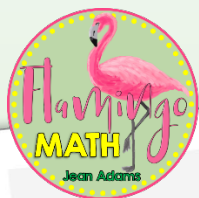


AP[®] Precalculus

Conic Sections



Finding an Equation of an Ellipse

EX #8: Find the equation in standard form for an ellipse whose foci are $(0, 3)$ and $(0, -3)$ with a major axis length 10.

$$a = 5 \quad b = 4 \quad c = 3$$

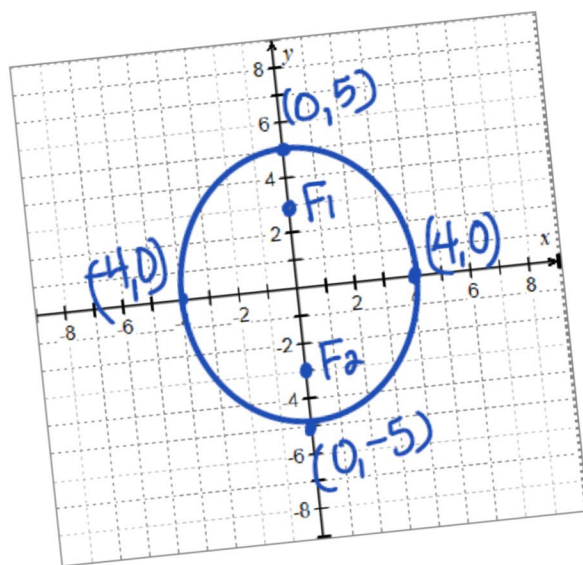
Major axis: vertical

Center: $(0, 0)$

Major vertices: $(0, 5)$ $(0, -5)$

Minor vertices: $(4, 0)$ $(-4, 0)$

Foci: $(0, 3)$ $(0, -3)$



$$x^2 + \frac{y^2}{25} = 1$$

Completely Editable

4.6

Lesson 6-A: Conic Sections

While the Greeks defined conics in terms of the intersections of planes and the double-napped cone, we will study each of the conics as a **locus** (collection) of points that satisfy some given geometric properties. For example, in our study of parabolas, you learned that a parabola is the graph of a quadratic function and can be written in the form $y = ax^2 + bx + c$ or $y = a(x - h)^2 + k$.

Visualizing Conic Sections by Slicing a Double-Napped Cone

The intersection of a plane and a double-napped cone will create the basic conic sections, **circle**, **parabola**, **ellipse** and **hyperbola**. However, when the plane intersects with the vertex a **degenerate conic** will occur. Examples: **point**, **single line**, or **intersecting lines**.



The conic sections can be defined algebraically as the graphs of **second-degree (quadratic) equations in two variables**, that is, equations of the form

Definition of a Parabola

A parabola is the set of all points (x, y) in a plane that are equidistant from the **directrix** (a fixed line) and the **focus** (a point not on the line).

Verbal Vocabulary

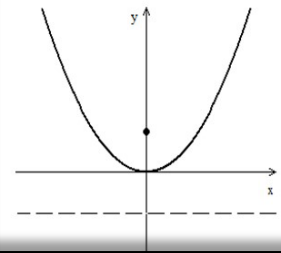
Exploration: Label the parabola in the visual vocabulary at the right, using the highlighted words below.

The **directrix** is a fixed line that is $\pm p$ units from the vertex. The line passing through the focus and perpendicular to the directrix is called the **axis of symmetry**.

The value p , the directed distance from the **vertex** to the **focus** of the **parabola** is called the **focal length**.

A line segment that passes through the focus and has endpoints on the parabola is the **focal width (chord)**. It is parallel to the directrix. The value $|4p|$ is called the **focal width**, and is the length of the

Visual Vocabulary



Standard Forms of Equations of Parabolas

The graph of a parabola can have a vertex at (h, k) when it has been translated from the origin. These translations will change the standard form equations shown in the table below.

Axis of Symmetry	Vertical	Horizontal
Equation	$(x - h)^2 = 4p(y - k)$	$(y - k)^2 = 4p(x - h)$
Direction: $p > 0$	Upward	Right
Direction: $p < 0$	Downward	Left
Vertex	(h, k)	(h, k)
Focus	$(h, k + p)$	$(h + p, k)$
Directrix	$y = k - p$	$x = h - p$
Axis	$x = h$	$y = k$
Focal Length	p	p
Focal Width	$ 4p $	$ 4p $

Finding an Equation of a Parabola

EX #3: Use the graph at right to find the standard form of the equation for the parabola.

Graphing Translations of a Parabola

EX #4: Analyze and graph $(x - 1)^2 = 8(y + 2)$

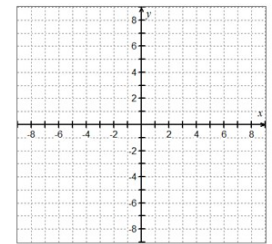
$$h = \quad k = \quad p = \quad$$

Vertex: _____

Focus: _____

Directrix: _____

Focal width: _____



Writing Equations of a Parabola

EX #5: Find the standard form of the equation for the parabola with focus $(6, 0)$ and directrix $x = 2$.

Lesson 6-B: Conic Sections

In the second day of our lesson on conic sections, you will explore the properties of an ellipse. Recall, that a circle is a special case of an ellipse where the vertical and horizontal distances from the center are equal.

Definition of an Ellipse

An ellipse is the set of all points (x, y) in a plane whose distance from two fixed points in the plane have a constant sum. The fixed points are the **foci** (plural of focus) of the ellipse. The line through the foci is the **focal axis**. The point on the focal axis midway between the foci is the **center**. The points where the ellipse intersects its axis are the **vertices** of the ellipse.

Standard Forms of the Equations of an Ellipse

Pythagorean Relations

The standard form of the equation of an ellipse with center at the origin, and major and minor axes of lengths $2a$ and $2b$, where a and b are positive. The foci are on the major axis, c units from the center and $b^2 = a^2 - c^2$. Also, $c^2 = a^2 - b^2$, and $a^2 > b^2$.

Major Axis is Horizontal:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Center: $(0, 0)$

Major Vertices: $(-a, 0), (a, 0)$

Minor Vertices: $(0, -b), (0, b)$

Foci: $c = \sqrt{a^2 - b^2}$

Major Axis is Vertical:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Center: $(0, 0)$

Major Vertices: $(-a, 0), (a, 0)$

Minor Vertices: $(0, -b), (0, b)$

Foci: $c = \sqrt{b^2 - a^2}$

Graphing Basic Ellipses

EX #7: $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$$a = \quad b = \quad c = \quad$$

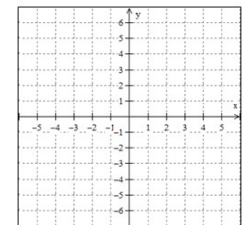
Major axis: _____

Center: _____

Major vertices: _____

Minor vertices: _____

Foci: _____



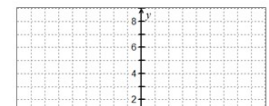
Equation: _____

Finding an Equation of an Ellipse

EX #8: Find the equation in standard form for an ellipse whose foci are $(0, 3)$ and $(0, -3)$ with a major axis length 10.

$$a = \quad b = \quad c = \quad$$

Major axis: _____



Equation: _____

Guided Notes



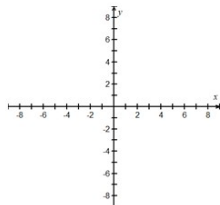
EX #9: Graph the ellipse: $9x^2 + 36y^2 = 324$

Center: _____

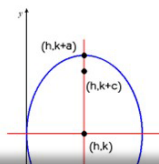
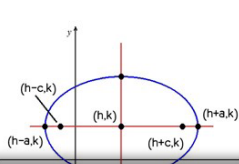
Major vertices: _____

Minor vertices: _____

Foci: _____



Ellipses with Center (h, k)



Finding Vertices and Foci

EX #10: $9(x+3)^2 + 4(y+2)^2 = 36$

$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}}$

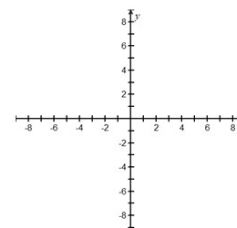
Major axis: _____

Center: _____

Major vertices: _____

Minor vertices: _____

Foci: _____



EX #11: Prove that the graph of the equation is an ellipse, find its vertices, foci by completing

Lesson 6-C: Conic Sections

The last conic section to explore will be hyperbolas. In the study of analytic geometry, a hyperbola is formed by intersecting a right circular cone with a plane at an angle such that both halves of the cone are intersected. This intersection creates two separate unbounded curves (branches) that are mirror images of each other.

Definition of a Hyperbola

A **hyperbola** is the set of all points in a plane whose distances from two fixed points in the plane have a constant difference. The fixed points are the **foci** of the hyperbola. The line through the foci is the **focal axis**. The point on the focal axis midway between the foci is the **center**. The points where the hyperbola intersects its focal axis are the **vertices** of the hyperbola.

Standard Form of a Hyperbola

Major Axis is Horizontal:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Center: (0, 0)

Major Vertices: $(-a, 0), (a, 0)$

Major Axis is Vertical:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Center: (0, 0)

Major Vertices: $(0, -a), (0, a)$

Graphing Basic Hyperbolas

EX #12: Find the vertices, foci, and asymptotes of the hyperbola and sketch.
 $9x^2 - 16y^2 = 144$

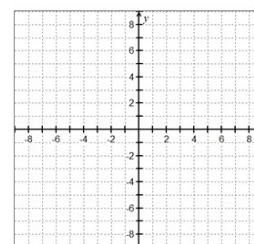
$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}}$

Major vertices: _____

Minor vertices: _____

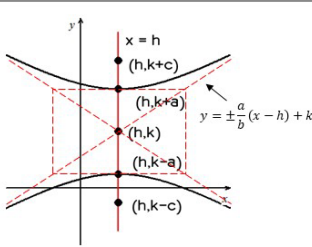
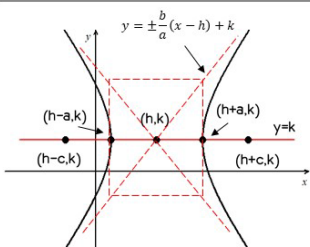
Foci: _____

Asymptotes: _____



Writing an Equation of a Hyperbola

Hyperbolas with Center (h, k)



Major Axis	Horizontal	Vertical
Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Focal Axis	$y = k$	$x = h$
Foci	$(h \pm c, k)$	$(h, k \pm a)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Pythagorean Relation	$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$
Asymptotes	$y - k = \pm \frac{b}{a}(x - h)$	$y - k = \pm \frac{a}{b}(x - h)$

EX #14: Graph the hyperbola. $\frac{(y-3)^2}{9} - \frac{(x-2)^2}{16} = 1$

Finding Equations from a Graph

EX #15: Write an equation for the hyperbola. Then sketch the location for each focus point.

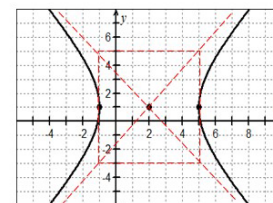
$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}}$

Equation: _____

(h, k): _____

Foci: _____

Asymptotes: _____

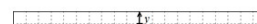


Finding an Equation of a Hyperbola

EX #16: Find the equation in standard form for a hyperbola whose transverse axis endpoints are $(-1, -3)$ and $(-1, 7)$ and the conjugate axis length is 8.

$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}}$

Equation: _____

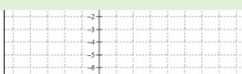


Guided Notes

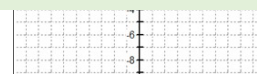
Minor vertices: _____

Foci: _____

Asymptotes: _____



Asymptotes: _____



4.6 Conic Sections Practice Set 1

Name _____

Date _____ Period _____

Problems 1 – 6, the graph of a parabola is given. Match each graph to its equation.

A. $(y - 2)^2 = (x + 2)$

B. $(x - 1)^2 = 4(y + 1)$

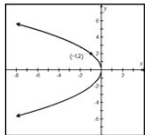
C. $y^2 = -4x$

D. $y^2 = 4x$

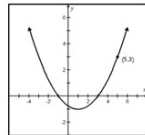
E. $(x - 2)^2 = -4(y - 2)$

F. $(x + 1)^2 = -8(y - 1)$

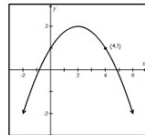
1.



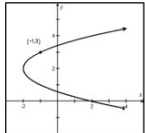
2.



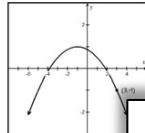
3.



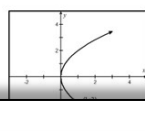
4.



5.



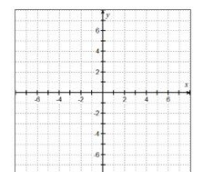
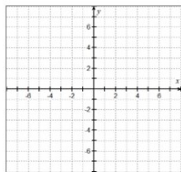
6.



Problems 7 – 8, find the equation of the parabola. Find the vertex, focus, and directrix, and graph the equation.

7. Focus at (2, 0); vertex at (0, 0)

8. Vertex at (0, 0); focus at (0, 2)



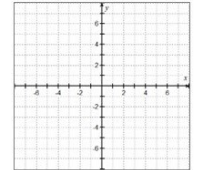
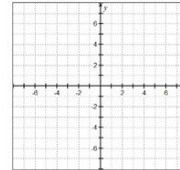
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Problems 9 – 12, find the vertex, focus, and directrix of each parabola. Graph the equation.

9. $(y - 1)^2 = 8(x + 2)$

10. $(x + 1)^2 = 16y$



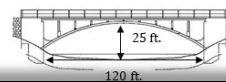
11. $x^2 + 8x = 4y - 12$

12. $y^2 + 6y + 4x + 1 = 0$

Problems 13-15, solve.

13. A radio telescope has a parabolic dish with a diameter of 110 feet. The radio signals are reflected to the focus of the parabola. If the focal length is 25 feet, find the depth of the dish, rounded to two decimal places.

14. The parabolic arch of a bridge over a small river is 25 feet above the water at the center and 120 feet wide at the base. Find the height of the arch at a distance 10 feet from the center.



15. A football is kicked from the ground and reaches a height of 25 feet and lands on the ground 120 feet from the origin. Find the path of the ball, assuming the origin is at the point where the ball was kicked.

4.6 Conic Sections Practice Set 2

Name _____

Date _____

Problems 1 – 6, the graph of an ellipse is given. Match each graph to its equation.

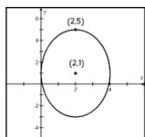
A. $(x - 2)^2 + (y + 3)^2 = 9$

B. $x^2 + \frac{y^2}{9} = 1$

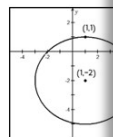
D. $\frac{(x+2)^2}{9} + \frac{(y-1)^2}{16} = 1$

E. $\frac{(x-2)^2}{4} + \frac{(y-1)^2}{9} = 1$

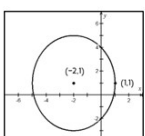
1.



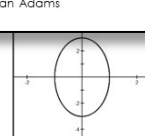
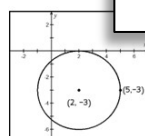
2.



4.



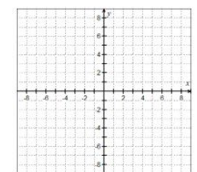
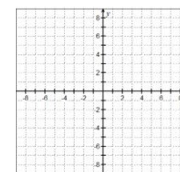
5.



Problems 9 – 14, find the center, foci, and vertices of each ellipse. Graph the equation.

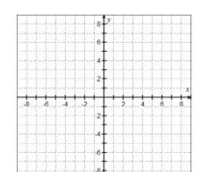
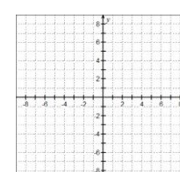
9. Focus at (0, 4) vertices at $(0, \pm 6)$

10. $(x - 2)^2 + (y - 4)^2 = 16$



11. Focus at (4, 0) vertices at $(\pm 5, 0)$

12. $x^2 + 3y^2 = 12$



Practice Sets

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Problems 15-17, solve.

15. Madison is standing at one focus of a whispering gallery and is 6 feet away from the nearest wall. Her friend, Julia, is standing at the other focus, 100 feet away. What is the length of the whispering gallery? How high is its elliptical ceiling at the center?

16. A water feature in the shape of an ellipse is built in front of City Hall. The architect plans to place fountains at the foci. If the pool of water is 20 feet long and 12 feet wide, how far from the center should the fountains be located? (Round to the nearest hundredth.)

17. The moon travels around Earth in an elliptical orbit with Earth at one focus. The major axis of the orbit is 768,800 kilometers and the minor axis is 767,640 kilometers. Find the apogee and perigee (greatest and smallest distances) from Earth's center to the moon's center.

4.6

Conic Sections Practice Set 3

Name _____

Date _____ Period _____

Problems 1–6, the graph of an ellipse is given. Match each graph to its

A. $\frac{x^2}{9} - y^2 = 1$

B. $x^2 - \frac{y^2}{9} = 1$

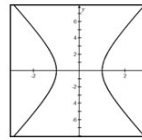
C. $\frac{y^2}{4} - \frac{x^2}{9} = 1$

D. $\frac{(x-1)^2}{9} - \frac{(y+2)^2}{16} = 1$

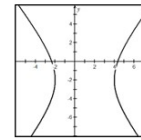
E. $\frac{(x-2)^2}{9} - \frac{(y-1)^2}{4} = 1$

F. $\frac{(y-2)^2}{16} - \frac{(x+2)^2}{9} = 1$

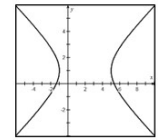
1.



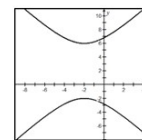
2.



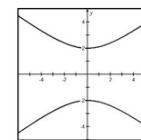
3.



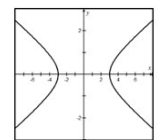
4.



5.

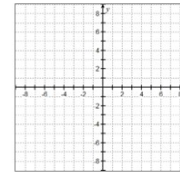


6.

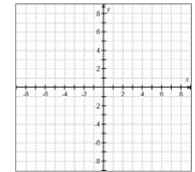


Problems 7–8, find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation.

7. $\frac{x^2}{25} - \frac{y^2}{16} = 1$



8. $9x^2 - 4y^2 = 36$

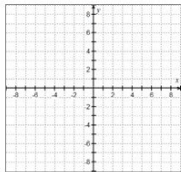


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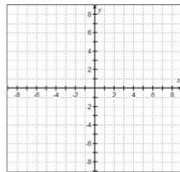
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Problems 9–16, find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation.

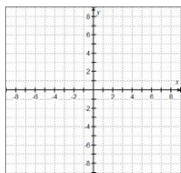
9. Asymptotes $y = \pm \frac{2}{3}x$, Vertices $(0, \pm 2)$



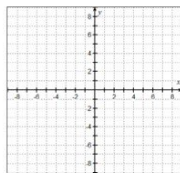
10. $25y^2 - 16x^2 = 400$



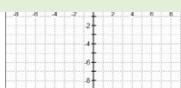
11. Foci at $(\pm 5, 0)$ vertex at $(3, 0)$



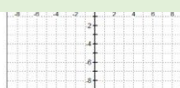
12. $\frac{(x+2)^2}{9} - \frac{(y-3)^2}{16} = 1$



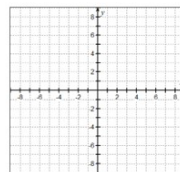
13. $\frac{(y-3)^2}{25} - \frac{(x+2)^2}{16} = 1$



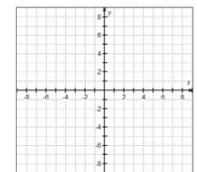
14. Focus at $(0, 6)$ vertices at $(0, \pm 2)$



15. $x^2 - y^2 - 4x + 4y - 9 = 0$



16. $x^2 - y^2 + 6x + 4y - 4 = 0$



Problems 17-18, solve.

17. Voyager 2, a space probe, was launched by NASA in August, 1977 to study the outer planets. As it approached Saturn the gravitational pull of the planet changed the probe's path to a hyperbola that diverges from its asymptote. Assuming that the center of the hyperbola is at the origin and the transverse axis is horizontal, find an equation that models the path of the Voyager given that $a = 332,956$ km and $c = 492,788.2$ km.

18. A panoramic photo can be taken using a hyperbolic mirror. A camera is pointed toward the vertex

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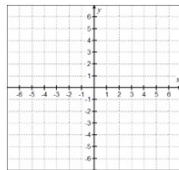
Practice Sets

4.6 Conic Sections Set 1

Form A

Name _____
Date _____ Period _____

1. Find the equation of the parabola. Find the endpoints of the focal chord (latus rectum) and graph the equation.
Vertex at $(1, -2)$; Directrix $x = -1$



2. Find the vertex, focus, and directrix of the parabola. $x^2 + 6x = 8y - 1$

3. A parabolic dish with a diameter of 90 feet, has a focal length of 20 feet. Find the depth of the dish. Round to 3 decimal places.

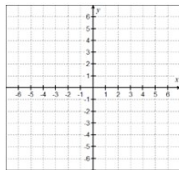
Grade:

4.6 Conic Sections Set 1

Form B

Name _____
Date _____ Period _____

1. Find the equation of the parabola. Find the endpoints of the focal chord (latus rectum) and graph the equation.
Vertex at $(4, -1)$; Directrix $x = 2$



2. Find the vertex, focus, and directrix of the parabola. $x^2 + 4x = 8y + 20$

3. A parabolic dish with a diameter of 100 feet has a focal length of 22 feet. Find the depth of the dish. Round to 3 decimal places.

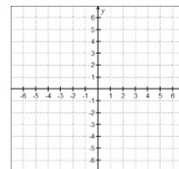
Grade:

4.6 Conic Sections Set 2

Form A

Name _____
Date _____ Period _____

1. Find the center, foci, and vertices of the ellipse. Graph the equation.
 $4(x - 1)^2 + 9(y + 2)^2 = 36$



2. Leah is standing at one focus of a whispering gallery and is 8 feet from the nearest wall. Her friend, Ashleigh, is standing at the other focus, 110 feet away. What is the length of the whispering gallery? How high is its elliptical ceiling at the center?

3. A water feature in the shape of an ellipse is built. The pool is 30 feet long and 14 feet wide. If fountains are placed at the foci, how far from the center should the fountains be located?

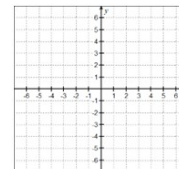
Grade:

4.6 Conic Sections Set 2

Form B

Name _____
Date _____ Period _____

1. Find the center, foci, and vertices of the ellipse. Graph the equation.
 $9(x + 1)^2 + 4(y - 2)^2 = 36$



2. Kim is standing at one focus of a whispering gallery and is 4 feet from the nearest wall. Her friend, Sydney, is standing at the other focus, 108 feet away. What is the length of the whispering gallery? How high is its elliptical ceiling at the center?

3. A water feature in the shape of an ellipse is built. The pool is 40 feet long and 22 feet wide. If fountains are placed at the foci, how far from the center should the fountains be located?

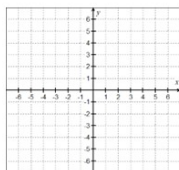
Grade:

4.6 Conic Sections Set 3

Form A

Name _____
Date _____ Period _____

1. Find the center, transverse axis, vertices, foci and asymptotes. Graph the equation.
 $\frac{(y + 1)^2}{4} - \frac{(x - 1)^2}{9} = 1$



2. Put the equation in standard form. Identify the center, and the direction of opening.
 $x^2 - y^2 + 4x + 10y - 30 = 0$

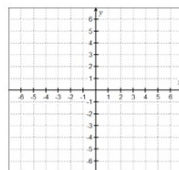
Grade:

4.6 Conic Sections Set 3

Form B

Name _____
Date _____ Period _____

1. Find the center, transverse axis, vertices, foci, and asymptotes. Graph the equation.
 $\frac{(y - 1)^2}{9} - \frac{(x + 1)^2}{4} = 1$



2. Put the equation in standard form. Identify the center and the direction of opening.
 $x^2 - y^2 + 4x + 6y - 15 = 0$

Grade:

Daily Lesson Quiz