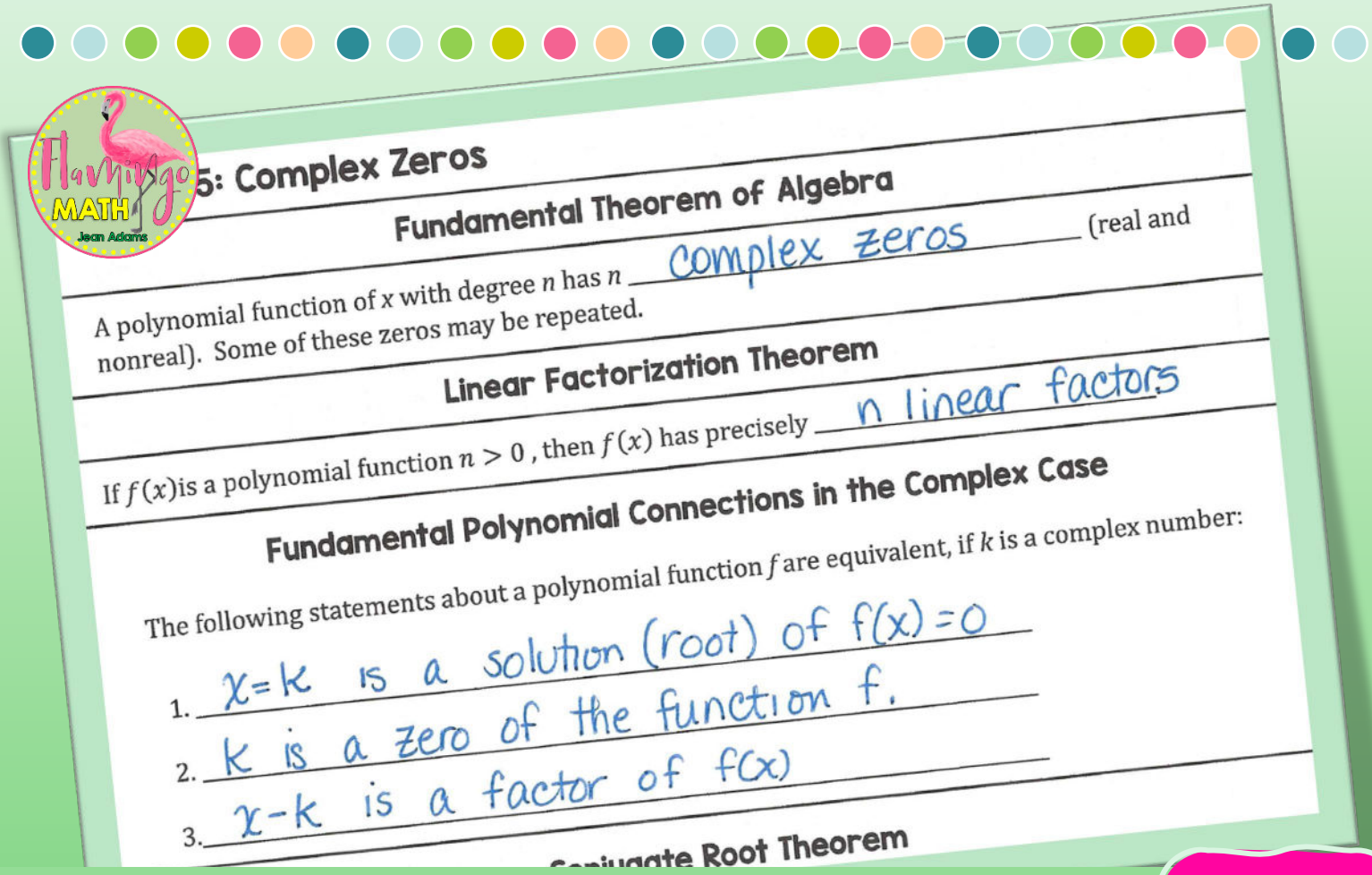
Precalculus Honors

Complex Zeros



Completely Editable

2.5

Lesson 5: Complex Zeros

Fundamental Theorem of Algebra

A polynomial function of x with degree n has n _____ (real and nonreal). Some of these zeros may be repeated.

Linear Factorization Theorem

If $f(x)$is a polynomial function $n > 0$, then $f(x)$ has precisely _____.

Fundamental Polynomial Connections in the Complex Case

The following statements about a polynomial function f are equivalent, if k is a complex number:

1. _____
2. _____
3. _____

Conjugate Root Theorem

If $P(x)$ is a polynomial with rational coefficients, then the irrational roots of $P(x) = 0$ occur in conjugate pairs. That is, when $(a + \sqrt{b})$ is a root, then _____ is also a root.

Also, if $(a + bi)$ is a zero of a function, then _____ is also a zero.

Fundamental Polynomial Connections

EX #1: Find a cubic polynomial equation with the roots at $x = \sqrt{2}$ and $x = 5$. Leave in factored form.

EX #2: Write the polynomial function in standard form and identify the zeros of the function and the x-intercepts of its graph. $f(x) = (x + 2)(x - i\sqrt{3})(x + i\sqrt{3})$

Finding a Polynomial Given Complex Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include the following:

EX#3: $\{-3, -4, (2 - i)\}$

EX #4: $x = 1$ (multiplicity 2); $x = 1 - i$

Factoring a Polynomial with Complex Zeros

EX #5: Find all the zeros of $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$ and factor $f(x)$ over the set of complex numbers.

EX #6: The complex number $z = 2i$ is a zero of the polynomial. Find the remaining zeros of $f(x)$ and factor over complex numbers: $f(x) = 2x^4 + x^3 - 7x^2 + 4x - 60$

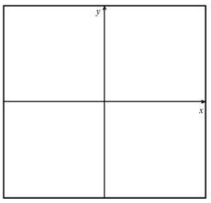
Factoring Over Real Numbers

EX #7: Write $f(x) = 3x^3 - 2x^2 + x - 2$ as a product of linear and irreducible quadratic factors, each with real coefficients. That is, factor over real numbers.

Sketching A Graph with Repeating Zeros

EX #8: Sketch the graph of the polynomial function with the given zeros, multiplicities, and sign of leading coefficient

A. -3 (multiplicity 2); 4 (multiplicity 3); $a > 0$



B. -1 (multiplicity 2); 3 (multiplicity 2); $a < 0$



NOTES

2.5 Complex Zeros Homework

Name _____

Date _____ Period _____

Problems 1 – 5, Find the remaining zeros of f using the given information about the polynomial. Then write the linear factorization of the polynomial.

1. Degree 3; zeros: $2, 5 - i$

2. Degree 4; zeros: $3i, \sqrt{7}$

3. Degree 4; zeros: $-i, 2 - i\sqrt{3}$

4. Degree 5; zeros: $-3, i, 2i$

5. Degree 6; zeros: $0, -4, \sqrt{3}, -2 + i$

Problems 6 – 9, Write a polynomial with real coefficients having the given degree and zeros.

6. Degree 4; zeros: $2 - 3i$; -1 (multiplicity 2)

7. Degree 4; zeros: $-i$; 3 (multiplicity 2)

8. Degree 4; zeros: $\sqrt{3}; 2i$

9. Degree 5; zeros: $0, \sqrt{2}, -i$

Problems 10 – 12, Use the given zero to find the remaining zeros of each function.

10. $f(x) = x^3 - 5x^2 + 4x - 20$; zero: $2i$

11. $f(x) = 2x^3 + x^2 - 6x - 3$; zero: $-\sqrt{3}$

12. $f(x) = x^4 + 9x^2 - 400$; zero: $5i$

Problems 13 – 16, Find the complex zeros of each polynomial function. Use your results to write the polynomial as a product of linear factors.

13. $f(x) = x^3 + 3x^2 - 2x - 6$

14. $f(x) = x^4 + 10x^2 + 9$

Free Response Question

Given $f(x) = ax^3 + x^2 + 12x + c$. A table of values for the function is shown below. Use the equation of $f(x)$ and the table of values to answer the following question.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	105	36	5	??	9	20	21	0	-55

A.) Can you determine the end behavior of $f(x)$ using the table? Give a reason.

B.) Can the multiplicity of the zero at $x = 3$ be determined from the table of values? Give a reason why or why not.

C.) What is the value of c? How do you know.

D.) Use the zero from the table to find the value of a. Show your work.

E.) Use synthetic division to find the other zeros for $f(x)$. Show your work.

Practice Sets

Lesson Warm-Ups

Name: Date: Period:

Lesson 2.5: Complex Zeros **Warm-Up # 1**

1. Write the polynomial in standard form.
$f(x) = (x + 2i)(x - 2i)(x + 3)$

2. Write the linear factorization of a polynomial whose degree is 3 and zeros are $x = 3$ (multiplicity of 1) and $x = \sqrt{3}$.

3. Find the complex zeros for:
$f(x) = x^4 + 17x^2 + 16$

4. Use the given zero to find the remaining zeros.
$f(x) = x^3 - 5x^2 + 9x - 45$; zero: $3i$

Name: Date: Period:

Lesson 2.5: Complex Zeros **Warm-Up # 2**

1. Write the polynomial in standard form.
$f(x) = (x + i)(x - i)(x - 2)$

2. Write the linear factorization of a polynomial whose degree is 4 and zeros are $x = 3$ (multiplicity of 2) and $x = \sqrt{5}$.

3. Find the complex zeros for:
$f(x) = x^4 + 5x^2 + 4$

4. Use the given zero to find the remaining zeros.
$f(x) = 2x^3 + 2x^2 - 4x - 4$; zero: $\sqrt{2}$

2.5 Complex Zeros **Form A**

Name _____

Date _____ Period _____

1. Write the polynomial in standard form
$f(x) = (x + 3i)(x - 3i)(x - 2)$

2. Write the linear factorization of a polynomial whose degree is 4 and zeros are $x = 4$ (multiplicity 2) and $x = \sqrt{2}$

3. Find the complex zeros for:
$f(x) = x^3 - 9x^2 + 20x - 12$

4. Use the given zero to find the remaining zeros. $f(x) = x^3 - 5x^2 + 9x - 5$ zero: $2 - i$

Grade:

2.5 Complex Zeros **Form B**

Name _____

Date _____ Period _____

1. Write the polynomial in standard form
$f(x) = (x + 2i)(x - 2i)(x + 5)$

2. Write the linear factorization of a polynomial whose degree is 4 and zeros are $x = 3$ (multiplicity 2) and $x = \sqrt{5}$

3. Find the complex zeros for:
$f(x) = x^3 - 6x^2 + 11x - 6$

4. Use the given zero to find the remaining zeros. $f(x) = x^3 - 4x^2 + 6x - 4$ zero: $1 - i$

Grade:

Daily Lesson Quiz