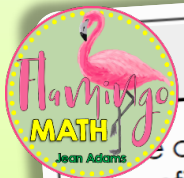


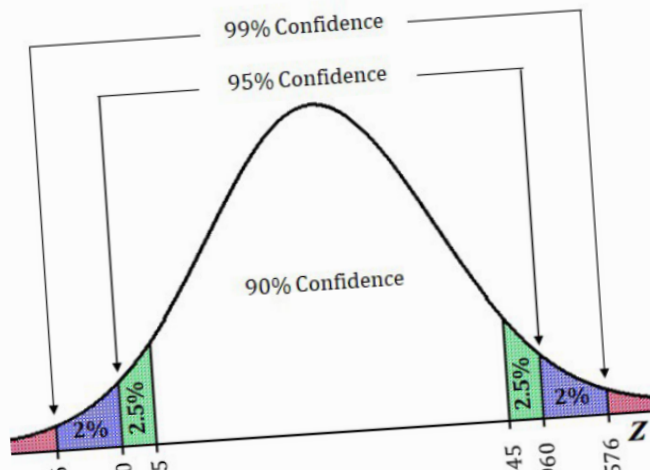
Algebra 2 Honors

Confidence Intervals



Confidence Levels and Critical Values

A confidence level refers to the percentage of probability, or certainty, that the confidence interval would contain the true population parameter when you draw a random sample many times.



The area under the standard normal curve is the percentage of confidence, C .

To calculate the percentage of the area for each tail:

$$A = \frac{1 - C}{2}$$

Use the formula above to confirm that the area in the tails for a 95% confidence interval is 2.5% of the area lies to the left of $-z$ and 2.5% lies to the right of z .

Completely Editable

13.6

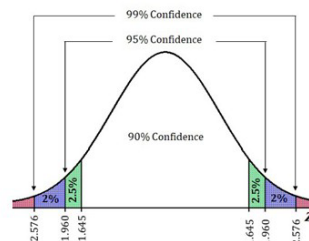
Lesson 13.6: Confidence Intervals

Confidence intervals and hypothesis tests are inferential methods that allow statisticians to draw conclusions about a population using a sample.

Confidence Intervals	An estimate of a population parameter stated as a range with a specific degree of certainty. For a normal distribution, the confidence interval is equal to the area under the standard normal curve between $-z$ and z . Usually considered 90%, 95%, and 99% confidence intervals. $CI = \bar{x} \pm E \quad \text{or} \quad \bar{x} \pm z \cdot \frac{s}{\sqrt{n}}$
Statistical Inference	Uses information from a sample to draw conclusions about a population.
Inferential Statistics	A technique used to draw conclusions about a population using a sample.
Maximum Error	In order to find the confidence interval, we will use the maximum error of estimate and the sample mean. The maximum error of estimate E is the maximum difference between the estimate of the population mean, μ and its actual value using the formula

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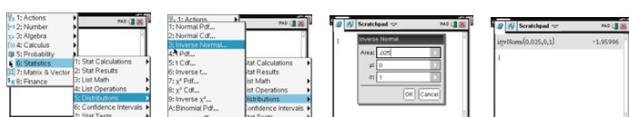
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TI-Nspire CX-CAS Example: For a 95% Confidence $A = \frac{1 - 0.95}{2} = 0.025$

Use Keystrokes: Menu > Statistics (or Probability) > Distributions > Inverse Normal



Finding the Maximum Error of Estimate

In order to find the confidence interval, we will use the maximum error of estimate and the sample mean. The maximum error of estimate E is the maximum difference between the estimate of the population mean, μ and its actual value using the formula

$$E = z \cdot \frac{s}{\sqrt{n}}$$

Where z is the z -value that corresponds to a particular confidence level, s is the standard deviation of the sample, and n is the sample size; $n \geq 30$.

EX #2: A poll of 214 randomly selected high school students showed that they spend an average of 16 minutes per day on a social network for sharing user-generated videos. There is a standard deviation of 2.8 minutes.

A. Use a 95% confidence interval to find the maximum error of estimate for the

Confidence Intervals

Once the maximum error of estimate E is found, a confidence interval CI for the population mean can be found by adding $\pm E$ to the sample mean.

A confidence interval for a population mean is given by

$$CI = \bar{x} \pm E \quad \text{or} \quad \bar{x} \pm z \cdot \frac{s}{\sqrt{n}}$$

where $\bar{x}$ is the sample mean and E is the maximum error of estimate.

EX #3: A sample of 200 high school teachers was asked the average amount of time they spend outside the classroom on school-related work. The mean time reported was 16.6 hours per week with a standard deviation of 2.7 hours. Determine a 99% confidence interval for the population mean.

Hypothesis Testing

To estimate the population mean, we have used a confidence interval. In order to assess a specific claim about the mean we will use a hypothesis test. There are two parts to a hypothesis test.

Null Hypothesis - H_0 a statement of equality to be tested

Alternative Hypothesis - H_a a statement of inequality that is the complement of the

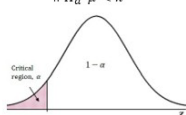
Z-Tests

Tests of Significance (Z-Tests)

The type of hypothesis test we are using is known as a Z-Test. The value μ represents the claim about the population mean μ .

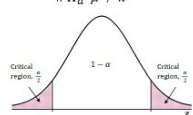
Left-Tailed Test

If $H_a: \mu < k$



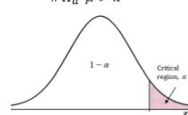
Two-Tailed Test

If $H_a: \mu \neq k$



Right-Tailed Test

If $H_a: \mu > k$



Steps for Hypothesis Testing

1. State the hypotheses and identify the claim.
2. Determine the critical value(s) and critical region.
3. Calculate the z -statistic using $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
4. Reject or fail to reject the null hypothesis.
5. Make a conclusion about the claim.

EX #5: Oliver thinks there are more than 8 commercials during each 30-minute Primetime TV program. Using a sample of 45 programs, he calculated an average of 8.5 commercials with a standard deviation of 1.8. Test the

Real-World Connections

EX #6: A food label states that there are 150 calories per serving of your favorite cookie. Using a sample of 40 servings, You calculated a mean of 150.8 calories with a standard deviation of 2.1. Test the hypothesis at 5% significance.

EX #7: A soda manufacturer claims there are 12 ounces of natural juice in every can of Sparkling Berry Fizz. Using a sample of 65 cans, Jeff calculated a mean of 12.2 ounces with a standard deviation of 0.95 ounce. Test the hypothesis at 1% significance.

NOTES

Confidence Intervals Practice**Problems 1 - 2.** Use a 95% confidence interval to find the maximum error of estimate for each scenario.

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| 1. A sample of 40 randomly selected avocados was taken from a large population with a mean diameter of the sample was 97 mm and the standard deviation was 6 mm. | 2. A poll of 246 high school seniors showed that they spend an average of 6.5 hours per week writing college essays with a standard deviation of 1.25 hours. |
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Problems 3 - 6. Find the confidence interval for each set of data.

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| 3. A random sample of size of 625 is drawn from a large population. The population standard deviation is 15.2. The sample mean is 82.7. Find a 90% confidence interval for the population mean, μ . | 4. A random sample of size of 465 is drawn from a large population. The population standard deviation is 8.4. The sample mean is 78. Find a 95% confidence interval for the population mean, μ . |
| 5. A random sample of size of 250 is drawn from a large population. The population standard deviation is 2.6. The sample mean is 43.2. Find a 99% confidence interval for the population mean, μ . | 6. A random sample of size of 136 is drawn from a large population. The population standard deviation is 5.5. The sample mean is 23. Find a 99% confidence interval for the population mean, μ . |

Problems 7 - 10. Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

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| 7. Hayden sends at least 12 text messages to his friend every day. | 8. Jennifer says she plays 5 different word search games on her phone each night. |
| 9. A pizza delivery states that it takes no more than 18 minutes to deliver a pizza to your house. | 10. Your digital escape room takes no more than 45 minutes to complete. |

Problems 11 - 14. Identify the hypothesis and claim, decide whether to reject the null hypothesis, and make a conclusion about the claim.

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| 11. A sandwich shop promises to deliver your sub in less than 30 minutes. Using a sample of 68 deliveries, Jon calculated a mean delivery time of 29.4 minutes with a standard deviation of 3.8 minutes. Test the hypothesis at 1% significance. |
| 12. Among 158 adults, aged 65-70, the mean total cholesterol levels is 167 mg/dL with a standard deviation of 26. We want to know if, on the basis of these data, we can conclude that the mean total cholesterol levels for the population of adults is greater than 170. Test the hypothesis at 10% significance. |

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| 13. The mean lifetime of a sample of 150 LED lightbulbs is 1,620 hours. Test the hypothesis at 5% level of significance so that the mean lifetime of the LED bulbs is 1,650 hours with a standard deviation of 80 hours. |
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| 14. The average sales from a sporting goods store from a sample of 50 sales receipts is \$138. If the standard deviation is \$26.50. Test the hypothesis at a significance of 5% to determine whether or not the average sales at the store is ore than \$150. |
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Problems 15 - 16 Find the confidence interval for each scenario.

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| 15. Commercial fisherman in a coastal town hold an Oyster and Seafood Festival each winter. They harvest oysters and have a sample of 225 oysters that measure an average width of 12.7 centimeters with a standard deviation of 2.1 centimeters. Find a 90% confidence interval for the mean width of oysters harvested for the festival. |
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Practice Sets

Lesson Warm-Ups

2. Estimate the probability that a randomly selected student scored less than 85.

3. Find the percent of data within an interval between 76 and 88.

Name _____ Date _____ Period _____

Lesson 13.6 Confidence Intervals

Warm-Up #2

1. Sketch a normal curve for mean 72 and a standard deviation 10. Label the x-axis at one, two and three standard deviations from the mean.

Scores on your Algebra quiz are normally distributed with a mean of 72 and a standard deviation of 8.

2. Estimate the probability that a randomly selected student scored less than 80.

3. Find the percent of data within an interval between 64 and 72.

Form A

Period _____

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13.6 Confidence Intervals

Form B

Name _____

Date _____ Period _____

A sample of 40 randomly selected oranges was taken from a large population. The mean weight of the sample was 131 grams, and the standard deviation was 6 grams.

1. Find the maximum error of estimate for the population using a 95% confidence interval.

2. Find a 95% confidence interval for the population mean, μ .

A local radio station plays no more than 8 commercials each hour. In a random sample of 40 hours on air, the mean of 8.1 commercials were played with a standard deviation of 0.4 commercials.

3. Identify the null and alternative hypotheses for the statement. Then identify the claim.

2. Find a 90% confidence interval for the population mean, μ .

A local radio station plays at least 14 songs each hour. In a random sample of 50 hours on air, the mean of 13.8 songs were played with a standard deviation of 0.8 songs.

3. Identify the null and alternative hypotheses for the statement. Then identify the claim.

Daily Lesson Quiz

Grade: _____

Grade: _____