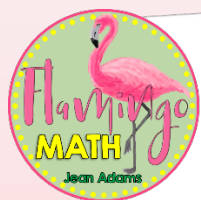


AP[®] Calculus-BC

Differential Equations



5. Radioactive radium has a half-life of approximately 1599 years.
- A. If 50 milligrams of pure radium is present, initially, when will the amount be 20 milligrams? Give your answer to the nearest year.

$$20 = 50 \left(\frac{1}{2} \right)^{t/1599}$$

$$\ln \left(\frac{2}{5} \right) = \frac{t}{1599} \left(\ln \frac{1}{2} \right)$$

$$\frac{\ln \left(\frac{2}{5} \right)}{\ln \left(\frac{1}{2} \right)} = \frac{t}{1599}$$
$$t \approx 2113.763$$

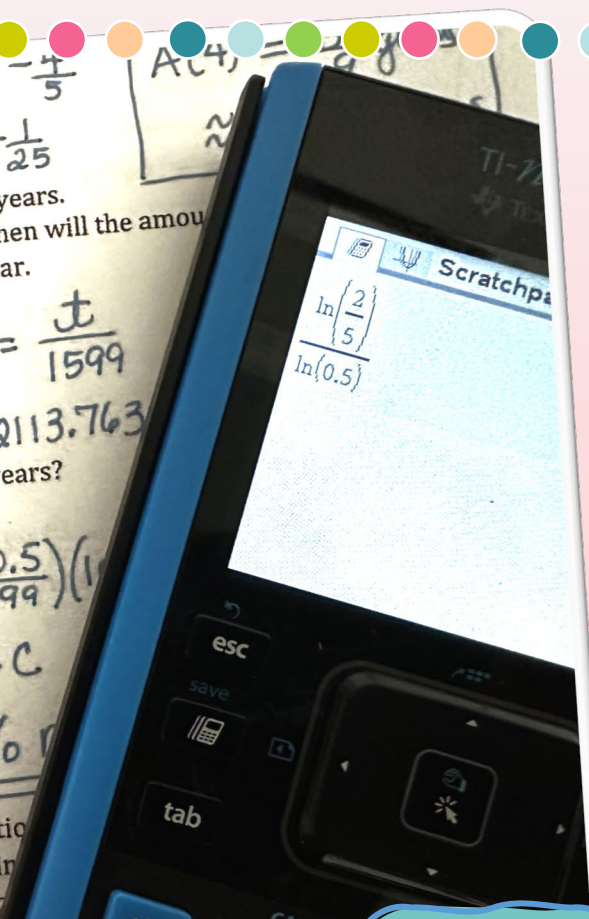
- B. What percent of a given amount remains after 100 years?

$$y = Ce^{kt}$$
$$\frac{1}{2} = 1e^{1599k}$$
$$k = \frac{1}{1599} \ln \left(\frac{1}{2} \right)$$

when $t=0$

$$y = Ce^{\left(\frac{\ln 0.5}{1599} \right)(100)}$$
$$y = 0.9576C$$
$$\therefore 95.76\%$$

6. Water is leaking out of a large barrel at a rate proportional to the water level at that time. If the water level starts at 36 inches, how long will it take for the water to drain out of the barrel?



Homework

7

Completely Editable

AP CALCULUS BC CURRICULUM

Scope and Sequence

Unit 7: Differential Equations

Lesson	Skills & Learning Objectives
[7.1] Introduction to Differential Equations	☐ Interpret verbal statements to write differential equations and verify solutions.
[7.2] Slope Fields	☐ Use slope fields to approximate solutions of differential equations. ☐ Identify slope fields for differential equations ☐ Use initial conditions to find particular solutions of differential equations.
[7.3] Euler's Method	☐ Use a differential equation and its slope field to calculate points on the graph iteratively using Euler's Method.
[7.4] Separation of Variables	☐ Use separation of variables to solve simple differential equations.
[7.5] Growth and Decay	☐ Use exponential functions to model growth and decay in applied problems.
[7.6] Logistic Growth	☐ Use logistic differential equations to model and solve applied problems including Newton's Cooling Law.

Problems 1 – 8, Write a differential equation that describes each relationship.

- | | |
|---|--|
| 1. The rate of growth of a certain bacteria B , in a petri dish is proportional to the number of bacteria in the dish. | 2. An object moves along a straight line. Its speed S , is inversely proportional to the square of the distance S , the object traveled. |
| 3. Every month the balance, B , of Rachel's car loan increases by 4.5% and decreases by \$375.00. | 4. The fraction of the student body P , who has heard a rumor increases at a rate proportional to the fraction of the population who has not yet heard the rumor. |
| 5. The rate of growth of a population of grasshoppers is inversely proportional to the square root of the population. | 6. The rate of decay of a certain substance R , is inversely proportional to the amount of substance remaining. |
| 7. Your sweet potato casserole C , is removed from a hot oven. The room temperature of the kitchen is 75°F . The rate at which the casserole cools is proportional to the difference between the room temperature and the temperature of the casserole. | 8. Your swimming pool bacteria level is elevated and needs to be diluted by pumping fresh water into the pool and pumping out the bacteria ridden water. The volume of water in the pool at any time t is $2t + 15$. The amount of bacteria b , in the pool decreases at a rate proportional b and inversely proportional to the volume of water in the pool. |

7.3

Euler's Method
Homework

Name _____

Date _____ Period _____

Problems 1 - 4, Use Euler's Method to approximate the given value of dy/dx , with the given step size Δx .

1. Find $y(0.5)$

$$\frac{dy}{dx} = 2y; y(0) = 3; \Delta x = 0.1$$

x	$\Delta y = y'(previous)$	New y
0	-----	3

2. Find $y(3.25)$

$$\frac{dy}{dx} = x^2 - y; y(3) = 1; \Delta x = 0.05$$

x	$\Delta y = y'(previous)$	New y
3	-----	1

3. Find $y(2)$

$$\frac{dy}{dx} = x \sin y; y(1) = 3; \Delta x = 0.2$$

x	$\Delta y = y'(previous)$	New y
1	-----	3

4. Find $y(1)$

$$\frac{dy}{dx} = \cos x + \sin y; y(0) = 4; \Delta x = 0.25$$

x	$\Delta y = y'(previous)$	New y
0	-----	4

7.4**Separation of Variables
Homework**

Name _____

Date _____ Period _____

Problems 1 - 6, Find the solutions to the differential equations, subject to the given initial conditions.

1. $6x + 5y \frac{dy}{dx} = 0$, $y(3) = 2$. Write your answer in standard form: $Ax^2 + By^2 = C$.

2. $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$, $y(4) = 1$

3. $\frac{dy}{dx} = (y + 8)$, $y(0) = 4$

4. $e^x - y \frac{dy}{dx} = 0$, $y(0) = 3$

7.6

Logistic Growth Homework

Name _____

Date _____ Period _____

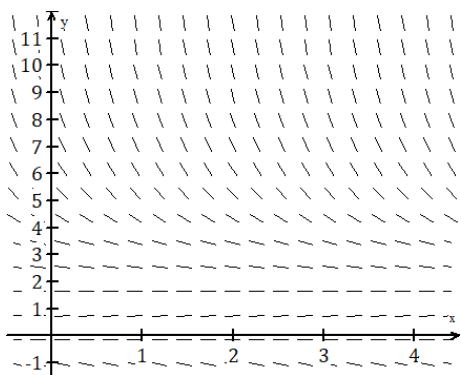
Problems 1 – 3, Match each of these differential equations with one of the slope fields shown below.

1. $\frac{dy}{dx} = 0.45y$

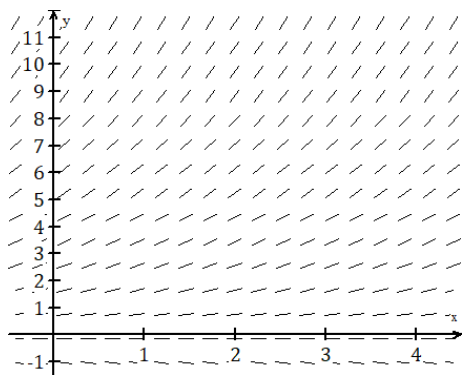
2. $\frac{dy}{dx} = 0.4y(5 - y)$

3. $\frac{dy}{dx} = 0.4y\left(1 - \frac{y}{2}\right)$

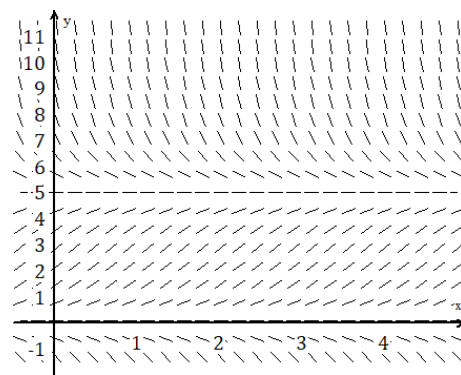
A.



B.



C.



4. Given the logistic equation $P(t) = \frac{2000}{1 + 18e^{-0.5t}}$:

A. Find the carrying capacity.

B. Find the value of k .

C. Find the initial population.

D. Find the time at which the population reaches 800.

E. Give the logistic differential equation.