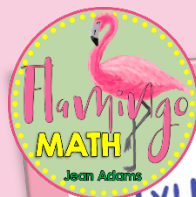


AP[®] Calculus

Differentiation-Part 2

Composite, Implicit, and Inverse Functions

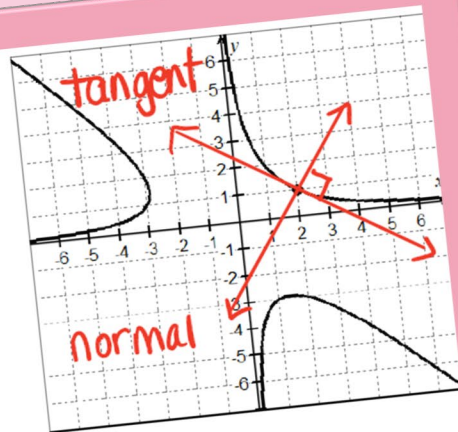


$$y^2x = 6 \text{ at } (2,1)$$

$$\begin{aligned} 2xy + x^2y' + 2yy'x + y^2 &= 0 \\ y'[x^2 + 2xy] &= -2xy - y^2 \\ y' &= \frac{-2xy - y^2}{x^2 + 2xy} \end{aligned}$$

$$\left. \frac{dy}{dx} \right|_{(2,1)} = \frac{-4-1}{4+4} = -\frac{5}{8}$$

$$\text{Tangent line: } y = -\frac{5}{8}(x-2)+1$$



Homework

3

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3.I**The Chain Rule
Homework**

Name _____

Date _____ Period _____

Problems 1 – 11, find the derivative of the function.

1. $y = (3x - 2)^4$

2. $f(x) = 5(2 - x^2)^3$

3. $y = (6 + 3x)^{2/3}$

4. $f(x) = \sqrt{x + 3}$

5. $g(x) = \sqrt[3]{x^2 - 9}$

6. $y = \frac{-2}{(x^2 - 3x - 4)^2}$

7. $f(x) = \left(\frac{3}{3-x}\right)^2$

8. $y = \sqrt[3]{2x^2 - 5x + 3}$

Problems 32 – 35, use the numerical values in the table below to evaluate the derivatives at the given values of x .

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	6	3	1	-2
3	2	-4	2π	5

32. $f(g(x))$ at $x = 2$

33. $\sqrt{f(x) \cdot g(x)}$ at $x = 2$

34. If $H(x) = (4f(x) + g(x))^{\frac{2}{3}}$, find the value of $H'(3)$.

35. Let $P(x) = \ln(4x - 5)^k$ for some positive-valued constant k . If $P'(2) = 8$, what is the value of k ?

3.2

Symbolic Differentiation Homework

Name _____

Date _____ Period _____

Let $f(x), f'(x), g(x), g'(x)$ be differentiable functions whose values are given in the table below. Find the symbolic differentiation. Then use the values from the table to find the derivative at the designated point.

x	-1	0	1	2
$f(x)$	-1	3	-2	0
$f'(x)$	-3	4	1	5
$g(x)$	2	1	5	-2
$g'(x)$	7	-3	0	-1

1. $\frac{d}{dx}(f + g)$ Evaluate when $x = 1$

2. $\frac{d}{dx}(f - g)^2$ Evaluate when $x = 0$

3. $\frac{d}{dx}\left(\frac{1}{f-g}\right)$ Evaluate when $x = 2$

4. $\frac{d}{dx}\sqrt{f+g}$ Evaluate when $x = 1$

5. $\frac{d}{dx}(f \cdot g)$ Evaluate when $x = 0$

6. $\frac{d}{dx}(f^2)$ Evaluate when $x = 2$

Problems 17 – 20, use the table of values to calculate the derivative of the function at the given point.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	9	-2
3	0	-1	2	3
6	-1	1/2	3	4

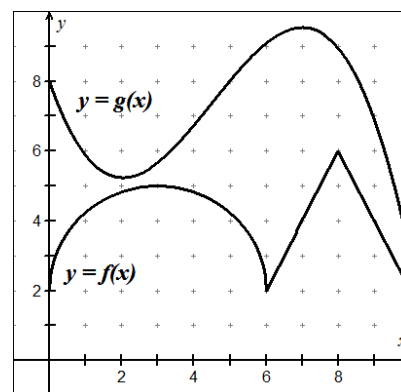
17. $H(x) = f(g(x))$. Find $H'(6)$

18. $K(x) = g(\sqrt{x})$. Find $K'(9)$

19. $J(x) = e^{f(x)}$. Find $J'(3)$

20. $R(x) = \ln(x^2 \cdot \sqrt{g(x)})$. Find $R'(1)$

Problems 21– 22, let $H(x) = f(g(x))$, where the graphs of f and g are shown at right. Estimate each derivative. Justify your analysis algebraically.



21. $g'(f(6))$

22. $H'(6)$

3.4**Inverse Functions
Homework**

Name _____

Date _____ Period _____

Problems 1 - 4, find $(f^{-1})'(b)$ for each function.

1. $f(x) = x^3 - 1, b = 7$

2. $f(x) = x^3 + 2x - 1, b = 2$

3. $f(x) = \sin x, b = \frac{1}{2}$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

4. $f(x) = \cos 2x, b = 1$ on $\left[0, \frac{\pi}{2}\right]$

Problems 5 - 6, Multiple Choice.

5. If $f(x) = \frac{4}{x} + 2$ and g is the inverse of f , then $g'(10) =$

- A.
- -16
- B.
- $-\frac{1}{2}$
- C.
- $-\frac{1}{16}$
- D.
- $\frac{1}{2}$

6. If $(3,5)$ is a point on the graph of $y = f(x)$, a one-to-one function, and $f'(3) = 2$, then which of the following must be true?

- | | |
|--|---|
| A. $\frac{d}{dx}[f^{-1}(3)] = \frac{1}{2}$ | B. $\frac{d}{dx}[f^{-1}(3)] = -\frac{1}{2}$ |
| C. $\frac{d}{dx}[f^{-1}(5)] = \frac{1}{2}$ | D. $\frac{d}{dx}[f^{-1}(3)] = -2$ |

Problems 7 – 10, Short Answer.

7. If $f(3) = 8$ and $f'(3) = 5$ what do you know about f^{-1} ?

8. Let $f(x) = x^3 + x - 2$, and let g be the inverse function. Evaluate $g'(0)$.

9. A function f and its derivative take on the values shown in the table. If g is the inverse of f , find $g'(6)$.

x	$f(x)$	$f'(x)$
2	6	$\frac{1}{3}$
6	8	$\frac{3}{2}$

10. Your parents bought you a brand new car at graduation! Well, we can dream, right? Assume that the value of the car is given by the function, $V(t) = 25,000e^{-0.1t}$, where v is the value, in dollars, of the car and t is the time, in years, from the date of purchase.

A. How much was the car worth at the dealership ($t = 0$)?

B. How much did the car depreciate during its fourth year (from $t = 3$ to $t = 4$)?

C. When you graduate from college, assuming you are on the 4-year plan, what is the instantaneous rate of change (in dollars per year) of the value at $t = 4$? Why is this different than your answer in part B?

D. When will the value of the car reach \$12,500?