

AP[®] Calculus

Differentiation

Basic Derivative Rules



Did Neil Armstrong step off the Lunar Module and pick up a moon rock from the surface of the moon on July 20, 1969? If he did, scientists say that the rock would have a velocity of 24 meters per second (about 86 km/h). The height of the moon rock can be modeled by the function $s(t) = 24t - 0.8t^2$ meters in t seconds.

- A. Find the velocity and acceleration of the moon rock as a function of time. (Note: The acceleration will be the acceleration of gravity on the moon.)

$$v(t) = 24 - 1.6t$$

$$a(t) = -1.6$$

- B. How long will it take for the rock to reach a maximum height?

$$24 - 1.6t = 0$$

$$1.6t = 24$$

$$t = \underline{\underline{15 \text{ sec.}}}$$

- C. How high did the rock go? When did it hit the ground?

$$s(15) = 24(15) - 0.8(15)^2$$



Photo Courtesy: NASA.gov

Guided Notes

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AP CALCULUS CURRICULUM

Scope and Sequence

Unit 2: Differentiation - Basic Derivative Rules

Lesson	Skills & Learning Objectives
[2.1] Rates of Change and the Tangent Line Problem	☐ Interpret rates of change over an interval or at a single point by graphical and numerical methods ☐ Write equations of tangent lines and normal lines
[2.2] Tangent Lines and the Derivative	☐ Use limit definition of the derivative, proper notation to find the general form of a derivative ☐ Analyze function behavior using the derivative
[2.3] Understanding the Derivative and Linear Approximations	☐ Explore the concept of local linearity ☐ Use the tangent line to investigate function behavior ☐ Estimate derivatives graphically and from tables
[2.4] Differentiability and Continuity	☐ Understand the relationship between differentiability and continuity
[2.5] Basic Differentiation Rules	☐ Find derivatives using basic rules of differentiation for polynomial, power, sine, cosine, tangent, exponential and logarithmic functions
[2.6] Product and Quotient Rules	☐ Apply the product and quotient rules algebraically for polynomial, power, sine, cosine, tangent, exponential and logarithmic functions
[2.7] Velocity and Other Rates of Change	☐ Solve basic rectilinear motion problems involving position, speed, velocity and acceleration ☐ Interpret the effect of a one-unit change

Lesson I: Rates of Change and the Tangent Line Problem

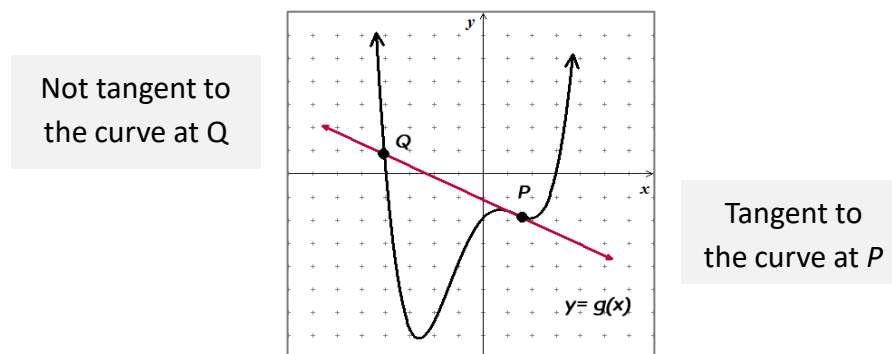
Topic 2.1: Defining Average and Instantaneous Rates of Change at a Point

In plane geometry, we learned that a SECANT LINE is a line that intersects a circle, or some other curve, in two places. We also learned that a TANGENT LINE is a line that intersects a circle at one point.

In our study of differential calculus, we need to clarify, classify, and truly understand these two big ideas. For general curves, we need a better definition. We will be concerned with the tangent line drawn at a point on a curve.

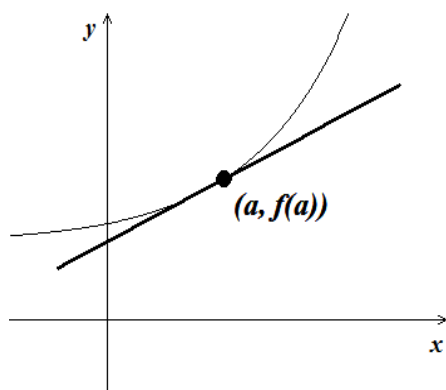
Most of us believe that a tangent to a curve is a line that intersects the curve at exactly one point. However, in the figure below, you can see that the tangent line touches the curve at several points. So a better way to describe a tangent line would be to say,

A tangent to a curve at a point is the line that best models, or approximates, the behavior of the curve at that point.



This idea will help us find the *slope of the curve at a point*. Also, that this slope of the curve at a specified point will be referred to as the **SLOPE OF THE TANGENT LINE, INSTANTANEOUS RATE OF CHANGE, or DERIVATIVE**.

The most important thing to note here is that the tangent to a curve at any given point is unique. A function f that has a defined slope at $x = a$ has two things in common with its tangent at $x = a$.



1. The point of tangency, $(a, f(a))$
2. The slope at the point of tangency, denoted by $f'(a)$, read f prime of a .

Any function with a defined slope at a specific point will have one and only one tangent line at that point.

Understanding the Derivative Graphically

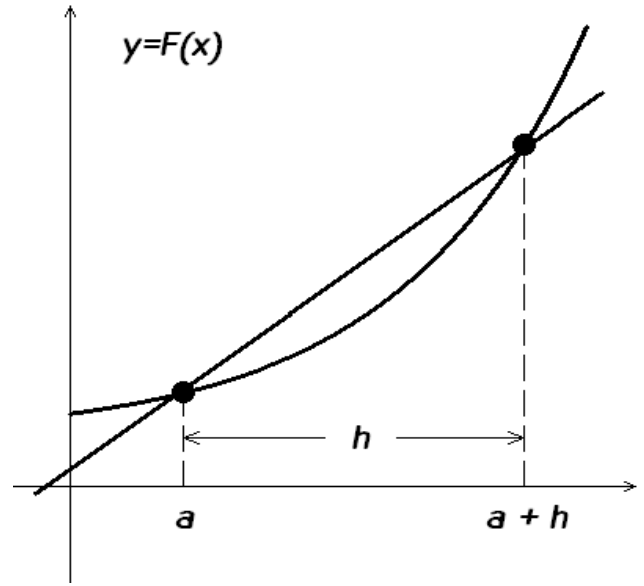
EX #5: Grab some colored pencils and a ruler.

The graph of the function $y = F(x)$ at right, is differentiable on the interval $[a, a + h]$.

Instructions:

Use the expression in Column I:

1. State whether the expression represents a length or a slope.
2. Draw the geometric representation of the expression on the graph of F using
 - A blue pencil to represent any length
 - A red pencil to represent any slope



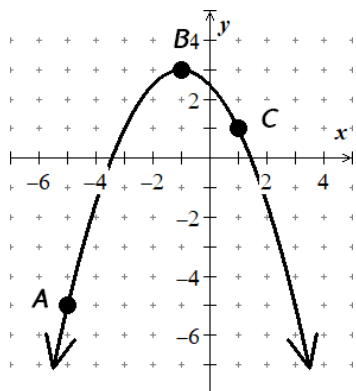
Notation	Length or Slope?	Terminology
1. $f(a)$		
2. $f(a + h)$		
3. $f(a + h) - f(a)$		
4. h		
5. $\frac{f(a+h)-f(a)}{h}$		
6. $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$		
7. $f'(a)$		

Function Behavior Related to Tangent Line Position

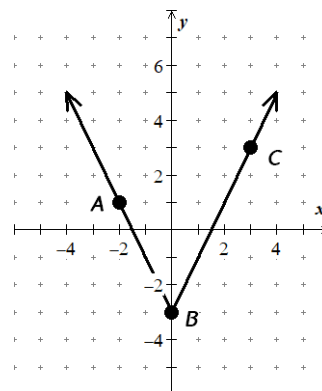
Topic 2.3: Estimating Derivatives of a Function at a Point

EX #2: Sketching Tangent Lines In each figure below, use a straight edge to draw a tangent line beyond the point of tangency in each direction.

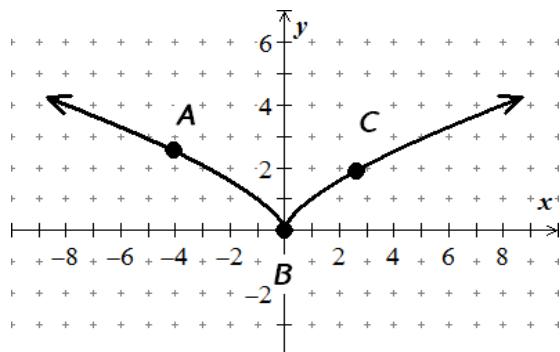
A.



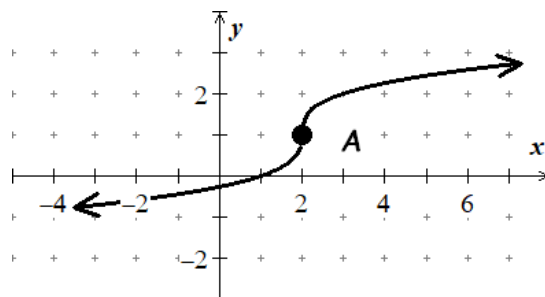
B.



C.



D.



EX #3: The position of the tangent line (relative to the function it models) depends on the concavity of the function at the point of tangency. Throughout the course you will be required to explain, justify, and defend your knowledge using proper terminology and precision. Use the graphs from EX #2, above to complete the following statements.

- A. When the curve is *concave up*, the tangent line lies _____ (except at the point of tangency)
- B. When the curve is *concave down*, the tangent line lies _____ (except at the point of tangency)
- C. At a “turning point,” max/min, the slope of the tangent line is _____ or _____.
- D. At a “point of inflection” concavity changes, the tangent is _____.
- E. At a “sharp” corner or “cusp”, the tangent _____.
- F. At a point on a linear function, the tangent is _____.

E. $h(x) = 3x^2 - \frac{4}{x^2} + 2\sqrt{x}$

F. $f(x) = \frac{8x^3 + 4x^2 - 3}{x}$

Topic 2.7: Derivatives of $\cos(x)$, $\sin(x)$, e^x and $\ln(x)$

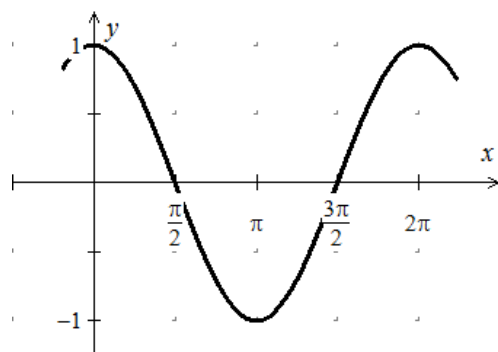
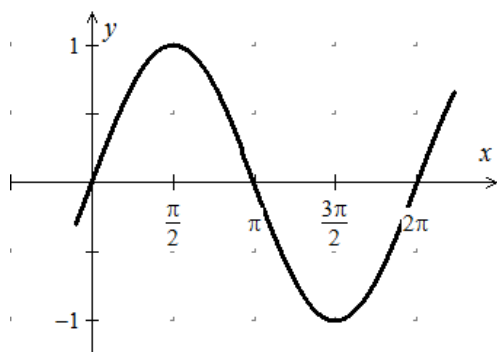
6. Derivatives of Sine, Cosine, and Tangent Functions

EX #4: The derivative of the sine function is the cosine function. Consider where the sine function increases, the cosine function is positive and vice versa.

$$\frac{d}{dx} [\sin x] =$$

$$\frac{d}{dx} [\cos x] =$$

$$\frac{d}{dx} [\tan x] =$$

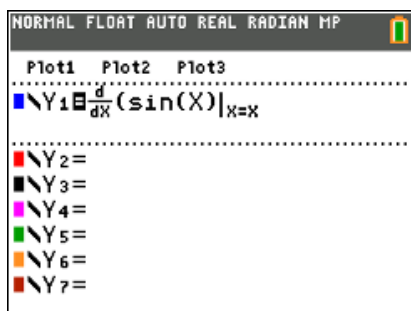


Graphing Derivatives with a Calculator

You will enter the function as a function of x , to be evaluated at the value of x , instead of an actual number value.

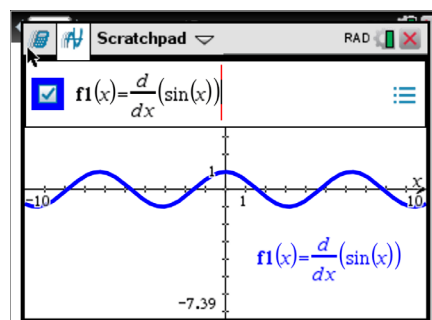
TI-83+ or TI-84+

Use the numeric derivative operation (MATH > 8) in the graph editor screen.



TI-Nspire CX

Use the template menu in the graph editor screen



2. The Quotient Rule:

If a function is the quotient of two differentiable functions, then the derivative is
“the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the denominator, squared.

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) f'(x) - f(x) g'(x)}{[g(x)]^2} ; g'(x) \neq 0$$

$$\frac{d}{dx} \left[\frac{HI}{LO} \right] = \frac{\text{Low di Hi} - \text{Hi di Low}}{\text{Low} \cdot \text{Low}} ; \text{Low} \neq 0$$

EX #2: Use the quotient rule to differentiate.

A. $g(x) = \frac{2x^3 + 4x^2 - 7}{x-5}$

B. $g(x) = \frac{e^x}{x^2 + 1}$

C. $f(x) = \frac{\ln(x)}{x-2}$

D. $y = \frac{\sin(x)}{1 + \cos(x)}$

More Things to Contemplate!

Subtraction is not commutative, you have to be careful with the quotient rule.
The quotient rule is NOT to be interpreted as follows: “The derivative of a quotient of two functions f and g is the quotient of the derivatives of f and g .

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] \neq \frac{f'(x)}{g'(x)} ; g(x) \neq 0, g'(x) \neq 0$$

Another Fun Mnemonic:

$$\frac{d}{dx} \left[\frac{HI}{HO} \right] = \frac{\text{HO di HI} - \text{HI di HO}}{\text{HO} \cdot \text{HO}} ; \text{HO} \neq 0$$

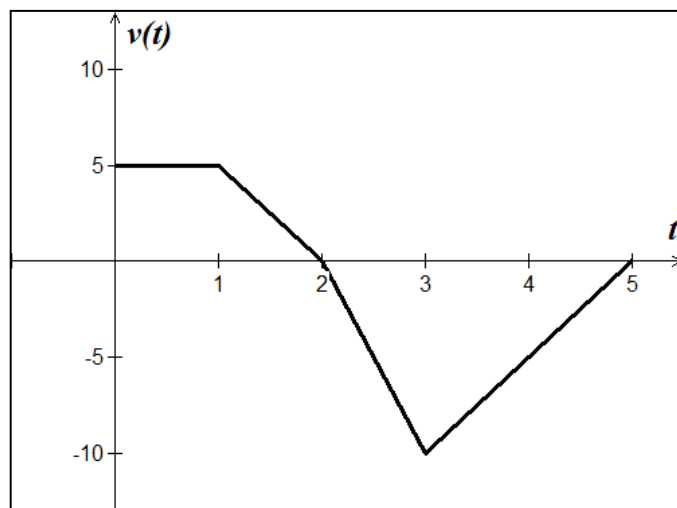
EX #5: The graph at right shows the velocity of a particle moving along a coordinate line. Assume that the positive direction is to the right, and $v(t)$ is measured in feet per second, find the following.

1. When is the particle moving at a constant speed?

2. When is the particle moving to the right?

3. When is it moving to the left?

4. When is the particle speeding up?



5. When is it slowing down?

6. When is the velocity increasing?

7. When is the velocity decreasing?

8. Are your answers to questions 6 & 7 the same as your answers to 4 & 5 respectively? Explain.

9. How fast is the particle moving at time $t = 4$, and in what direction?

10. What is the particle's velocity at time $t = 4$?

11. What is the average velocity over the time interval $2 < t < 3$?

12. When does the particle change direction?

13. How far does the particle move during the first second and which way?

14. How far to the right does the particle go and when does it arrive there?

15. What is the total distance traveled by the particle?

16. What is the particle's position after 5 seconds?