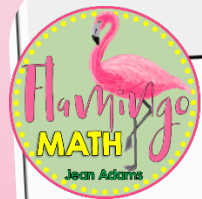


AP[®] Calculus BC

Comparison of Series



The Limit Comparison Test

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with **positive** terms. *for all $n=1,2,3,\dots$*

If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ such that $0 < L < \infty$, *positive and finite*
then **either both series converge or both series diverge.**

This test is a great tool for comparing a “messy algebraic series” with a p -series. Just beware that you must choose a p -series with an n th term of the same magnitude as the messy series under consideration.

For example, given $\sum_{n=1}^{\infty} \frac{1}{3n^2 - 5n + 7}$ choose the p -series $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Consider the **highest powers of n** in the numerator and the denominator.

Completely Editable

10.3

Lesson 10.3: Comparison of Series

Topic 10.6: Comparison Tests for Convergence

Let's explore two new tests for positive-term series that will help us determine convergence or divergence. These new tests will allow us to *compare* a "complicated series" that is *similar* to a "simpler series." We will look at the **Direct Comparison Test** and the **Limit Comparison Test**.

Direct Comparison Test Exploration

Let $a_n = \frac{1}{2^n}$ and $b_n = \frac{1}{2^{n+1}}$. Write out the first five terms of each series.

1. $\sum_{n=1}^{\infty} a_n =$

2. $\sum_{n=1}^{\infty} b_n =$

3. Complete the statement with $\leq$, $\geq$, or $=$. The series a_n _____ b_n for all values of $n \geq 1$.

4. Notice that a_n is a geometric series. Find the sum.

5. Using your findings from questions 3 and 4, what can you conclude about $\sum_{n=1}^{\infty} b_n$?

Direct Comparison Test

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms. Let $0 < a_n \leq b_n$ for all n .

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Verbally Speaking

☐ If you think the series converges, compare it to a converging series. The series you compare it to, should be larger.

☐ If you think the series diverges, compare it to a diverging series. The series you compare it to, should be smaller.

EX #1: Determine whether the following series converge or diverge.

A. $\sum_{n=1}^{\infty} \frac{7}{\sqrt[3]{n}}$

B. $\sum_{n=1}^{\infty} \frac{2^n}{1+3^n}$

C. $\sum_{n=1}^{\infty} \frac{n-1}{n^3+3}$

D. $\sum_{n=1}^{\infty} \frac{8n^2+1}{2n^3-1}$

Can the Direct Comparison Test Fail?

Let $a_n = \frac{1}{2^n-1}$ and $b_n = \frac{1}{2^n}$. Write out the first five terms of each series.

1. $\sum_{n=1}^{\infty} a_n =$

2. $\sum_{n=1}^{\infty} b_n =$

3. What do you notice about the terms of each series?

4. Will the series converge?

The Limit Comparison Test

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with **positive** terms.

If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ such that $0 < L < \infty$,

then **either both series converge or both series diverge**.

This test is a great tool for comparing a "messy algebraic series" with a p -series. Just beware that you must choose a p -series with an n th term of the same magnitude as the messy series under consideration.

For example, given $\sum_{n=1}^{\infty} \frac{1}{3n^2-5n+7}$ choose the p -series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

NOTE: You only need to consider the **highest powers of n** in the numerator and the denominator.

EX #2: Apply the Limit Comparison Test to each of the series below.

A. $\sum_{n=1}^{\infty} \frac{2}{n^3+4}$

NOTES

10.3

Comparison of Series Homework

Name _____

Date _____ Period _____

Problems 1 - 6, Use the Direct Comparison Test to determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{3n^2 + 2}$

2. $\sum_{n=1}^{\infty} \frac{3^n}{4^n + 5}$

3. $\sum_{n=1}^{\infty} \frac{1}{n(3^n)}$

4. $\sum_{n=1}^{\infty} \frac{\ln n}{n + 1}$

5. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt[n]{n} - 1}$

6. $\sum_{n=2}^{\infty} \frac{5^n}{2^n - 3}$

Problems 7 - 12, Use the Limit Comparison Test to determine the convergence or divergence of the series.

7. $\sum_{n=1}^{\infty} \frac{5}{3^n + 1}$

8. $\sum_{n=1}^{\infty} \frac{\sqrt{n} + 2}{n^2 + n + 3}$

9. $\sum_{n=1}^{\infty} \frac{5n^2}{2n^3 + 4}$

10. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 1}}$

11. $\sum_{n=1}^{\infty} \frac{2 + 3^n}{2^n + 5}$

12. $\sum_{n=1}^{\infty} \frac{3n^2 - 1}{4n^5 + 2n + 1}$

Problems 13 - 20, Use each test at least once to test the series for convergence or divergence. Specify the test that was applied.

- (A) n th term test
(C) Telescoping Series Test
(E) Integral Test
(G) Limit Comparison Test

- (B) Geometric Series Test
(D) p -Series Test
(F) Direct Comparison Test

13. $\sum_{n=0}^{\infty} 5\left(-\frac{1}{5}\right)^n$

14. $\sum_{n=1}^{\infty} \frac{2n}{6n - 3}$

15. $\sum_{n=1}^{\infty} \frac{1}{3^n + 1}$

16. $\sum_{n=1}^{\infty} \frac{4}{n(n + 4)}$

17. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n^2}$

18. $\sum_{n=3}^{\infty} \frac{1}{n^2 - 4}$

Practice Sets

10.3 Comparison of Series Form A

Name _____

Date _____ Period _____

Determine if the series converges or diverges. Give a reason for your answer.

1. $\sum_{n=1}^{\infty} \left(\frac{1}{3^n + 5} \right)$

2. $\sum_{n=1}^{\infty} \frac{3n^2 - 2n + 4}{6n^5 + n - 3}$

3. $\sum_{n=1}^{\infty} \frac{1}{n}$

10.3 Comparison of Series Form B

Name _____

Date _____ Period _____

Determine if the series converges or diverges. Give a reason for your answer.

1. $\sum_{n=1}^{\infty} \left(\frac{1}{5^n + 2} \right)$

2. $\sum_{n=1}^{\infty} \frac{4n^2 + 5n - 1}{8n^7 - n + 2}$

3. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Daily Lesson Quiz