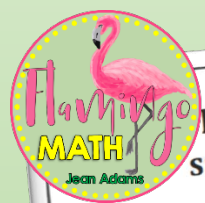


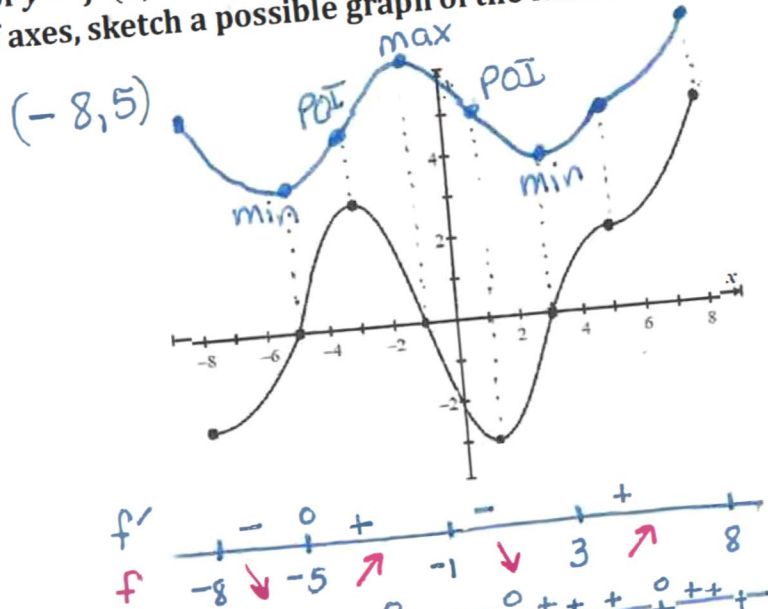
AP[®] Calculus

Curve Sketching

Connecting a Function, its First Derivative
and its Second Derivative



The graph of $y = f'(x)$ is shown below, on the interval $[-8, 8]$. If $f(-8) = 5$. On the same set of axes, sketch a possible graph of the function $y = f(x)$.



Completely Editable

5.5

Lesson 5: Curve Sketching

Topic 5.8: Sketching Graphs of Functions and Their Derivatives

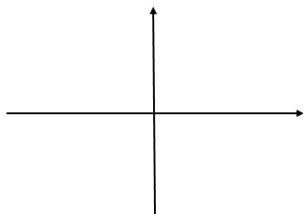
Curve Sketching is a procedure for analyzing a function and its behavior without the aid of a graphing utility. We can use the relationships of f , f' , and f'' over specific intervals to quickly sketch the graph of the function $y = f(x)$. We already know how to find domain, range, intercepts and asymptotes. In the last two lessons we learned how the first and second derivatives can be used to determine four basic behaviors and curvature of a function. If we know the sign of f' and f'' over some given interval, we know exactly what the graph of $f(x)$ looks like on that interval.

EX #1: Complete the table below by sketching the general shape of the curve when each condition is met.

	$f'(x) > 0$	$f'(x) < 0$
$f''(x) > 0$		
$f''(x) < 0$		

EX #2: Sketch a possible graph of the function $y = g(x)$ that satisfies the following conditions:

- $g'(x) > 0$ on $(-\infty, -1) \cup (3, \infty)$ and $g'(x) < 0$ on $(-1, 3)$
- $g''(x) > 0$ on $(-\infty, -3) \cup (-1, 3)$ and $g''(x) < 0$ on $(-3, -1) \cup (3, \infty)$
- $\lim_{x \rightarrow -\infty} g(x) = -4$ and $\lim_{x \rightarrow \infty} g(x) = 4$



Sketching Function Graphs from the Derivative Graph

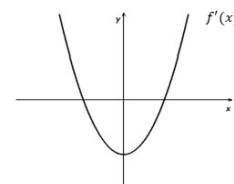
The Function Behavior Chart from our previous lesson can be a visual aid to using proper vocabulary as well as helping us make graphical representations from verbal descriptions.

Function Behavior				
$f(x)$	U	∩	↗	↘
$f'(x)$	↗	↘	+	-
$f''(x)$	+	-		

Symbol	Vocabulary
+	Positive
-	Negative
↗	Increasing
↘	Decreasing
U	Concave Up
∩	Concave Down

Discovering Key Concepts of Analyzing f' Graphs

EX #3: Can you sketch a possible graph of f given f' ?

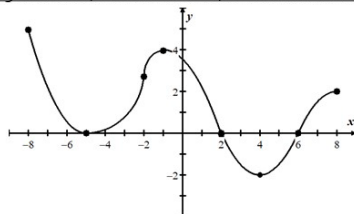


- Zeros of $f'(x)$ may be _____ of $f(x)$.
- Extrema of $f'(x)$ are possible _____ for $f(x)$.
- When $f'(x)$ is above the x -axis then $f(x)$ is _____.
- When $f'(x)$ is below the x -axis then $f(x)$ is _____.
- If $f'(x)$ is increasing then $f''(x) > 0$, so $f(x)$ is _____.
- If $f'(x)$ is decreasing then $f''(x) < 0$, so $f(x)$ is _____.

Reading Function Graphs and Behaviors

Topic 5.9: Connecting a Function, Its First Derivative, and Its Second Derivative

EX #4:



A. Suppose the graph shown above is the graph of a function $y = f(x)$ on the interval $[-8, 8]$:

- On what open interval(s) is $f(x)$ both increasing and concave up?
- On what open interval(s) is $f(x)$ both decreasing and concave down?
- At what x -value(s) does $f(x)$ have inflection points?

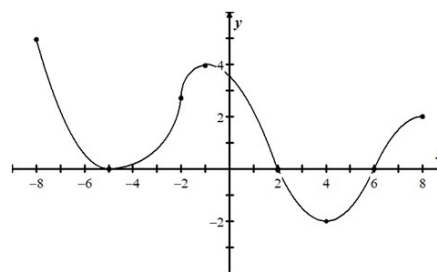
B. If the graph above is the graph of the derivative $y = f'(x)$ on $[-8, 8]$:

- On what open interval(s) is $f(x)$ decreasing? Justify.
- At what x -value(s) does $f(x)$ have a local maximum? Justify.
- On what open interval(s) is $f(x)$ concave down? Justify.

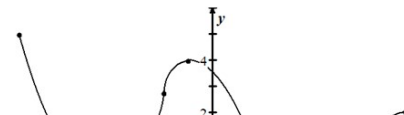
Sketching Graphs of Functions and Their Derivatives

EX #4 (continued)

D. Suppose the graph below is the graph of a function $y = f(x)$ on the interval $[-8, 8]$. On the same set of axes, sketch and label a possible graph of $f'(x)$ in red.



E. If the graph below is the graph of the derivative $y = f'(x)$ on $[-8, 8]$. Sketch and label a possible graph of $y = f(x)$ given that $f(-8) = -2$.



NOTES

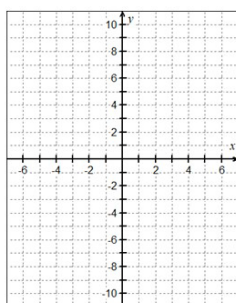
- At what x -value(s) does $f(x)$ have an inflection point? Justify.

Guidelines for Analyzing the Graph of a Function:

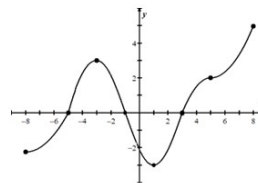
1. Determine the domain, intercepts, asymptotes, symmetry of the graph.
2. Find critical points and intervals where the function is increasing and decreasing.
3. Determine local maximum and minimum points.
4. Determine concavity and find points of inflection.
5. Sketch the curve.

EX #5: Analyze and sketch the graph of $g(x) = (x+2)(x-1)^2$

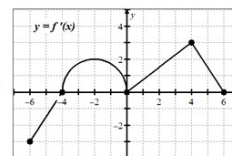
- A. Find the x and y-intercepts.
- B. Find the first and second derivatives.
- C. Complete a sign chart for $g'(x)$ to find intervals where $g(x)$ is increasing or decreasing.
- D. Determine any relative extrema.
- E. Complete a sign chart for $g''(x)$ to find intervals where $g(x)$ is concave up or concave down.
- F. Identify any points of inflection.
- G. Sketch the graph of $g(x)$.



EX #6: The graph of $y = f'(x)$ is shown below, on the interval $[-8, 8]$. If $f(-8) = 5$. On the same set of axes, sketch a possible graph of the function $y = f(x)$.



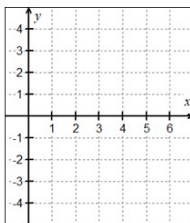
EX #7: The function $f(x)$ is defined and differentiable on the closed interval $[-6, 6]$. The graph of $y = f'(x)$, the derivative of f , consists of three line-segments and a semicircle, shown in the figure at right. For each question below find the values for $y = f(x)$ on the interval $-6 < x < 6$ and justify your reason.



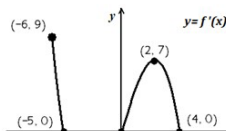
- A. Find the x-coordinate of each critical point for $y = f(x)$.
- B. Find the x-coordinate of each relative extrema of $y = f(x)$ and label as a maximum or minimum.
- C. Find the open interval(s) over which the function $y = f(x)$ is increasing or decreasing.
- D. Find the x-coordinate of each point of inflection for $y = f(x)$.
- E. Find the interval(s) over which the function $y = f(x)$ is concave up or concave down.

EX #8: Let $h(x)$ be a function that is continuous on the interval $[0, 6]$. The function h is twice differentiable except at $x = 3$, where the derivative of h does not exist. The function and its derivatives have the properties indicated in the table below. Use the table to sketch a graph of $y = h(x)$.

x	0	$0 < x < 2$	2	$2 < x < 3$	3	$3 < x < 5$	5	$5 < x < 6$
$h(x)$	3	Positive	0	Negative	-3	Negative	0	Positive
$h'(x)$	-3	Negative	-3	Negative	DNE	Positive	1	Positive
$h''(x)$	-2	Negative	0	Positive	DNE	Negative	2	Positive



EX #9: The function f is defined and differentiable on the closed interval $[-6, 5]$. The graph of $y = f'(x)$ the derivative of f , is shown in the figure below and has horizontal tangent lines at $x = -3$ and $x = 2$.



- A. Find the x-coordinate(s) of each relative extrema on $-6 < x < 5$. Identify as a max or min. Justify.

NOTES

C. Let $g(x) = x^3 - f(x)$. Find $g'(-3)$.

5.5

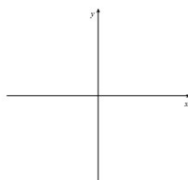
Curve Sketching and
Connecting f , f' , f''
Homework

Name _____

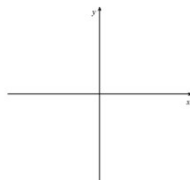
Date _____ Period _____

Problems 1 – 4, Analyze and sketch a graph of the function. Label any intercepts, relative extrema, asymptotes, and points of inflection.

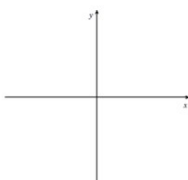
1. $y = x - 4\sqrt{x}$



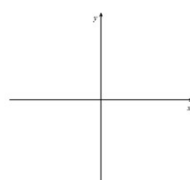
2. $y = \frac{x-3}{x}$



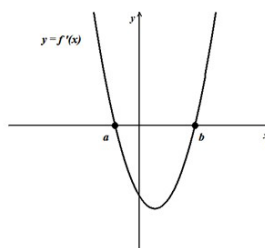
3. $y = 2 \cos(x) - x ; [0, 2\pi]$



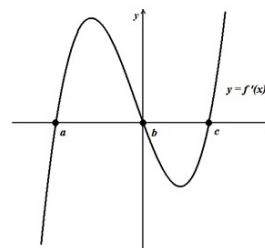
4. $y = x - 2 \ln(x)$

Problems 5 – 6, Use the graph of the f' to sketch the graph of f and the graph of f'' .

5.

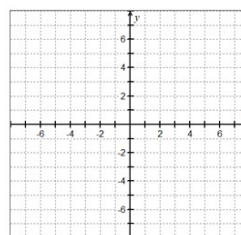


6.

Problems 7 – 8, Sketch the graph of the function, f , given the following information.

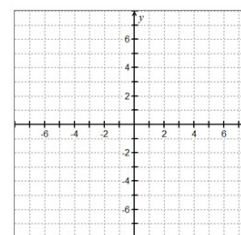
7.

- A. continuous for all real numbers
- B. $f'(x) < 0$ on $(-\infty, -3)$ and $(0, 3)$
- C. $f'(x) > 0$ on $(-3, 0)$ and $(3, \infty)$
- D. $f''(x) > 0$ on $(-\infty, 0)$ and $(0, 6)$
- E. $f''(x) < 0$ on $(6, \infty)$
- F. $f'(0)$ does not exist



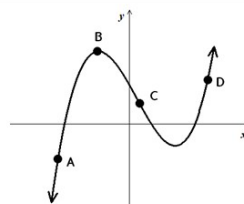
8.

- A. continuous for all real numbers
- B. $f'(2)$ does not exist
- C. $\lim_{x \rightarrow \infty} f(x) = 2$
- D. $f'(x) > 0$ on $(-\infty, -1)$ and $(2, 5)$
- E. $f'(x) < 0$ on $(-1, 2)$ and $(5, \infty)$
- F. $f''(x) > 0$ on $(-\infty, -3)$
- G. $f''(x) < 0$ on $(-3, 2)$ and $(2, \infty)$

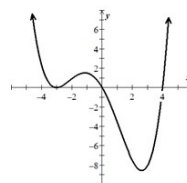


9. The graph of a function f is shown on the right. Fill in the chart with +, -, or 0.

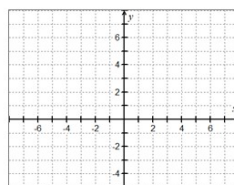
Point	$f(x)$	$f'(x)$	$f''(x)$
A			
B			
C			
D			



10. The graph of $f''(x)$, the second derivative of f is shown at right. Use the graph to determine the x -coordinate of the inflection points. Justify your answer:



11. Use the information from g , g' , and g'' to sketch a graph of the function $g(x) = \frac{x}{x^2 - 9}$.



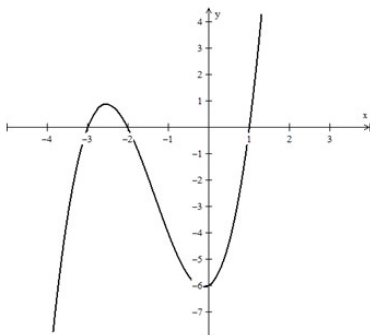
Practice Sets

5.5 Curve Sketching

Form A

Name _____
Date _____ Period _____

1. The graph of the **derivative of f** is shown in the figure below. Sketch the graph of $f(x)$.



2. Sketch the graph of a function having the indicated characteristics.

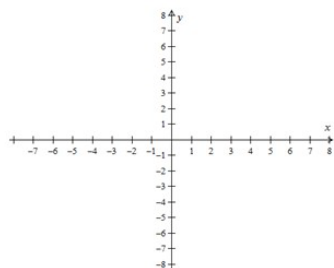
$$f'(x) < 0 \text{ if } x < 1$$

$$f'(x) > 0 \text{ if } x > 1$$

$$f''(x) < 0 \text{ for } (-\infty, \infty)$$

$$f'(1) \text{ Does Not Exist}$$

$$f(1) = -3$$

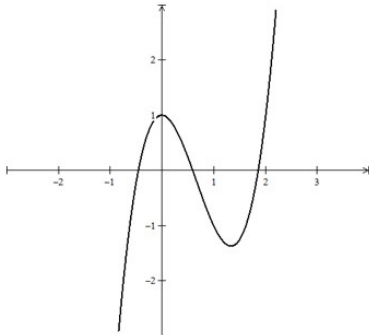


5.5 Curve Sketching

Form B

Name _____
Date _____ Period _____

1. The graph of the **derivative of f** is shown in the figure below. Sketch the graph of $f(x)$.



2. Sketch the graph of a function having the indicated characteristics.

$$f'(x) < 0 \text{ for } (-\infty, \infty)$$

$$f''(x) < 0 \text{ if } x < -2$$

$$f''(x) > 0 \text{ if } x > -2$$

$$\lim_{x \rightarrow -2} f(x)$$

$$f(-2) = -3$$

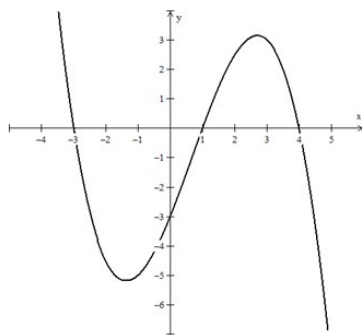


5.5 Curve Sketching

Form C

Name _____
Date _____ Period _____

1. The graph of the **derivative of f** is shown in the figure below. Sketch the graph of $f(x)$.



2. Sketch the graph of a function having the indicated characteristics.

$$f'(x) > 0 \text{ if } x < 2$$

$$f'(x) < 0 \text{ if } x > 2$$

$$f''(x) > 0 \text{ for } (-\infty, \infty)$$

$$f'(2) \text{ Does Not Exist}$$

$$f(2) = 5$$

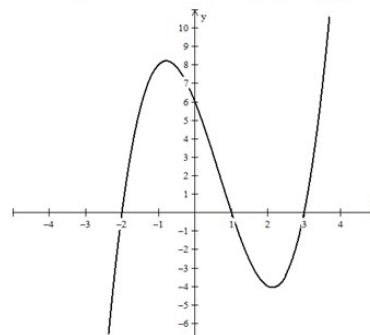


5.5 Curve Sketching

Form D

Name _____
Date _____ Period _____

1. The graph of the **derivative of f** is shown in the figure below. Sketch the graph of $f(x)$.



2. Sketch the graph of a function having the indicated characteristics.

$$f'(x) > 0 \text{ for } (-\infty, \infty)$$

$$f''(x) > 0 \text{ if } x < -1$$

$$f''(x) < 0 \text{ if } x > -1$$

$$\lim_{x \rightarrow -1} f(x) \text{ Does Not Exist}$$



Daily Lesson Quiz

