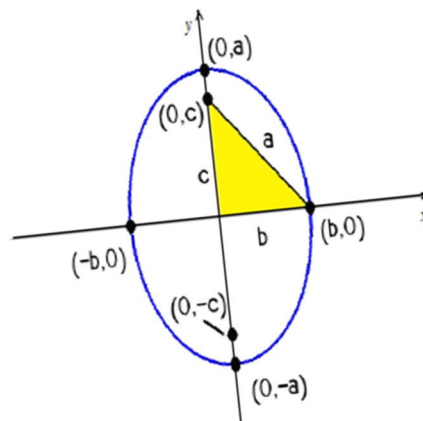
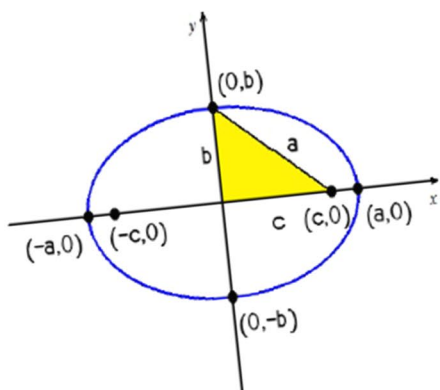


Precalculus Honors

Conic Sections & Analytic Geometry



Pythagorean Relations



Standard Forms of the Equations of an Ellipse

the equation of an ellipse with center at the origin, and major and minor axes of lengths $2a$ and $2b$ respectively. The foci are on the major axis, at a distance c from the origin, where $c^2 = a^2 - b^2$.

Guided Notes

8

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PRECALCULUS CURRICULUM

Scope and Sequence

Unit 8: Conic Sections & Analytic Geometry

Lesson	Skills & Learning Objectives
[8.1] Parabolas	☐ Find the equation, focus and directrix of a parabola ☐ Write parabolas in standard form by completing the square ☐ Graph parabolas given standard form ☐ Write the equation of a parabola given key features
[8.2] Circles	☐ Find the center and radius of a circle ☐ Write the equation of a circle by completing the square ☐ Graph circles given standard form
[8.3] Ellipses	☐ Find center, foci, and vertices of an ellipse ☐ Write the equation of an ellipse by completing the square ☐ Graph ellipses given standard form ☐ Write the equation of an ellipse given key features
[8.4] Hyperbolas	☐ Find center, foci, vertices of a hyperbola ☐ Find the asymptotes of a hyperbola ☐ Write the equation of a hyperbola by completing the square ☐ Graph hyperbolas given standard form ☐ Write the equation of a hyperbola given key features
[8.5] Parametric Equations	☐ Graph parametric equations ☐ Write parametric equations given a rectangular equation and vice versa
[8.6] Parametric Applications	☐ Solve real-world applications of parametrically defined functions

Lesson 8.1: Parabolas

DEFINITION OF A PARABOLA

A parabola is the set of all points (x, y) in a plane that are equidistant from the **directrix** (a fixed line) and the **focus** (a point not on the line).

The intersection of a plane and a double-napped cone will create the basic conic sections, **circle**, **parabola**, **ellipse** and **hyperbola**. However, when the plane intersects with the vertex a **degenerate conic** will occur. Examples: **point**, **single line**, or **intersecting lines**.

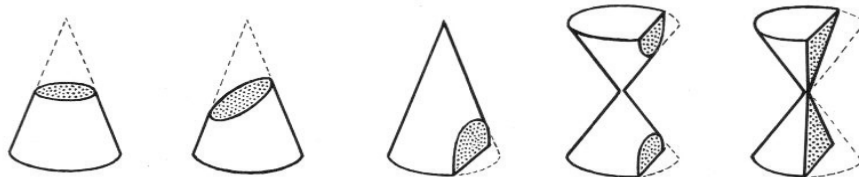


Image used with permission: Pearson Scott Foreseman

The conic sections can be defined algebraically as the graphs of **second-degree (quadratic) equations in two variables**, that is, equations of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

where A , B , and C are not all zero.

Verbal Vocabulary

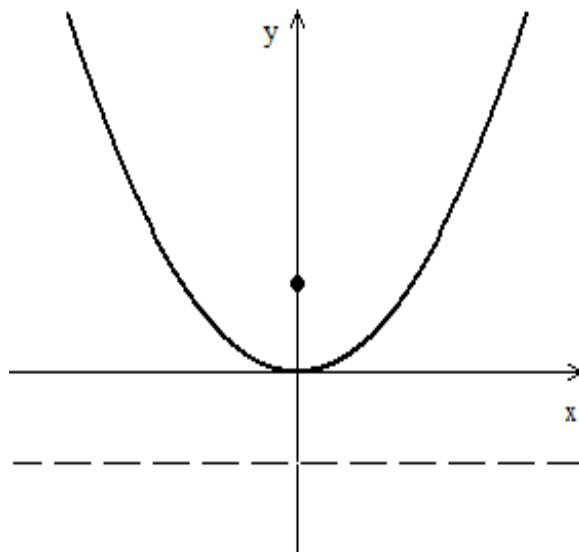
Exploration: Label the parabola in the visual vocabulary at the right, using the highlighted words below.

The **directrix** is a fixed line that is $\pm p$ units from the vertex. The line passing through the focus and perpendicular to the directrix is called the **axis of symmetry**.

The value p , the directed distance from the **vertex** to the **focus** of the **parabola** is called the **focal length**.

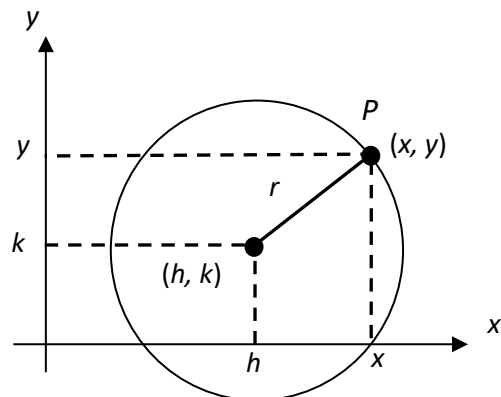
A line segment that passes through the focus and has endpoints on the parabola is the **focal width (chord)**. It is parallel to the directrix. The value $|4p|$ is called the **latus rectum** and is the length of the chord through the focus and **perpendicular** to the axis.

Visual Vocabulary



Pythagorean Relations

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Circle Formulas

**Standard Form
center at (0, 0)**

**Standard Form
Center at (h, k)**

Area

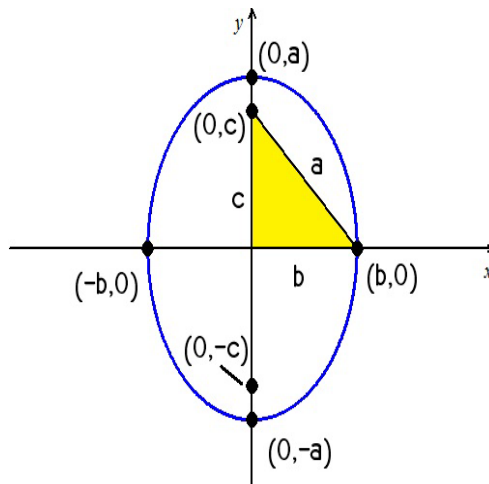
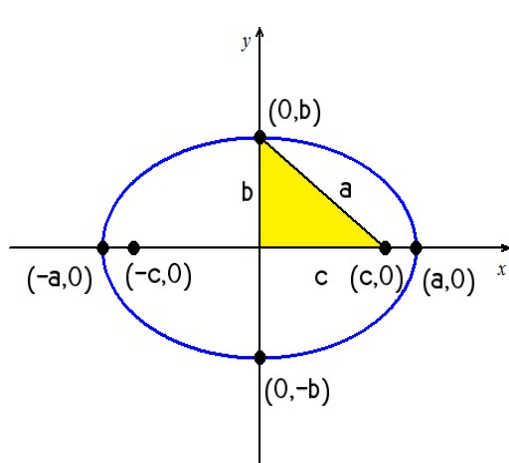
Circumference

Diameter

**Length of circular arc
Angle in degrees**

**Area of Circle Sector
Angle in degrees**

Pythagorean Relations



Standard Forms of the Equations of an Ellipse

The standard form of the equation of an ellipse with center at the origin, and major and minor axes of lengths $2a$ and $2b$, where a and b are positive. The foci are on the major axis, c units from the center and $b^2 = a^2 - c^2$. Also, $c^2 = a^2 - b^2$, and $a^2 > b^2$.

Major Axis is Horizontal:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Major Axis is Vertical:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Graphing Basic Ellipses

EX #1: $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$a =$ _____ $b =$ _____ $c =$ _____

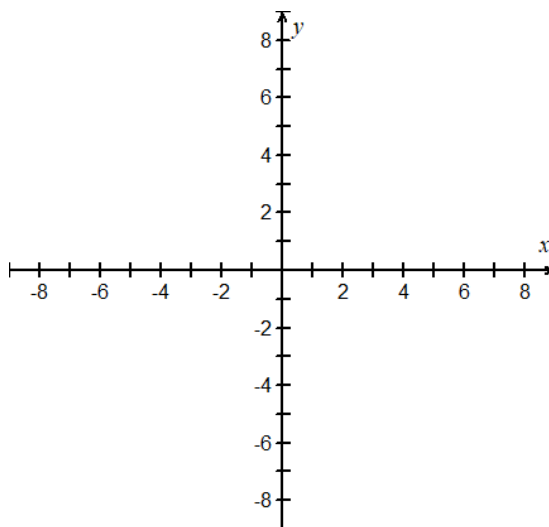
Major axis _____

Center _____

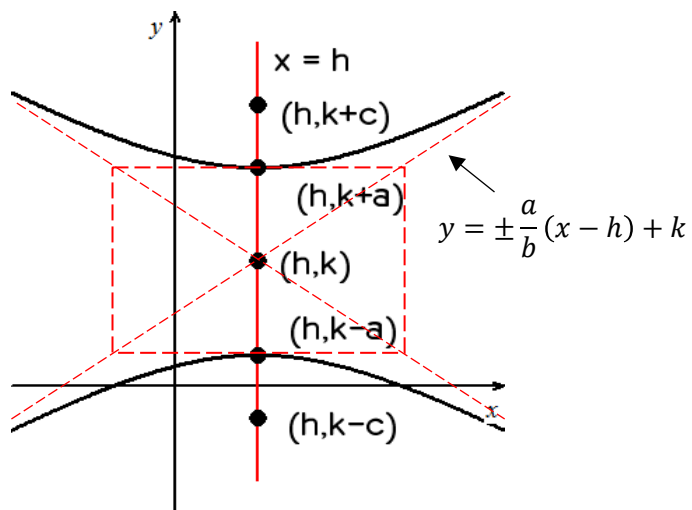
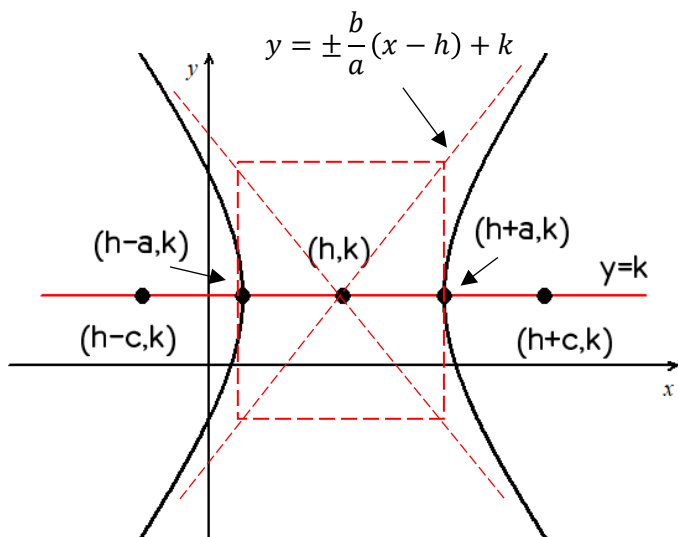
Major vertices _____

Minor vertices _____

Foci _____



Hyperbolas with Center (h, k)



Major Axis	Horizontal	Vertical
Equation		
Focal Axis		
Foci		
Vertices		
Semi -transverse		
Semi- conjugate		
Pythagorean Relation		
Asymptotes		

Finding the Rectangular Equation of a Parametric Curve

EX #2: Eliminate the parameter. $\begin{cases} x = t - 1 \\ y = t^2 - 2 \end{cases}$

STEP 1: Solve for one variable in terms of t .

STEP 2: Substitute into the other equation

STEP 3: Simplify.

Analyze the function over the real numbers:

Domain:

Range:

Max/Min:

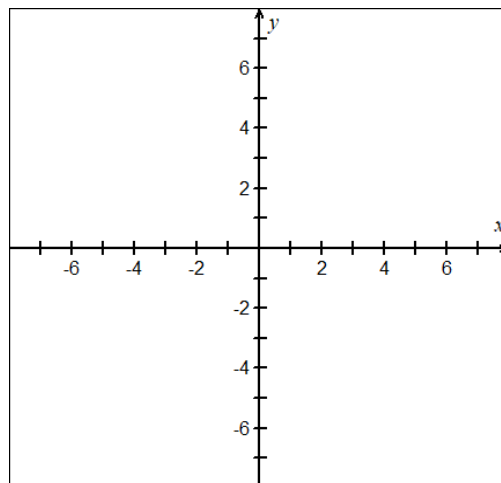
Increases:

Decreases:

EX #3: Graph the plane curve on $-2 \leq t \leq 2$

$$\begin{cases} x = t^2 + 1 \\ y = 3t \end{cases}$$

t	x	y	(x, y)
-2			
-1			
0			
1			
2			



Lesson 8.6: Parametric Applications

In algebra we explore how things are connected or related through similarities and differences. We have discussed functions and relations to find input and output values. However, there are some situations that need a different approach.

For example, let's consider when a hurricane is heading toward the southern United States. The closer the storm approaches the coast the more gas and bottled water is sold. Suppose for every 100 miles the storm moves that a convenience store sells x gallons of gas and x^2 bottles of water. Algebraically, that relationship would be written as follows:

$$\text{bottles of water} = (\text{gallons of gas})^2$$

Even though this is correct, it is misleading. The equation implies that gas is changing the demand for bottled water, when really it is the proximity of the storm, in miles, to a region that changed both items. It would make better sense to write two separate equations where we considered either miles from shore or time as the changing parameter. We could write two separate equations to model the situation.

$$\begin{aligned}\text{Gallons of gas} &= \text{miles} - 100 \\ \text{Bottled water} &= (\text{miles} - 100)^2\end{aligned}$$

In this way you can see that miles from shore is the cause for changes in the sale of gas and water due to the impending storm. Parametric equations help us look deeper at hidden parameters that affect change.

Graphing Parametric Equations

EX #1:

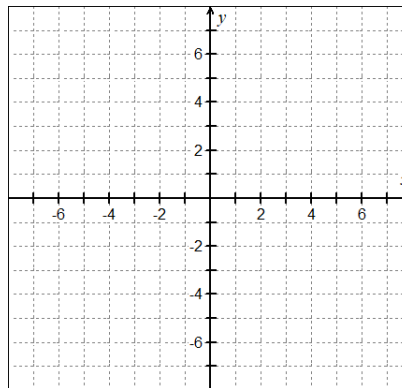
Set MODE to Parametric and graph $\begin{cases} x = t + 1 \\ y = t^2 - 3 \end{cases}$

Sketch curve shown.

What are the settings for the time parameter?

Tmin = _____ Tmax = _____ Tstep = _____

What changes to the graph occur if Tmin = -6 ?



Add these changes to your sketch above in a different color.

What happens if you change Tstep = 5? Explain.