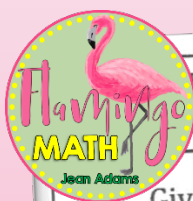


AP[®] Calculus

Derivatives of Inverse Trigonometric Functions



Use the Pythagorean Theorem, properties of right triangle trigonometry and implicit differentiation to derive the inverse trigonometric function formulas.

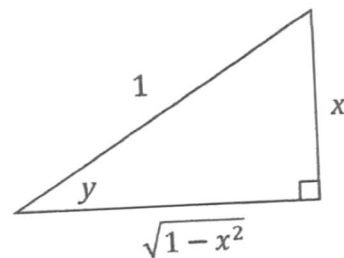
A. Given $y = \sin^{-1}x$, find $\frac{d}{dx}(\sin^{-1}x)$

$$\sin y = x$$
$$\frac{d}{dx}[\sin y] = \frac{d}{dx}[x]$$

$$\cos y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$



$$\cos y = \frac{A}{H}$$

B. Given $y = \cos^{-1}x$, find $\frac{d}{dx}(\cos^{-1}x)$

$$\cos y = x$$

$$\frac{dy}{dx} = -1$$



Completely Editable

3.5

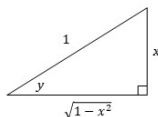
Lesson 5: Derivatives of Inverse Trigonometric Functions

Topic 3.4: Differentiating Inverse Trigonometric Functions

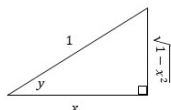
In this final lesson for the unit we can take the derivative of an inverse trigonometric function using the ratios of a right triangle and implicit differentiation. We will start with sine inverse. Let's derive!

EX #1: Use the Pythagorean Theorem, properties of right triangle trigonometry and implicit differentiation to derive the inverse trigonometric function formulas.

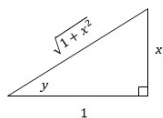
A. Given $y = \sin^{-1}(x)$, find $\frac{d}{dx}(\sin^{-1} x)$



B. Given $y = \cos^{-1}(x)$, find $\frac{d}{dx}(\cos^{-1} x)$



C. Given $y = \tan^{-1}(x)$, find $\frac{d}{dx}(\tan^{-1} x)$



Derivatives of Inverse Trigonometric Functions

NOTEWORTHY: On previous exams, it is most important to know the inverse formulas for sine, cosine, and tangent. You can memorize them or learn to derive them!

$$\frac{d}{dx} [\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} [\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} [\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} [\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

EX #2: Differentiate.

A. $y = \sin^{-1}(5x)$

B. $y = \tan^{-1}(\sqrt{2x+1})$

C. $y = \arccos(x^2)$

D. $y = \sin^{-1}(x^2 - 1)$

E. $y = \tan^{-1}(\cos x)$

F. $y = \sin^{-1}(e^x)$

EX #3: Find an equation of the tangent line to the graph of the function $y = \arctan\left(\frac{x}{2}\right)$ at the point $\left(2, \frac{\pi}{4}\right)$.

EX #4: Find $\frac{dy}{dx}$ when $\tan(4x - y) = 4x$. [Hint: all trig functions will disappear in your answer.]

EX #5: Let g be the inverse of the function $f(x) = 2 \cos x$, $0 \leq x \leq \pi$. Find the value of $g'(1)$.

NOTES

3.5

Derivatives of Inverse
Trigonometric Functions
Homework

Name _____

Date _____ Period _____

Problems 1 – 12, differentiate.

1. $y = \tan^{-1}(e^x)$

2. $y = \cos^{-1}\sqrt{x}$

3. $h(x) = \arccos\left(\frac{x}{2}\right)$

4. $y = \cos^{-1}(x^3)$

5. $y = \arctan\left(\frac{1}{x}\right)$

6.

7. $h(x) = (2x) \sin^{-1}(x)$

8. $y = \tan^{-1}(\cos x)$

9. $g(x) = (x^2) \tan^{-1}(3x)$

10. $f(x) = e^{\cos^{-1}(x)}$

11. $k(x) = 3 \arctan(e^{2x})$

12. $h(x) = \ln|\sin^{-1}(x)|$

13. Using a calculator, find an equation for the tangent line to the graph of $y = \sin^{-1}\left(\frac{x}{4}\right)$ at $x = 2$. Round your answers to three decimal places.



Practice Sets

3.5 Derivatives of Inverse Trigonometric Function **Form A**

Name _____

Date _____ Period _____

Find the derivative.

1. $f(x) = \cos^{-1}(3x^2)$

2. $f(x) = (3x) \tan^{-1}(x)$

3. $y = \sin^{-1}(4x)$

3.5 Derivatives of Inverse Trigonometric Function **Form B**

Name _____

Date _____ Period _____

Find the derivative.

1. $f(x) = \tan^{-1}(5x^2)$

2. $f(x) = (2x) \sin^{-1}(x)$

3. $y = \cos^{-1}(x)$

3.5 Derivatives of Inverse Trigonometric Function **Form C**

Name _____

Date _____ Period _____

Find the derivative.

1. $f(x) = \sin^{-1}(x^3)$

2. $f(x) = (-3x^2) \cos^{-1}(x)$

3. $y = \tan^{-1}(x - 1)$

3.5 Derivatives of Inverse Trigonometric Function **Form D**

Name _____

Date _____ Period _____

Find the derivative.

1. $f(x) = \cos^{-1}(2x)$

2. $f(x) = (5x) \tan^{-1}(x)$

3. $y = \sin^{-1}(3x^2)$

Daily Lesson Quiz