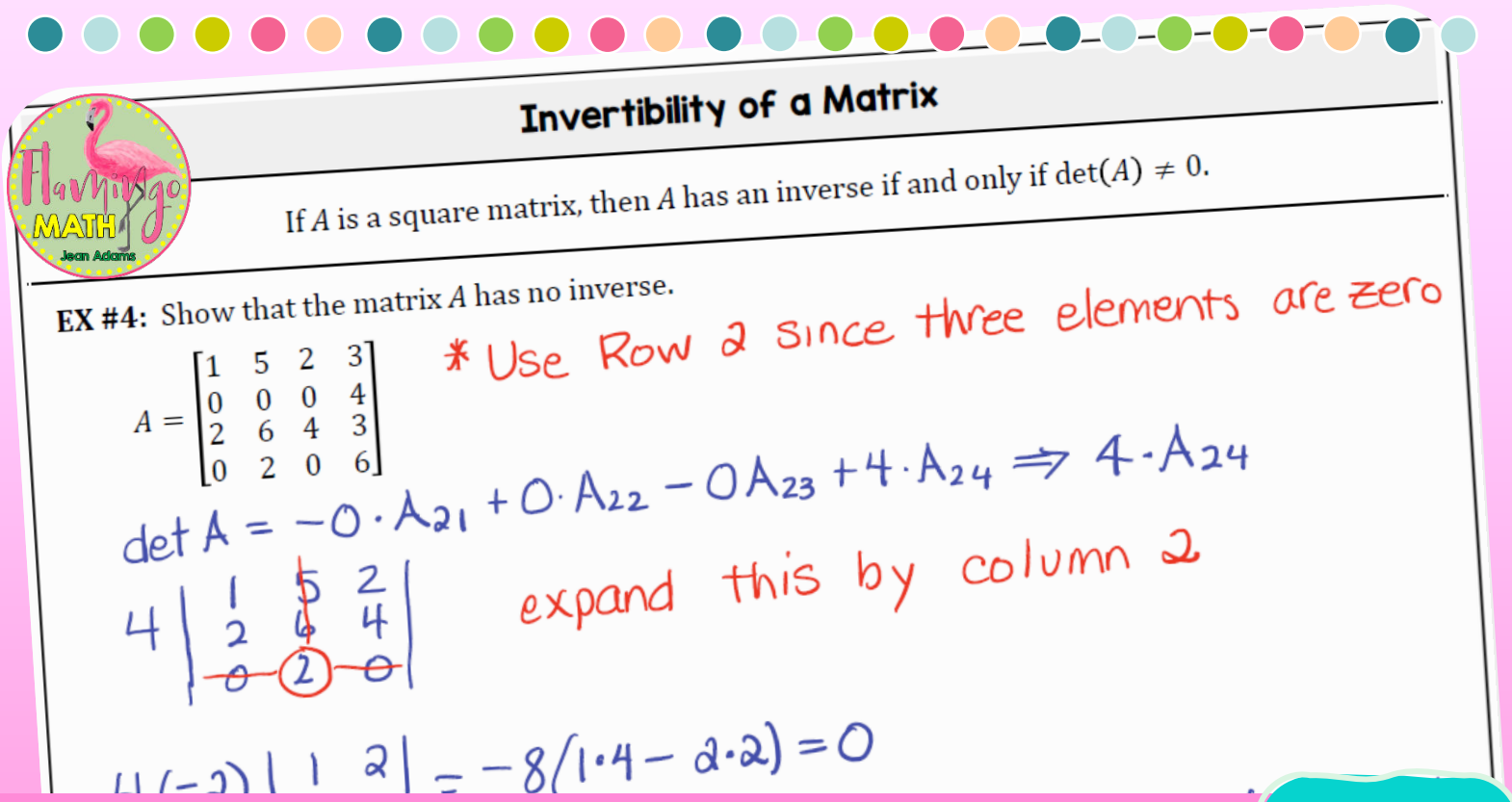
Precalculus Honors

The Determinant of a Square Matrix



Completely Editable 7.6

Lesson 7.6: The Determinant of a Square Matrix

The notation for the determinant of a square matrix A is given by the symbols $\det(A)$ or $|A|$. We can define $\det(A)$ for a 1×1 matrix as $A = [a]$, then $\det(A) = a$. The following definition provides our foundation for a 2×2 matrix.

Determinant of a 2 X 2 Matrix

The **determinant** of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

EX #1: Evaluate $|A|$ for $A = \begin{bmatrix} 8 & -2 \\ 3 & 3 \end{bmatrix}$

Minors and Cofactors

To find the determinant for any $n \times n$ matrix of order 3×3 or higher, let's consider the concepts of **minors** and **cofactors**.

If A is a square matrix, the **minor** $\boldsymbol{M_{ij}}$ of the entry element a_{ij} is the determinant of the matrix obtained by deleting the *ith row and the jth column of A.*

The **cofactor** $\boldsymbol{C_{ij} = (-1)^{i+j} M_{ij}}$

EX #2: Find all the minors and cofactors of the matrix.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 5 \\ -2 & 1 & 4 \end{bmatrix}$$

Determinant of a Square Matrix

If A is an $n \times n$ matrix, then the **determinant of A** is obtained by multiplying each element of the first row by its cofactor and then adding the results. Symbolically,

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + \cdots + a_{1n}A_{1n}$$

EX #2: Evaluate the determinant of the matrix.

$$A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & 2 & 3 \\ 6 & 4 & -1 \end{bmatrix}$$

Alternate Method for 3 X 3 Determinants

EX #3: Copy the first and second columns of the matrix to form a fourth and fifth column. The determinant of A is obtained by adding products of the three "downward diagonals" and then subtracting the products of the three "upward diagonals."

$$A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & 2 & 3 \\ 6 & 4 & -1 \end{bmatrix}$$

Invertibility of a Matrix

If A is a square matrix, then A has an inverse if and only if $\det(A) \neq 0$.

EX #4: Show that the matrix A has no inverse.

$$A = \begin{bmatrix} 1 & 5 & 2 & 3 \\ 0 & 0 & 0 & 4 \\ 2 & 6 & 4 & 3 \\ 0 & 2 & 0 & 6 \end{bmatrix}$$

Solving Determinant Equations

EX #5: Solve for x.

A. $\begin{vmatrix} x & 1 \\ 3 & x \end{vmatrix} = 6$

B. $\begin{vmatrix} (x+3) & 5 \\ 1 & (x+4) \end{vmatrix} = -3$

NOTES

7.6 **The Determinant of a Square Matrix Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, find the determinant of the matrix.

1. $[-5]$

2. $\begin{bmatrix} 3 & \frac{1}{2} \\ 8 & 7 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 2 & 4 \\ 2 & -1 & 0 \\ 1 & 2 & 4 \end{bmatrix}$

Problems 5 – 6, find all (a) minors and (b) cofactors of the matrix.

5. $\begin{bmatrix} -1 & 2 & -5 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{bmatrix}$

6. $\begin{bmatrix} 4 & 0 & 3 \\ 5 & -2 & 1 \\ 0 & -6 & -1 \end{bmatrix}$

Problems 7 – 8, find the determinant of the matrix. Expand by cofactors on the row or column that appears to make the computations easiest.

7. $\begin{bmatrix} 7 & 9 & 0 \\ 1 & 2 & 3 \\ -2 & 0 & 0 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \\ -2 & 0 & 4 & 0 \\ 3 & 6 & -5 & -3 \end{bmatrix}$

Problems 9 – 10, find (a) $|A|$, (b) $|B|$, (c) AB, and (d) $|AB|$

9. $A = \begin{bmatrix} 3 & 0 \\ 4 & -2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ -1 & 0 & 0 \end{bmatrix}$

Problems 11 – 14, solve for x.

11. $\begin{vmatrix} x & 5 \\ -1 & x \end{vmatrix} = 30$

12. $\begin{vmatrix} x & 6 \\ 3 & 4x \end{vmatrix} = 7$

13. $\begin{vmatrix} x & 5 & 9 \\ 0 & x-3 & 18 \\ 0 & 0 & x-2 \end{vmatrix} = 0$

14. $\begin{vmatrix} 1 & x & -3 \\ 1 & 2 & 2 \\ 0 & 3 & 2 \end{vmatrix} = 0$

Problems 15-16, The Area of a Triangle with Vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

Area $= \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ where the area is a positive area. Calculate the area for the following.

Practice Sets

Lesson Warm-Ups

3. Evaluate: $\begin{vmatrix} 2 & -4 & 2 \\ -1 & 0 & 5 \\ 3 & 0 & 4 \end{vmatrix}$

Name _____ Date _____ Period _____

Lesson 7.6 Determinant of a Square Matrix **Warm-Up # 2**

1. Evaluate the determinant: $\begin{vmatrix} -4 & 2 \\ 6 & -7 \end{vmatrix}$

2. Use matrix equations to solve $\begin{cases} x + y = 6 \\ 3x - 4y = 4 \end{cases}$

3. Evaluate: $\begin{vmatrix} -1 & 3 & -5 \\ -3 & 4 & 0 \\ 3 & 1 & 0 \end{vmatrix}$

7.6 Determinant of a Square Matrix **Form A**

Name _____

Date _____ Period _____

1. Find the minors and cofactors of the matrix.

$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

2. Find the determinant of the matrix by using expansion by cofactors . Expand about Row 1.

$\begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 5 \\ 1 & -4 & 3 \end{bmatrix}$

3. Solve for x. $\begin{vmatrix} 1 & x & -4 \\ 1 & 2 & 3 \\ 0 & 3 & -2 \end{vmatrix} = 3$

Grade:

7.6 Determinant of a Square Matrix **Form B**

Name _____

Date _____ Period _____

1. Find the minors and cofactors of the matrix.

$\begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$

2. Find the determinant of the matrix by using expansion by cofactors . Expand about Row 1.

$\begin{bmatrix} 2 & 3 & 0 \\ -1 & 4 & 2 \\ 3 & -2 & 1 \end{bmatrix}$

3. Solve for x. $\begin{vmatrix} 1 & x & -2 \\ 1 & 3 & 3 \\ 0 & 1 & -2 \end{vmatrix} = 3$

Grade:

Daily Lesson Quiz