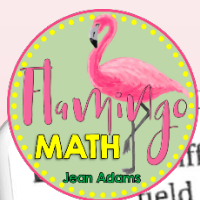


AP[®] Calculus-AB

Differential Equations



A differential equation, a point, and a slope field are given. Use the slope field to sketch a particular solution through the point. Then use integration to find the particular solution of the differential equation.

$$\frac{dy}{dx} = \frac{1}{x+2}; (0,1)$$

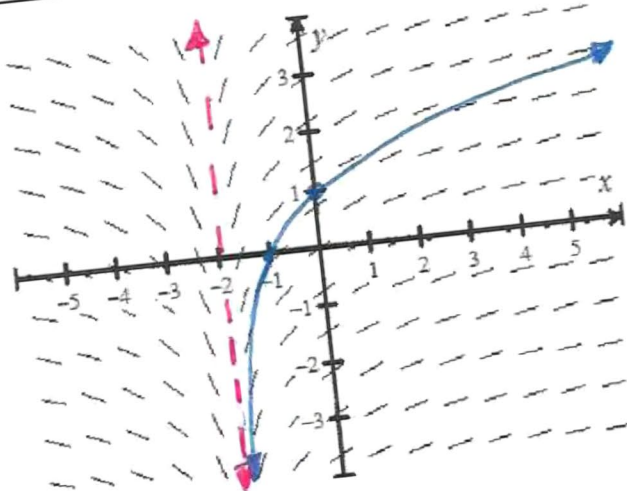
$$\int dy = \int \frac{1}{x+2} dx$$

$$y = \ln|x+2| + C$$

$$1 = \ln 2 + C$$

$$C = 1 - \ln 2$$

$$f(x) = \ln|x+2| + 1 - \ln 2$$



Guided Notes

7

Completely Editable

AP CALCULUS AB CURRICULUM

Scope and Sequence

Unit 7: Differential Equations

Lesson	Skills & Learning Objectives
[7.1] Introduction to Differential Equations	☐ Interpret verbal statements to write differential equations and verify solutions.
[7.2] Slope Fields	☐ Use slope fields to approximate solutions of differential equations. ☐ Identify slope fields for differential equations ☐ Use initial conditions to find particular solutions of differential equations.
[7.3] Separation of Variables	☐ Use separation of variables to solve simple differential equations.
[7.4] Growth and Decay	☐ Use exponential functions to model growth and decay in applied problems.

A Summer Internship

EX #5: You have been given a summer internship at a winery in the Tuscany region of Italy. Your task, each day, is to track the rate that red grape juice is being pumped into these large stainless-steel vats.



- A. The depth, y , is increased at a rate proportional to the square root of the depth at a given time. Write a rate equation that would represent this situation.
- B. When the grape juice reaches a height of 4 inches, the depth of the juice is increasing at a rate of 4 inches per hour. What can you find with this information?
- C. Write the (differential) rate equation using your new-found information.
- D. Complete the table below for the chosen depths using your differential equation. You notice that the rate the depth of the grape juice is changing does not depend on the amount of time.

Depth (inches)	0	4	9	16	25	36	49
Rate depth is changing (inches/hour)							

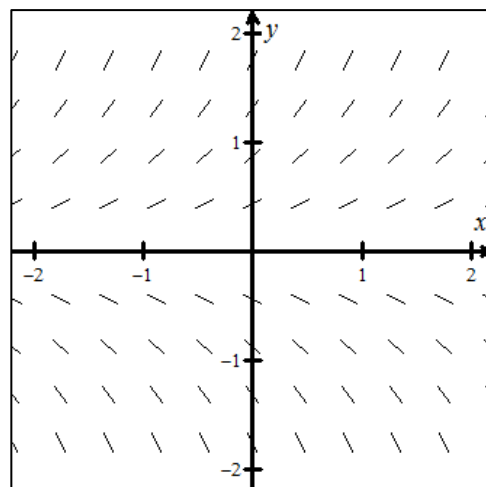
- E. The vintner needs to know the depth of the water at any given time. That's why he hired you. What fact do you need in order to find an answer for your boss?
- F. Suppose there was 4 inches of grape juice initially. In order to estimate the depth after 1 hour, we will assume the rate remains constant for the first hour. How did we arrive at an answer of 8 inches? Use this technique to complete the chart.

Time (hours)	0	1	2	3	4	5	6
Estimated depth of grape juice (inches)	4	8					

Lesson 7.2: Slope Fields

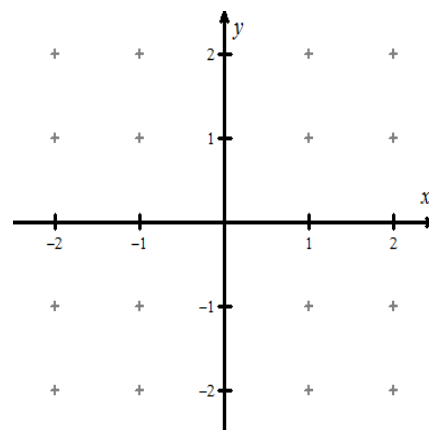
Topic 7.3: Sketching Slope Fields

Slope, vector, or direction fields are graphical representations of differential equations. You can use slope fields to visualize the parallel curves that solve the family of solutions to a first-order differential equation without algebra. For example, the equation $\frac{dy}{dx} = y$ has the property that the slope, at any point, is equal to the y coordinate at that point, see graph at right. We can draw many line segments at each marked point showing the slope of the curve there.



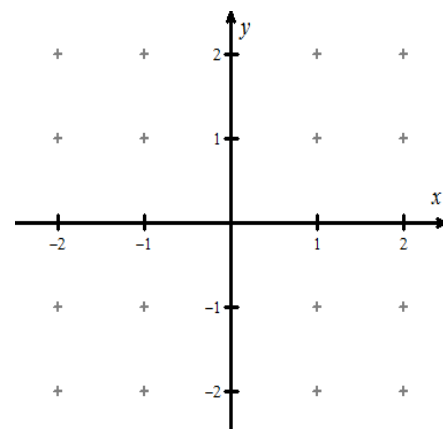
EX #1: Sketch the slope field for $\frac{dy}{dx} = 3x - 2$ using the table below.

x	dy/dx



EX #2: Sketch the slope field. Then use integration to solve for the general solution of the differential equation. $2x - y \frac{dy}{dx} = 0$

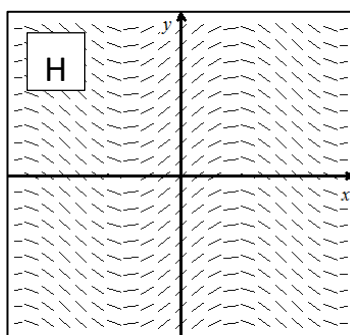
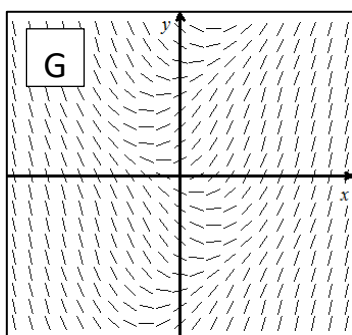
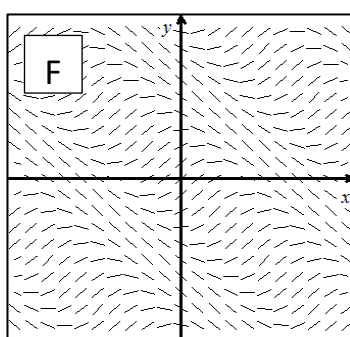
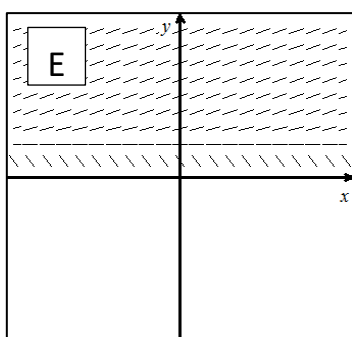
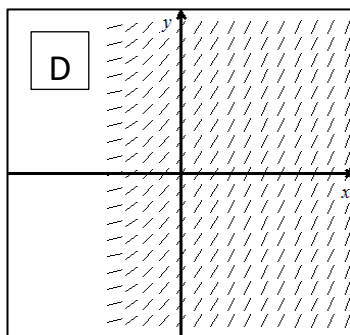
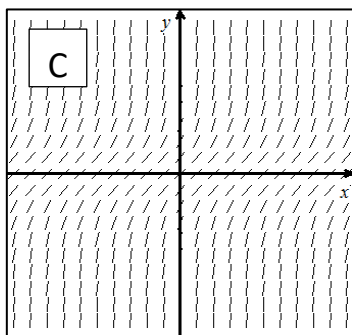
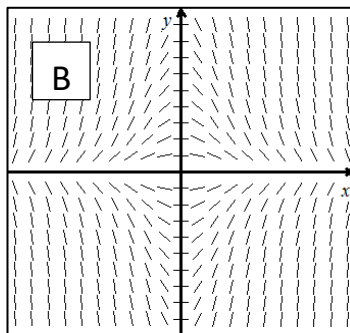
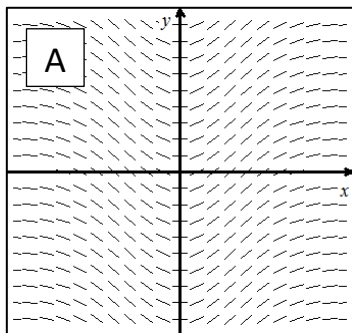
x	y	dy/dx



G. Explain what happens to the slope values as you move from left to right with a fixed y value.

H. Explain what happens to the slope values as you move up from the x -axis with a fixed x value.

EX #4: Matching



_____ 1.) $\frac{dy}{dx} = \sqrt{x+2}$

_____ 2.) $\frac{dy}{dx} = xe^{-0.2x^2}$

_____ 3.) $\frac{dy}{dx} = \sin(y) + x$

_____ 4.) $\frac{dy}{dx} = \cos x$

_____ 5.) $\frac{dy}{dx} = \frac{\ln y}{y}$

_____ 6.) $\frac{dy}{dx} = y^2 + 1$

_____ 7.) $\frac{dy}{dx} = -xy$

_____ 8.) $\frac{dy}{dx} = \cos(x+y)$

Lesson 7.3: Separation of Variables

Topic 7.6: Finding General Solutions Using Separation of Variables

Equations where all x - terms can be collected with dx and all y -terms with dy are said to be separable. The solution procedure is called *separation of variables*. A differential equation will be in the form of $\frac{dy}{dx} = f(x) \cdot g(y)$. To solve, you must put your solution in the form of $y = h(x)$.

EX#1: Find the general solution for each of the following:

A. $\frac{dy}{dx} = \frac{2x}{y}$

B. $\frac{dy}{dx} = y^2$

C. $\frac{dy}{dx} = xy$

D. $4yy' - 3e^x = 0$

EX #2: Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + 1}{3y^2}$

Lesson 7.4: Differential Equations - Growth and Decay

Topic 7.8: Exponential Models with Differential Equations

This lesson teaches how to use exponential functions to model growth and decay in applied problems. Consider the statement, “The rate of change of some quantity y is directly proportional to y .”

$$\frac{dy}{dt} = ky$$

The Law of Exponential Change

If y is a differentiable function of t such that $y > 0$ and “ **y changes at a rate proportional to the amount present**” (that is, if $dy/dt = ky$), and for some constant k , then

$$y = Ce^{kt}$$

Where C is the initial value of y , and k is the constant of proportionality.

- ☐ When $k > 0$ Exponential growth occurs, and
- ☐ When $k < 0$ Exponential decay occurs.

EX #1: Prove that $\frac{dy}{dt} = ky$ supports the model $y = Ce^{kt}$