

Precalculus Honors

Combinations of Functions



7: Combination of Functions

Two functions can be combined to create new functions.

Let f and g be two functions with overlapping domains.
Then, the algebraic combinations of f and g are defined for all x common to both domains:

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$.

NOTE ON THE DOMAIN:

... of all numbers that belong to both

Completely Editable

1.7

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NOTE ON THE DOMAIN:

In each case, the domain of the new function consists of all numbers that belong to both the domain of f and the domain of g . The zeros of the denominator are excluded from the domain of the quotient.

Defining New Functions Algebraically

EX #1: Let $f(x) = 5x - 8$ and $g(x) = 7 - 2x^2$

A: Find $(f - g)(x)$

B:

EX#2: Let $f(x) = x^3$ and $g(x) = \sqrt{x+1}$. Find the

Operation:

$f + g$:

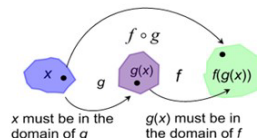
$f - g$:

$f \cdot g$:

$\frac{f}{g}$:

Composition of Functions

The composition of the function f with the function g is defined as $(f \circ g)(x) = f(g(x))$. The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f .



In the above function, the output of $g(x)$ is used as the input of f .

Composition of Functions

EX #3: Let $f(x) = x^2 + 2x - 1$, $g(x) = \frac{1}{x-2}$ and $h(x) = \sqrt{3x}$. Find a rule for each of the following. State the domain of each function.

A. $(f \circ g)(x)$

B. $(g \circ f)(x)$

D. $(h \circ f)(x)$

Decomposing Functions

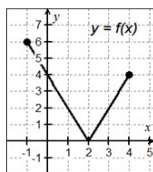
When you "decompose" a composite function you are finding the components of the composition. Look for an "inner" function and an "outer" function.

EX #4: For each function h , find functions f and g such that $h(x) = f(g(x))$.

Function	Inside	Outside
A. $h(x) = (x + 3)^2$		
B. $h(x) = \frac{1}{x-5}$		
C. $h(x) = \sqrt{x+2}$		
D. $h(x) = (x-2)^2 - 4(x-2) + 5$		

EX #5: Let $h(x) = \sqrt{3x^2 + 1}$, write h as a composition of functions in two different ways.

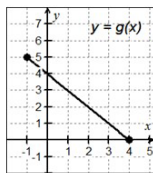
EX #6: Use the graphs of f and g to evaluate the functions.



A: $(f + g)(3)$

C: $\left(\frac{f}{g}\right)(0)$

E: $(f \circ g)(4)$



More Function Operations

EX #7: The table of values for three functions, f , g , and h is shown below. Find the composite function operation below.

x	-3	-2	-1
$f(x)$	-4	-1	0
$g(x)$	0	-1	-2
$h(x)$	1	0.5	1.5

A. $2f(1) + 3h(0)$

B. $h(g(-2)) \cdot f(g(2))$

D. If $k(x) = |3x - 4|$, for what value(s) of x is $k(x) = f(g(0))$?

Applications

EX #8: The volume V , in cubic meters, of a weather balloon is given by $V(r) = \frac{4}{3}\pi r^3$.

Applications

EX #9: The number N of bacteria in a container of leftover food stored in a refrigerator, where T is the temperature of the food in degrees Celsius can be modeled by $N(T) = 60T^2 - 140T + 76$. When the container is removed from the refrigerator the temperature is $T(t) = 5t + 3$; $0 \leq t \leq 3$, where t is the time in hours.

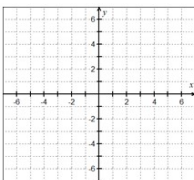
A. Find the composite $N(T(t))$ and explain its meaning in context of the scenario.

B. Find the number of bacteria present in the food after the container has been sitting out of refrigeration for 2 hours.

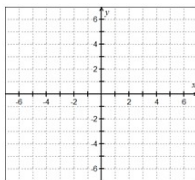
NOTES

Problems #1 - 2, Graph the functions f , g , and $f + g$ on the same set of axes.

1. $f(x) = \frac{1}{2}x^2$; $g(x) = x - 3$



2. $f(x) = x$; $g(x) = 4 - x^2$



Problems #3 - 8, Evaluate the following given $f(x) = x^2 + 3$; $g(x) = x - 2$, and $h(x) = \sqrt{x - 3}$

3. $(f - g)(-2)$

4. $(f + g)(t - 3)$

5. $(f \cdot g)(4)$

6. $\left(\frac{f}{g}\right)(0)$

7. $(f \circ h)(8)$

8. $h(f(g(5)))$

Problems #9 - 12, Find (A) $f \circ g$, (B) $g \circ f$, and (C) $f \circ f$. State the domain of each.

9. $f(x) = \sqrt{x + 1}$; $g(x) = x^2 - 1$

10. $f(x) = x^2 + 1$; $g(x) = 2 - x$

11. $f(x) = x^2 - 2$; $g(x) = \frac{2}{x}$

12. $f(x) = \sqrt{x - 2}$; $g(x) = 1 - 2x$

Problems #13 - 18, Use the functions below to find the equation for the indicated operations.

$f(x) = x^2 - 2x - 3$

$g(x) = \frac{x-3}{2x+3}$

$h(x) = 3x - 4$

13. $f(h(x))$

14. $g(h(x))$

15. $(f + g)(x)$

16. $(f \cdot h)(x)$

17. $2x \cdot h(x) - 3f(x)$

18. $\left(\frac{g}{h}\right)(x)$

Problems #19 - 22, For each function h , find functions f and g so that $h(x) = f \circ g$.

19. $h(x) = (3x - 2)^3$

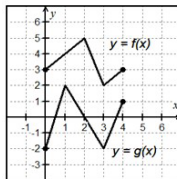
20. $h(x) = |x^5 - 3|$

21. $h(x) = \sqrt{x^4 - 2x + 3}$

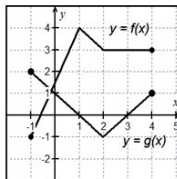
22. $h(x) = (x^2 + 1)^2$

Problems #23 - 24: Use the graphs of f and g to graph $h(x) = (f + g)(x)$.

23.



24.



Problems #25 - 28: Answer each question.

25. A rock is thrown into a calm lake, causing ripples to form in concentric circles. The radius, in feet, of the outer ripple is $r(t) = 0.8t$, where t is the number of seconds after the rock hits the

26. Angioplasty is a cardiac procedure where doctors insert a catheter into a heart vein and then inflate a small, spherical balloon on the tip of the catheter. If the balloon is inflated at a constant rate of 40 cubic millimeters per second,

A. Find the volume after t seconds.

B. When the volume is $V = \frac{4}{3}\pi r^3$, what is the radius r ?

C. Write an equation that expresses the radius as a function of time.

D. Find the radius of the balloon after 3 seconds.

27. A circular puddle of liquid is evaporating and slowly shrinking in size. After t minutes, the radius of the surface of the puddle measures $r(t) = \frac{18}{2t+9}$ inches. The area A of the surface of the puddle is given by $A = \pi r^2$. Express the area as a function of time by finding $(A \circ r)(t)$ and compute the area of the surface of the puddle at $t = 12$ minutes.

28. A water-filled balloon is dropped vertically from a window 124 ft. above the ground. Its height above the ground after t seconds is $124 - 16t^2$. Jessica is standing on the ground 50 ft. from the

Practice Sets

C.) Explain the meaning in the context of the scenario.

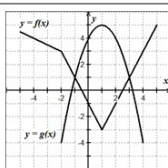
Lesson Warm-Ups

1.7 Combination of Functions Form A

Name _____

Date _____ Period _____

Use the graphs of $f(x)$ and $g(x)$ to answer each question.



1. The values of x such that $f(x) = g(x)$

2. $f(g(0))$

3. $\frac{3g(0) + f(4)}{f(g(1))}$

4. If $h(x) = |x - 4| - 3$ for what value(s) of x is $h(x) = g(f(2))$?

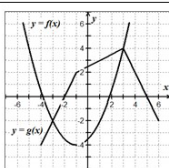
Grade: _____

1.7 Combination of Functions Form B

Name _____

Date _____ Period _____

Use the graphs of $f(x)$ and $g(x)$ to answer each question.



1. The values of x such that $f(x) = g(x)$

2. $g(f(3))$

3. $\frac{4g(1) + f(-3)}{f(g(1))}$

4. If $h(x) = |x + 5| - 2$ for what value(s) of x is $h(x) = g(f(-5))$?

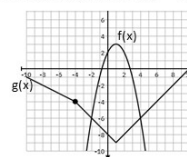
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Name _____ Date _____ Period _____

Lesson 1.7: Combination of Functions

Warm-Up # 1

1. What are the values of x such that $f(x) = g(x)$



2. $f(g(8))$

3. $\frac{3(g(8)) + f(2)}{f(g(6))}$

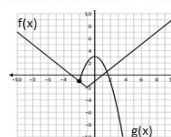
4. If $h(x) = |x + 1| - 3$, for what value(s) of x is $h(x) = g(f(2))$?

Name _____ Date _____ Period _____

Lesson 1.7: Combination of Functions

Warm-Up # 2

1. What are the values of x such that $f(x) = g(x)$



2. $f(g(-2))$

3. $\frac{3(g(-1)) + f(3)}{f(g(6))}$

4. If $h(x) = |x + 1| - 3$, for what value(s) of x is $h(x) = g(f(0))$?

Daily Lesson Quiz