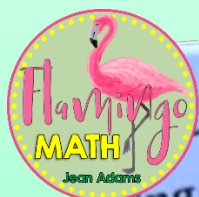


AP[®] Calculus-BC

Contextual Applications of the Derivative



moving along a horizontal line (it could be your kitchen floor) in a continuous manner. His velocity, v , in inches per second, has been tracked by your home security system for $0 \leq t \leq 10$ minutes in the table below.

2	2.5	4	5	6.5	8	10
0.75	-0.5	-0.25	0.5	1.25	1.75	0.25

Is he moving to the left? Explain how you know.

at where the



Homework

4

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AP CALCULUS CURRICULUM

Scope and Sequence

Unit 4: Contextual Applications of Differentiation

Lesson	Skills & Learning Objectives
[4.1] Interpretations of the Derivative in Context	☐ Determine the meaning of a derivative as it relates to real-world scenarios
[4.2] Straight Line Motion: Connecting Position, Velocity & Acceleration	☐ Solve rectilinear motion problems involving speed, velocity and acceleration
[4.3] Rates of Change in Other Applications	☐ Apply the derivative to solve problems involving rates of change in applied contexts.
[4.4] Related Rates	☐ Calculate related rates in applied contexts ☐ Find a rate at which one quantity is changing by relating it to other quantities whose rates of change are known. ☐ Interpret related rates in applied contexts
[4.5] Local Linearity and Linearization	☐ Use tangent lines to approximate values near a point of tangency ☐ Determine whether a tangent line approximation overestimates or underestimates the corresponding function value
[4.6] L'Hospital's Rule	☐ Determine limits of functions that result in indeterminate forms.

4.2

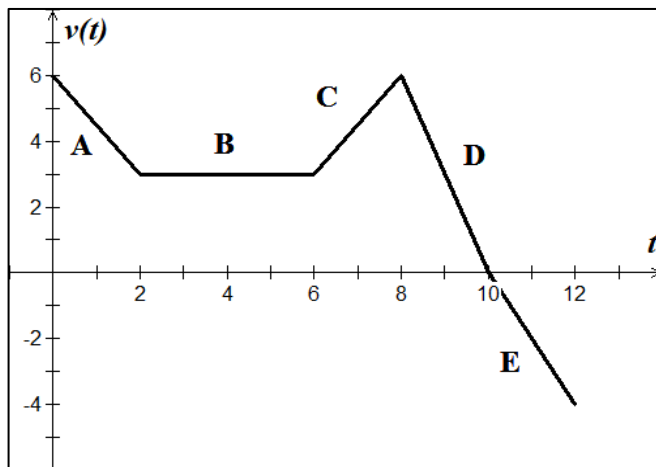
Straight Line Motion - Connecting Position, Velocity & Acceleration Homework

Name _____

Date _____ Period _____

Problems 1 – 2, Use the velocity vs. time graph below to answer the questions.

1. In the velocity graph, at right, let the units for velocity be in feet per second.



	A	B	C	D	E
Acceleration (slope) (+, -, 0)					
Acceleration (yes/no)					
Velocity/Acceleration (+, -)					
Speed up, slow down, constant					

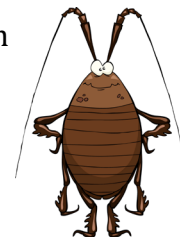
2. Use the graph of the velocity function above to find the following. Show the computations that lead to your answer, include units.

A. How far did the particle move in the first 10 seconds?

B. What is the average velocity over the 12 second time interval?

C. When does the particle change directions?

9. Randy the Roach is back. He is moving along a horizontal line (it could be your kitchen counter at 2 a. m.) in a smooth and continuous manner. His velocity, v , in inches per minute, is a differentiable function and has been tracked by your home security system. Randy's velocity is shown for $0 \leq t \leq 10$ minutes in the table below.



t	0	0.5	1	2	2.5	4	5	6.5	8	10
$v(t)$	-1.7	0.5	0.5	.75	-0.5	-0.25	0.5	1.25	1.75	0.25

- A. At what time(s) is Randy moving to the left? Explain how you know.
- B. Is there any time(s) during the interval $0 \leq t \leq 10$ minutes when Randy is at rest? Explain your answer.
- C. Use the data from the table to find an approximation for $v'(7)$. Explain the meaning of $v'(7)$ in terms of Randy's movement. Show the computations that lead to your answer, and indicate the units of measure.
- D. Find Randy's average acceleration for $4 \leq t \leq 5$ minutes. Explain what this means in terms of the Randy's velocity on the interval.

4.3

Rates of Change in Other Applications Homework

Name _____

Date _____ Period _____

Problems 1 – 7, evaluate the marginal analysis scenarios.

1. **Nole Quarters** is located just blocks away from Doak Campbell Stadium on the FSU Campus, in Tallahassee, FL. There are 65 student rental apartments available. If x apartments are rented then the monthly profit, in dollars, is modeled by the profit function, $\pi(x) = -7x^2 + 840x - 12,500$.



Photo by: Sabrina Adams

A. How many apartments should be rented to maximize profits?

B. Evaluate profit for $\pi(0)$ and $\pi(65)$

C. What is the maximum monthly profit earned?

2. You are listening to a lecture on the Battle of Gettysburg in your APUSH class; and are reminded of the laws of retention from psychology. You love math, so you tune out and begin to calculate the percentage of information you will forget, t minutes after the lecture began, using the formula for retention, given by $R(t) = \frac{44}{1 + e^{0.02t}}$. A maximum occurs when $t = 0$ and decreases as time passes. Hey, are you paying attention to the lesson?



A. Find how the retention rate is changing 15 minutes after the lesson begins.

B. Explain the meaning of your answer in context to the problem.

3. A paper manufacturer has 150 employees at a paper mill that can produce $Q(x)$ units of paper, for $100 \leq x \leq 200$ employees at the mill. Their production output is modeled by the function $Q(x) = 18000 + 2x^2$ units. What is the instantaneous rate of change of paper production in units per employee?

4.4**Related Rates Homework**

Name _____

Date _____ Period _____

-
1. Assume x and y are both differentiable functions of t . Find $\frac{dx}{dt}$ given $x = -1$ and $\frac{dy}{dt} = 8$. For the relation: $4x^2 + 3y^3 = 28$.
-
2. Water is falling on a surface, wetting a circular area that is expanding at a rate of $10 \text{ mm}^2/\text{sec}$. How fast is the radius of the wetted area expanding when the radius is 116 mm ? (Round approximations to four decimal places.)
-
3. A spherical balloon is being inflated. The radius of the balloon is increasing at a rate of $\frac{1}{\pi}$ inches per second. Find the rate at which the volume is increasing when the radius is 1 inch.
-
4. All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is 10 centimeters?
-

4.5

Linear Approximation
Homework

Name _____

Date _____ Period _____

Problems 1 - 4, Find the tangent line approximation of each function at the given value $x = a$.

1. $f(x) = \tan x$; at $a = \frac{\pi}{4}$

2. $y = e^{\sqrt{x}}$; at $a = 1$

3. $g(x) = \frac{1}{\sqrt{1+x}}$; at $a = 3$

4. $y = \tan^{-1} x$; $a = 1$

5. Let f be a differentiable function such that $f(-2) = 3$ and $f'(-2) = 6$. The tangent line to the graph of f at $x = -2$ can be used to find an approximation to a zero of the function, f . Find the approximation for the value of x such that $f(x) = 0$.

6. Find the linear approximation for $f(\theta) = \sin^2 \theta$ for $\theta = \frac{\pi}{4}$. Use the linear approximation to estimate $\theta = 0.8$ and calculate the error. Determine whether the approximation overestimates or underestimates the function.

9. Use the table, at right, to complete the following.

Given $P(x) = f(g(x))$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	-2	1	4
-1	1	-1	-2	1
0	-1	2	-1	-2

A., Write the equation of the tangent line at $x = -1$.

B. Use the method of linear approximation to estimate the value of $P(-1.5)$.

C. If $P(-2) = 7$, and $P(x)$ is monotonic (always decreasing) on the interval $[-2, -1]$ what does this value tell you about the behavior of the graph of $P(x)$ when $x = -1.5$? Explain how you know.

10. Use linearization to estimate the given numbers. Find the error of the approximation.

A. $\ln(1.06)$

B. $(8.05)^{2/3}$

11. The local linear approximation to the function g at $x = -3$ is $y = 4x - 7$.
What is the value of $g(-3) + g'(-3)$?

4.2 Straight Line Motion

Form A

Name _____

Date _____ Period _____

1. For time $t \geq 0$, the velocity of a particle is given by $v(t) = 3t^4 - 3t^3 + 5$. How many times is the particle's acceleration equal to zero?

2. For time $t \geq 0$, a particle moves along the x -axis, its position is given by the function $s(t) = -t^4 - 4t^3 + 18t^2 + 4$. At what time does the particle obtain its maximum velocity?

3. For time $t \geq 0$, a particle moves along the x -axis, its position is given by the function $s(t) = -t^3 + 3t^2 - 6$. What is the velocity when acceleration is zero?

Grade:

4.2 Straight Line Motion

Form B

Name _____

Date _____ Period _____

1. For time $t \geq 0$, the velocity of a particle is given by $v(t) = -t^4 + 8t^2 + 12$. How many times is the particle's acceleration equal to zero?

2. For time $t \geq 0$, a particle moves along the x -axis, its position is given by the function $s(t) = -t^4 + 2t^3 + 36t^2 + 5$. At what time does the particle obtain its maximum velocity?

3. For time $t \geq 0$, a particle moves along the x -axis, its position is given by the function $s(t) = -t^3 + 6t^2 + 18$. What is the velocity when acceleration is zero?

Grade:

**4.3 Rates of Change in
Other Applications****Form A**

Name _____

Date _____ Period _____

A manufacturer has a monthly fixed cost of \$175,000 and a production cost of \$20 for each unit produced. The product sells for \$32 per unit.

1. What is the cost function, $C(x)$?
2. What is the revenue function, $R(x)$?
3. What is the profit function, $\pi(x)$?
4. Compute the profit (loss) for a production level of 20,000 units. Interpret your findings.
5. How many units should the company produce in order to make a profit of \$50,000?

Grade:**4.3 Rates of Change in
Other Applications****Form B**

Name _____

Date _____ Period _____

A manufacturer has a monthly fixed cost of \$56,000 and a production cost of \$12 for each unit produced. The product sells for \$18 per unit.

1. What is the cost function, $C(x)$?
2. What is the revenue function, $R(x)$?
3. What is the profit function, $\pi(x)$?
4. Compute the profit (loss) for a production level of 5,000 units. Interpret your findings.
5. How many units should the company produce in order to make a profit of \$30,000?

Grade:

4.6 L'Hospital's Rule**Form A**

Name _____

Date _____ Period _____

Find each limit below. Use L'Hospital's Rule, if necessary.

$$1. \lim_{x \rightarrow 0} \frac{3 \sin(2x)}{5x}$$

$$2. \lim_{x \rightarrow \infty} \frac{e^x}{x^2}$$

$$3. \lim_{x \rightarrow \infty} \frac{3x + 13}{12 - 4x}$$

$$4. \lim_{x \rightarrow 1} \frac{\ln(x) + x - 1}{2 - 2x^2}$$

Grade:**4.6 L'Hospital's Rule****Form B**

Name _____

Date _____ Period _____

Find each limit below. Use L'Hospital's Rule, if necessary.

$$1. \lim_{x \rightarrow 0} \frac{5 \tan(3x)}{2x}$$

$$2. \lim_{x \rightarrow \infty} \frac{x^3}{e^x}$$

$$3. \lim_{x \rightarrow -\infty} \frac{1 - 6x}{3 + 5x}$$

$$4. \lim_{x \rightarrow 1} \frac{e^x - e}{3 \ln(x)}$$

Grade: