much of **F** is pushing the crate down the ramp ($\mathbf{F_1}$) and how much is pressing the crate down against the ramp ($\mathbf{F_2}$) at a right angle to the ramp.



Suppose that **u** and **v** are two nonzero vectors with the same initial point *P* such that $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$ where $\mathbf{w_1}$ and $\mathbf{w_2}$ are orthogonal (perpendicular) and $\mathbf{w_1}$ is parallel to **v**. The vectors $\mathbf{w_1}$ and $\mathbf{w_2}$ are called vector components of **u**.

The vector $\mathbf{w_1}$ is the **projection of u onto v** and is denoted by $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\,\mathbf{u}$.

The vector $\mathbf{w_2}$ is given by $\mathbf{w_2} = \mathbf{u} - \mathbf{w1}$



θ is acute

Projection of u onto v

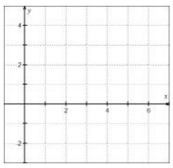
$\text{proj}_{\mathbf{v}}\,\mathbf{u} = \left(\frac{\mathbf{u} \bullet \mathbf{v}}{\mathbf{v}^2}\right)\mathbf{v}$ $\qquad \mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$

$\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ $\qquad \mathbf{w_2} = \mathbf{u} - \mathbf{w_1}$

EX #5: Given $\mathbf{u} = 4\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

A. Find the projection of **u** onto **v**

B. Find vector component of $\mathbf{u} \perp \mathbf{v}$



EX #6: Randy is pulling his niece Rachel up a driveway with a grade of 15°. Rachel weighs 80 pounds. What is the magnitude of the force needed to keep the wagon from rolling down the driveway?



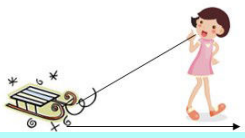
Work

In physics, the work *W* done by a constant force **F** in moving an object from point *A* to point *B* is defined as *W* =(magnitude of force)(distance)

1. $W = \|\text{proj}_{PQ}F\|\|AB\|$
2. $W = \mathbf{F}\mathbf{v}(\cos\theta)$ where $\mathbf{v} = AB$
3. $W = \mathbf{F}d$ d = displacement

EX #7: A 30-pound bag of feed is lifted 25 feet. Determine the work done.

EX #8: A child pulls a sled along level ground by exerting a force of 26 pounds on a rope that makes a 34° angle with the horizontal. How much work is done pulling the sled 225 feet?
Use: $W = F\,\mathbf{v}(\cos\theta)$



NOTES

6.6 **Dot Product of Vectors Homework**

Name _____

Date _____ Period _____

Problems 1 – 6, find the dot product of **u** and **v**.

1. $\mathbf{u} = \langle 4,3 \rangle$; $\mathbf{v} = \langle 8,5 \rangle$

2. $\mathbf{u} = \langle -6,1 \rangle$; $\mathbf{v} = \langle 14,-3 \rangle$

3. $\mathbf{u} = \langle -2,7 \rangle$; $\mathbf{v} = \langle -6,-9 \rangle$

4. $\mathbf{u} = -5\mathbf{i} + 7\mathbf{j}$; $\mathbf{v} = -2\mathbf{i} - 3\mathbf{j}$

5. $\mathbf{u} = 12\mathbf{i} - 5\mathbf{j}$; $\mathbf{v} = -8\mathbf{i} - 9\mathbf{j}$

6. $\mathbf{u} = -2\mathbf{i} - 9\mathbf{j}$; $\mathbf{v} = 14\mathbf{i} - 4\mathbf{j}$

Problems 7-10, find the angle between **v** and **w**. State whether the vectors are parallel, orthogonal, or neither.

7. $\mathbf{v} = 4\mathbf{i} + 4\mathbf{j}$, $\mathbf{w} = -2\mathbf{i}$

8. $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$, $\mathbf{w} = 3\mathbf{i} - 2\mathbf{j}$

9. $\mathbf{v} = \langle 1, \sqrt{3} \rangle$; $\mathbf{w} = \langle 1,-1 \rangle$

10. $\mathbf{v} = \langle 8,4 \rangle$; $\mathbf{w} = \langle -2,-1 \rangle$

Problems 11 – 14, find the values of k so that the vectors **v** and **w** are orthogonal.

11. $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$; $\mathbf{w} = -k\mathbf{i} + 2\mathbf{j}$

12. $\mathbf{v} = \mathbf{i} - k\mathbf{j}$; $\mathbf{w} = 2\mathbf{i} + 3\mathbf{j}$

13. $\mathbf{v} = \langle 4,6 \rangle$; $\mathbf{w} = \langle -2,k \rangle$

14. $\mathbf{v} = \langle 2,2 \rangle$; $\mathbf{w} = \langle k,1 \rangle$

Problems 15 – 18, decompose **v** into two vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ where $\mathbf{v_1}$ is parallel to **w** and $\mathbf{v_2}$ is orthogonal to **w**.

15. $\mathbf{v} = -3\mathbf{i} + 2\mathbf{j}$; $\mathbf{w} = 2\mathbf{i} + \mathbf{j}$

16. $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j}$; $\mathbf{w} = \mathbf{i} + 2\mathbf{j}$

17. $\mathbf{v} = \langle 3,4 \rangle$; $\mathbf{w} = \langle 8,2 \rangle$

18. $\mathbf{v} = \langle -5,-1 \rangle$; $\mathbf{w} = \langle -1,1 \rangle$

Problems 19 – 23, solve.

19. Find the work done lifting a 3250-pound car from a loading dock to a ship deck 50 feet above.

20. Find the work done by a force **F** of 28 pounds acting in the direction $\langle 4,5 \rangle$ in moving an object 10 feet from (0,0) to (10,0).

21. A passenger jet maintains a constant airspeed of 550 miles per hour in a northwesterly direction. The velocity of the Jetstream is 80 miles per hour from the east. Find the actual speed and direction of the aircraft.

22. A Ford F-250 with a gross weight of 4525 pounds is parked on a street with a slope of 10°.

A. Find the force required to keep the truck from rolling down the hill.

B. What is the force perpendicular to the hill?

Practice Sets

Lesson Warm-Ups

3. How much work is done to lift a 2825-pound car 7 feet? Answer to the nearest unit.

Name _____ Date _____ Period _____

Lesson 6.6: Dot Product of Vectors. **Warm-Up # 2**

1. Let $\mathbf{u} = 2i - 5j$; $\mathbf{v} = 3i + j$; $\mathbf{w} = 4i + 2j$
   Find $\mathbf{u} \cdot \mathbf{v} - \mathbf{u} \cdot \mathbf{w}$

2. Use dot product to find if vectors are orthogonal, parallel, or neither. $\mathbf{v} = 4i - 3j$; $\mathbf{w} = 6i + 8j$

3. How much work is done to lift a 3875-pound car 14 feet? Answer to

6.6 The Dot Product **Form A**

Name _____

Date _____ Period _____

1. Let $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$; $\mathbf{v} = 5\mathbf{i} + \mathbf{j}$; $\mathbf{w} = \mathbf{i} + 6\mathbf{j}$
   Find $\mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$

2. Use the dot product to find if vectors are orthogonal, parallel, or neither.
   $\mathbf{v} = 8\mathbf{i} - 3\mathbf{j}$ and $\mathbf{w} = 5\mathbf{i} - 12\mathbf{j}$

3. How much work is done to lift a 2750-pound car 6.5 feet? Answer to the nearest unit.

Grade:

6.6 The Dot Product **Form B**

Name _____

Date _____ Period _____

1. Let $\mathbf{u} = 5\mathbf{i} - 3\mathbf{j}$; $\mathbf{v} = 4\mathbf{i} + \mathbf{j}$; $\mathbf{w} = 3\mathbf{i} + \mathbf{j}$
   Find $\mathbf{u} \bullet \mathbf{v} - \mathbf{u} \bullet \mathbf{w}$

2. Use the dot product to find if vectors are orthogonal, parallel, or neither.
   $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{w} = 6\mathbf{i} + 9\mathbf{j}$

3. How much work is done to lift a 3850-pound pickup truck 12 feet? Answer to the nearest unit.

Grade:

Daily Lesson Quiz