

AP[®] Calculus

Exponential Models Differential Equations



Prove that $\frac{dy}{dt} = ky$ supports the model $y = Ce^{kt}$

$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k \, dt$$

$$\ln|y| = kt + C$$

$$e^{\ln|y|} = e^{kt+C}$$

$$|y| = e^{kt+C}$$

$$* |y| = e^{kt} \cdot e^C$$

$$\int \frac{u'}{u} = \ln|u| + C$$

Definition:

$$e^x = y \iff \ln|y| = x$$

exponentiation

property of exponents
 $b^{m+n} = b^m \cdot b^n$

$$\pm e^C = C \text{ (constant)}$$

* Note: absolute value is

Completely Editable

7.4

Lesson 7.4: Differential Equations - Growth and Decay

Topic 7.8: Exponential Models with Differential Equations

This lesson teaches how to use exponential functions to model growth and decay in applied problems. Consider the statement, "The rate of change of some quantity y is directly proportional to y ."

$$\frac{dy}{dt} = ky$$

The Law of Exponential Change

If y is a differentiable function of t such that $y > 0$ and " y changes at a rate proportional to the amount present" (that is, if $dy/dt = ky$), and for some constant k , then

$$y = Ce^{kt}$$

Where C is the initial value of y , and k is the constant of proportionality.

- ☐ When $k > 0$ Exponential growth occurs, and
- ☐ When $k < 0$ Exponential decay occurs.

EX #1: Prove that $\frac{dy}{dt} = ky$ supports the model $y = Ce^{kt}$

EX #2: The rate of change of y is proportional to y . When $t = 0$, $y = 3$, and when $t = 2$, $y = 6$. What is the value of y when $t = 3$?

EX #3: The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{3}}$ per unit of time, t . At $t = 0$ the number of bacteria present was 7,500. Find the number present at $t = 5$.

EX #4: A slow economy caused a company's annual revenues to drop from \$530,000 in 2008 to \$386,000 in 2010. If the revenue is following an exponential pattern of decline, what is the expected revenue for 2013?

EX #5: Suppose a population of insects increases according to the law of exponential growth. There were 130 insects after the third day of the experiment and 380 after the seventh day. Approximately how many insects were in the initial population?

Half-Life

If $A(t) = A_0e^{kt}$ with $k < 0$, then the half-life of A is

$$\text{Half-life} = \frac{\ln 2}{k}$$

EX #6: A zircon sample contains 4000 atoms of the radioactive element ^{235}U . Given that ^{235}U has a half-life of 700 million years, how long would it take to decay to 125 atoms?

EX #7: A cup of hot chocolate placed on a table cools at a rate if $\frac{dH}{dt} = -0.05(H - 75)^\circ\text{F}$ per minute, where H represents the temperature of the hot chocolate and t is time in minutes. If the coffee was at 160°F initially, what will its temperature be 10 minutes later?

NOTES

7.4

Growth and Decay
Homework

Name _____

Date _____ Period _____

1. Write and solve the differential equation that models the following statement, "The rate of change of W with respect to x is proportional to $x + 18$."

2. In 10 years, the mass of a 200-gram sample of an element is reduced to 100 grams; this is the half-life of the element. Find k for this element.

3. Newtonium (Nt) is a highly radioactive element with a half-life of 625 years. How long would it take 62 grams to decay to 22 grams? (Round your answer to the nearest year.)

4. Bacteria from a lab culture grows in such a way that the instantaneous rate of change of the bacteria population is directly proportional to the number of bacteria present.

- A) Write a differential equation that expresses this relationship and solve the equation for the number of bacteria as a function of time.
B) Initially there are 250 bacteria present. Three hours later, the culture has grown to 625. Find the bacteria population after one day.
C) When will the bacteria culture grow to 100,000?

5. When processing raw sugar "inversion" causes the sugar's molecular structure to change. During inversion the rate of change of the amount of raw sugar, R , is proportional to the amount of raw sugar remaining. If 1000 kg of raw sugar reduces to 750 kg during the first 10 hours, how much raw sugar will remain at the end of a 24-hour period?

6. Let $P(t)$ represent the number of deer in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $700 - P(t)$, where the constant of proportionality is k .

- A) If $P(0) = 400$, find $P(t)$ in terms of t and k . B) If $P(2) = 600$, find k .

7. A bacteria colony is grown in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 5000 bacteria. At the end of the 5 hours there are 20,000 bacteria. How many bacteria were present initially?

8. A cup of coffee is placed in a 70°F room. The temperature of the coffee is 140°F, initially and 102°F one hour later. Find the temperature of the coffee after 3 hours if the cooling rate is

9. Warfarin™ sodium is a drug used for stroke victims as an anticoagulant. The half-life of the drug in the body is 38 hours. After the drug is stopped, the quantity remaining in a patient's body decreases at a rate proportional to the quantity remaining. How many days does it take for the drug level in the body to be reduced to 20% of the original level?

10. The half-life of morphine in the bloodstream of humans is 3 hours. If initially there is 0.4 mg of morphine in the bloodstream, find an equation for the amount in the bloodstream at any time. When does the amount drop below 0.01 mg?

11. Suppose a bacterial culture initially has 120 cells. After 2 hours, the population has increased to 500. Find an equation for the population at any time. What will the population be after 8 hours?

Practice Sets

7.4 Growth and Decay**Form A**

Name _____

Date _____ Period _____

1. Write and solve the differential equation that models the following statement: The rate if change of y is directly proportional to $5y$.

The number of bacteria in a culture is increasing according to the law of exponential growth. There are 130 bacteria in the culture after two hours and 360 bacteria after four hours.

2. Find k .

3. Find the initial population, to the nearest whole number.

7.4 Growth and Decay**Form B**

Name _____

Date _____ Period _____

1. Write and solve the differential equation that models the following statement: The rate if change of y is directly proportional to $\sqrt{y}$.

The number of bacteria in a culture is increasing according to the law of exponential growth. There are 110 bacteria in the culture after two hours and 275 bacteria after four hours.

2. Find k .

3. Find the initial population, to the nearest whole number.

7.4 Growth and Decay**Form C**

Name _____

Date _____ Period _____

1. Write and solve the differential equation that models the following statement: The rate if change of y is directly proportional to $2\sqrt{y}$.

The number of bacteria in a culture is increasing according to the law of exponential growth. There are 125 bacteria in the culture after two hours and 350 bacteria after four hours.

2. Find k .

3. Find the initial population, to the nearest whole number.

7.4 Growth and Decay**Form D**

Name _____

Date _____ Period _____

1. Write and solve the differential equation that models the following statement: The rate if change of y is directly proportional to $(y + 3)$.

The number of bacteria in a culture is increasing according to the law of exponential growth. There are 120 bacteria in the culture after two hours and 380 bacteria after four hours.

2. Find k .

3. Find the initial population, to the nearest whole number.

Daily Lesson Quiz